



Teacher to Teacher

Unexpected Riches from a Geoboard Quadrilateral Activity

BARBARA J. BRITTON
AND SHERYL L. STUMP

EXPLORING THE SIMPLE QUADRILATERALS IN A three-peg-by-three-peg section of a geoboard has proved to be a very stimulating activity in elementary mathematics content and methods classes. In the April 1998 issue of *Mathematics Teacher*, Kennedy and McDowell described a wonderful activity that focused on counting quadrilaterals on geoboards of various sizes. Our focus in this article is on the properties of quadrilaterals. The activity that we describe is often used as an introduction to a unit on geometry. Tom Lewis, who is with the Moline Public Schools in Illinois, also used these activities in his fifth-grade classroom and reports similar results.

Beginning the Activity

THE CLASSROOM ACTIVITY BEGINS WITH STUDENTS working in groups to find all the quadrilaterals that they can on the three-peg-by-three-peg geoboard section. They keep track of their figures on dot paper. The groups discuss the meaning of *different* quadrilaterals. Most groups understand that *different* means *noncongruent* and launch into the activity. Other groups are puzzled. Typically, these students ask the instructor whether two squares of different sizes are considered different, or whether they are the same because they are the same shape. The instructor asks whether the students would consider the squares to be the same. The group usually says no. Through discussion, the students decide that the shapes are different if they do not fit exactly on top of each other. The students begin to identify figures related by reflections, rotations, and translations as congruent, although they may not use that terminology.

When all groups think that they have found all the quadrilaterals, the class comes together to share results. The geoboards themselves can be propped up in the chalk tray for this discussion, or students can be asked to transfer their figures to large sheets of paper with three-by-three dot grids to allow the figures to be displayed and viewed easily. One by one, each group selects a figure and presents it to the class. The selected figure must be different from any other shapes that have already



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been presented. Occasionally, a congruent figure is presented, leading to a whole-class discussion of rotations, reflections, and translations. The names and properties of the various shapes are discussed as the shapes are presented.

When all the groups have presented their quadrilaterals, the instructor notes whether all sixteen have been found, usually stating how many are left, and the groups attack the problem of finding the missing shapes. One semester, none of the groups considered the possibility of outlining concave quadrilaterals. When one of the students discovered this possibility, the “Aha!” could be heard throughout the room. All the remaining concave quadrilaterals were quickly found.

One of the most interesting discussions stemming from this activity arose when a student claimed that the shape shown in **figure 1** was a quadrilateral. The instructor asked the class to decide whether the figure was, in fact, a quadrilateral. The class decided that it was not. The students realized that although the pegs had diameter, conceptually they were meant to be points and the rubber bands were meant to be lines. Hence, being on one side or the other of a peg did not mean that the rubber band was actually passing through a different point.

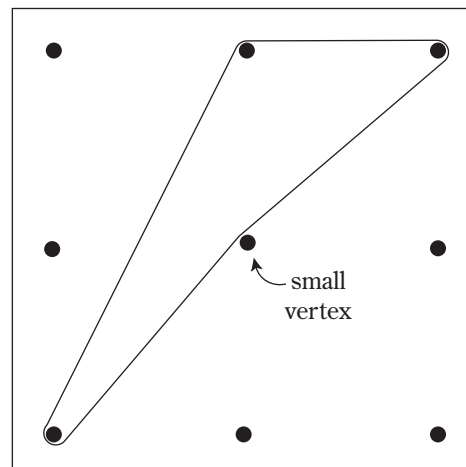


Fig. 1 The students debated whether this shape qualified as a quadrilateral.

BARBARA BRITTON, barbara.britton@emich.edu, is on the faculty at Eastern Michigan University, Ypsilanti, MI 48197. Her professional interests include the history of mathematics and the beliefs of preservice teachers. SHERYL STUMP, [sstump@gw.bsu.edu](mailto:ssump@gw.bsu.edu), teaches at Ball State University, Muncie, IN 47306. Her interests include teachers' knowledge and performance assessment.



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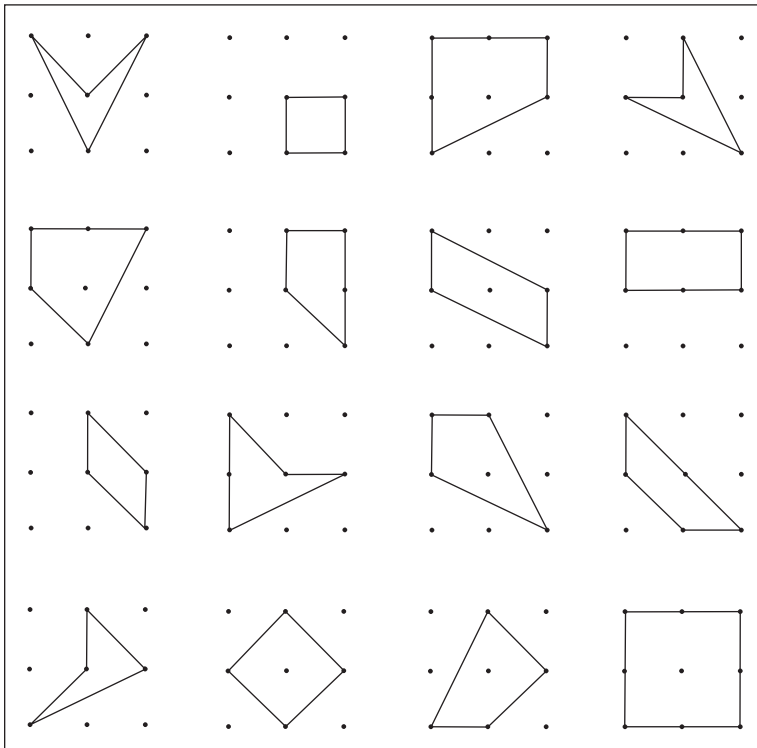


Fig. 2 Homework assignment for sorting quadrilaterals

Sorting the Quadrilaterals

AFTER THE STUDENTS HAVE FOUND AND DISCUSSED all the quadrilaterals, the instructor leads the class in a sorting activity. By moving the geoboards or large sheets of dot paper, the instructor sorts the sixteen quadrilaterals into two groups and tells the class that *all* the quadrilaterals in one group have a property that is shared by *none* of the quadrilaterals in the other group. The students then try to identify the property. Some examples of properties that have been used include having one pair of parallel sides, two pairs of parallel sides, a right angle, four right angles, an obtuse angle, a convex or nonconvex shape, and line symmetry. The sorting activity gives students an opportunity to communicate their mathematical thinking out loud. Some students struggle to find the language to describe the geometric properties that they see. The instructor may help the students connect their ideas with formal mathematical terminology. If time permits, individual students may come to the chalkboard to demonstrate different methods of sorting. The notion of sorting itself is a challenge for some students.

For homework, the students receive a sheet con-

taining the sixteen quadrilaterals drawn on three-by-three dot grids (see **fig. 2**). Students cut apart the set of sixteen quadrilaterals and sort the shapes by pasting them in two groups onto a sheet of paper. The students also write explanations of the properties used in their methods of sorting. Students must use a property that has not previously been discussed in the whole-class sorting activity.

Determining the Areas of the Quadrilaterals

AS A FURTHER EXTENSION OF THE ACTIVITY, STUDENTS are asked to find the areas of the quadrilaterals. This activity can be done in groups in class or as a homework assignment. After the students have found the areas, they share their answers and methods with the class. The variety of methods is astonishing. Students discuss numerous geometry concepts as they make convincing arguments that their methods are sound. Many students use a cut-and-paste approach (see **fig. 3**). They see that one part of the figure fits into an uncovered part of another square and must convince the rest of the class that the pieces are congruent. Their arguments lead to a review of many geometric principles about congruent triangles because the cut pieces are usually triangular. Occasionally, a student will say that he or she actually cut the figures into pieces and thus the student knows that the pieces fit.

Another approach to finding the area of these quadrilaterals is illustrated in the statement “That part looked like one-fourth and that part looked like one-half” (see **fig. 4**). Such statements are usually correct, but the reasoning behind them is not intuitively obvious. Teachers can introduce a problem-solving opportunity by having students prove that the section of the shape believed to be one-fourth or one-half or one, in some instances, does in fact have that area measure.

Of course, some students use area formulas to determine

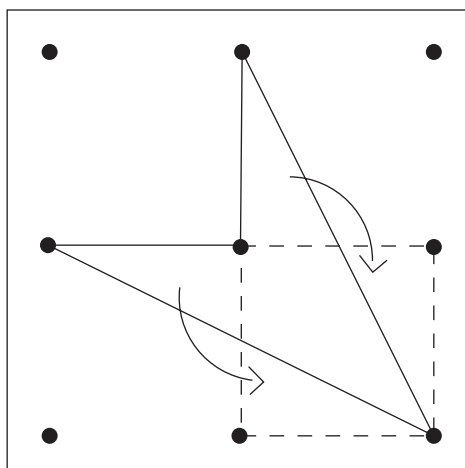


Fig. 3 Cut-and-paste method for finding the area of a quadrilateral

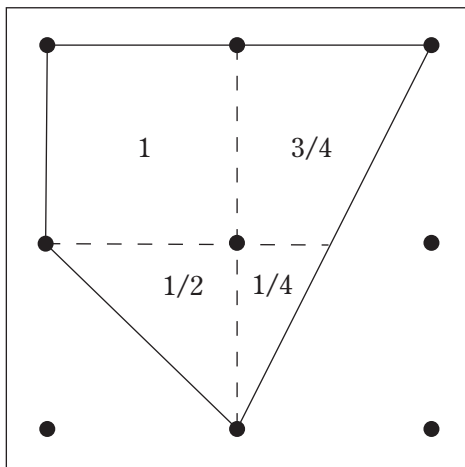


Fig. 4 Another student method for finding the area of a quadrilateral



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areas. Other approaches that we have seen include determining the area outside the figures and subtracting that value from 4, thinking of triangles as half-rectangles on the geoboard, and using previously found areas to aid in finding new ones. No doubt, students can come up with other strategies, as well. Every now and then, students surprise us with new strategies.

Conclusion

WHAT STARTED OUT AS AN ACTIVITY to introduce students to the geoboard turned out to be surprisingly rich. A recognition of shapes and their attributes; concepts of point and line, reflection, rotation, and translation; congruent figures; geometric proof; and area concepts are all represented in one problem. This activity also clearly shows the students that problems can be solved in a variety of ways.

Reference

Kennedy, Joe, and Eric McDowell. “Geoboard Quadrilaterals.” *Mathematics Teacher* 91 (April 1998): 288–90. ▲