Reflections on Practice

Perimeter and Area through the van Hiele Model

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IN MANY CLASSROOMS, MATHEMATICS IS TAUGHT by using examples to show students how to solve problems, then having the students complete large numbers of similar problems (Battista 1999). This process, called "parrot math" by O'Brien (1999), overlooks research showing that students (1) develop knowledge through interaction between the student and the knowledge, (2) do not think like adults, and (3) learn well through social interaction. Disregarding current research on how students learn mathematics and continuing the use of "parrot math" can be harmful to students' broader understanding of mathematical relationships (O'Brien 1999; Battista 1999).

Applying the van Hiele Model

HISTORICALLY, GEOMETRY TEACHERS INTRODUCED students to information about the Euclidean axiomatic system and deductive reasoning. Students were expected to understand geometric concepts and develop and reproduce proofs. This approach was problematic for many students and teachers, and both groups considered geometry to be the most dreaded high school mathematics class.

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"Reflections on Practice" is a look back at the end result of putting theory into practice. For those interested in submitting manuscripts that concern this theme, please send to editor Susan Friel at the address above.

Using the research of Dina van Hiele and Pierre van Hiele, we can help middle-grades students be more successful in high school geometry. We can change these negative feelings about geometry and prepare students to develop and reproduce proofs.

The van Hieles conducted research on how students think about and learn geometry. They observed that teachers often talked about geometry using language that students could not understand. The teachers and students were at different levels of thought about geometry. The van Hieles' research on levels of thinking and the role of insight in geometry defines five levels of thought that students experience as they study geometry (Fuys, Geddes, and Tischler 1988). See **figure 1.**

- **Level 0:** *Concrete,* in which the student identifies, names, compares, and operates on geometric figures
- **Level 1:** *Analysis,* in which the student analyzes figures in terms of their attributes and relationships among attributes and discovers properties and rules through observation
- **Level 2:** *Informal deduction,* in which the student discovers and formulates generalizations about previously learned properties and rules and develops informal arguments to show her or his generalizations to be true
- **Level 3:** *Deduction,* in which the student proves theorems deductively and understands the structure of the geometric system
- **Level 4:** *Rigor,* in which the student establishes theorems in different postulational systems and compares and analyzes the systems

Adapted from Fuys, Geddes, and Tischler (1988) and O'Daffer and Clemens (1991).

Fig. 1 Definition of van Hiele levels

Levels of Middle-Grades Students

BY THE TIME THAT STUDENTS ENTER THE MIDDLE grades, most of them are between the concrete and informal deduction levels defined by the van Hieles. In geometry, students should have a concept of what area and perimeter are and be able to describe their attributes. They should be able to derive the formulas for area and perimeter of some geometric shapes, such as rectangles, squares, and triangles, and use those formulas to find area and perimeter. Although many middle-grades students can solve these problems, they may not have fully conceptualized the meanings of the words. They become confused by the formulas and find area when they are asked for perimeter and perimeter when they are asked for area.

Different Strategies for Different Levels

Reflection:

What does students' behavior look like at the concrete, analysis, and informal deduction levels?

THE VAN HIELE MODEL for teaching geometry offers teachers strategies that are based on empirical research to guide instruction. Instruction—in both curriculum and pedagogy—that is based on the van Hiele model can help middle-grades students clarify their notions about perimeter and area. Consider the problem in **figure 2,** in which students are asked to add tiles until the perimeter of the figure is 16. Students at different van Hiele levels can plan strategies that help them find the solution. This problem is designed to help students see a relationship between area and the growth of perimeter. Even though students are not directed to find area as their investigations begin, relationships between area and perimeter naturally evolve in their conversations.

Using appropriate mathematical tasks accompanied by an inquiry-based pedagogy, students and teachers discover relationships in mathematics (Pugalee and Malloy 1999). As a result, they are able to make and justify conjectures about how and why these relationships exist. In a summer enrichment program, three sixth-grade students were asked to investigate this problem. They first tried to solve the problem individually, then worked as a group. Each student had plastic tiles available for the investigation. One of the students was at the concrete van Hiele level, another was at the analysis level, and the third was at the informal deduction level.

As might be expected from their levels of thought, the students used different strategies to find the solution. Terrence began by adding one tile and counting the units to find the perimeter. **Figure 3** shows that he added three squares before he arrived at a perimeter of 16. He did not notice that two tiles were enough, because the perimeter did not change when he placed a tile in the corner position.

Jarianne decided to guess. Without touching a tile, she predicted that the perimeter would change three units for each tile that she added. Her reasoning was that because a tile had four sides and she was covering one of those sides, she would lose one of the four sides with the addition of each tile (see **fig. 4**). By adding three sides to the perimeter with each tile, she decided that she would never get 16 as a perimeter. She could have 12 + 3 = 15 or 12 + 6 = 18 but not 16.

Misha placed brown tiles in the corners and counted the sides to find the perimeter just as Terrence had done. At first she was perplexed because the perimeter stayed the same. Then she realized that placing the tiles in the corners covered two sides of the original figure and two sides of the new tiles. No sides were gained. Next she decided to put green tiles on the outside of the figure and found that each tile added 2 to the perimeter. Using this in-

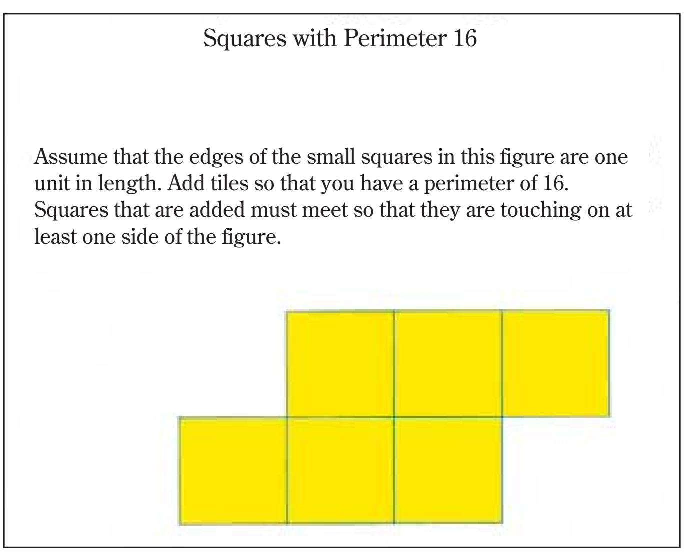

Fig. 2 Perimeter problem to promote van Hiele model development

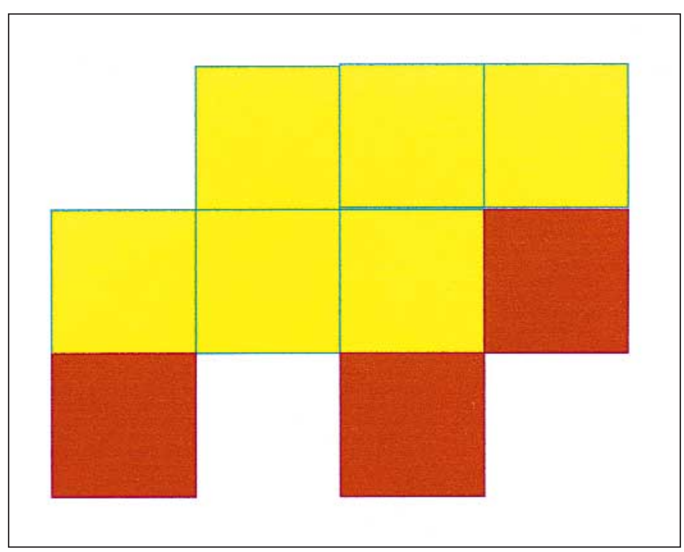

Fig. 3 Terrence's solution: Add three tiles

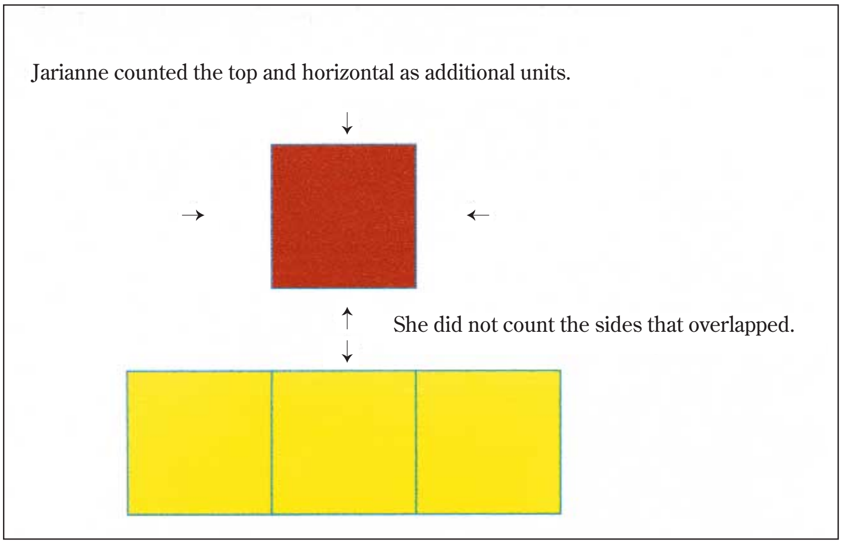

Fig. 4 Jarianne's solution: One tile adds 3; no solution

formation, Misha decided that many solutions were possible. She could place the two tiles anywhere around the shape and keep one or two of the brown tiles in the corners. (See **fig. 5.**)

Reflection:
How can you use the van Hiele levels to help students learn mathematics?

These students were focusing on different aspects of the problem for their thinking. Terrence was concentrating on the figure as the object, Jarianne was trying to classify the change in perimeter by manipulating the sides, and Misha was thinking about properties and relationships. Their work supported the research of the van Hieles. The students' *objects of thought* changed depending on their levels of thinking. At level 0, the object is the geometric figure. At level 1, it is the classification of objects. Properties are objects at level 2, and ordering relations is the object at level 3. When students progress to level 4, they are using the axiomatic systems, or the foundations of ordering relations, as the objects (Fuys, Geddes, and Tischler 1988).

Progressing from One Level to the Next

HOW CAN WE HELP STUDENTS PROGRESS from one van Hiele level to another? The van Hiele model suggests using five phases of instruction to help students in this progression. Students first gather information by working with examples (e.g., finding the perimeter of shapes), then they complete tasks that are related to the information, such as adding tiles to the figure to increase perimeter. The students become aware of relationships and are able to explain them. Finally, students are challenged to move to more complex tasks and to summarize and reflect on what they have learned. The language used by teachers and students is important for students' progression through the levels from concrete to visual to abstract (Fuys, Geddes, and Tischler 1988). (see text, next page)

The question in **figure 6** can be used to guide students who do not see the relationships between area and perimeter

Guiding Questions for Group Discussion

1. Where would you place a tile to increase the perimeter by 1? By 2? By 3?

2. How could you increase the area by 3 and not increase the perimeter?

3. What is the fewest number of tiles that can be added to increase the perimeter to 16 units? Describe this new shape. What is its area?

4. What is the greatest number of tiles that can be added to increase the perimeter to 16 units? Describe this new shape. What is its area?

5. Use the tiles to find all the noncongruent rectangles that have a fixed integral perimeter. Perimeters could vary from 12 to 24 units. (Note: Could a perimeter be an odd number?)

Fig. 6 Guiding questions

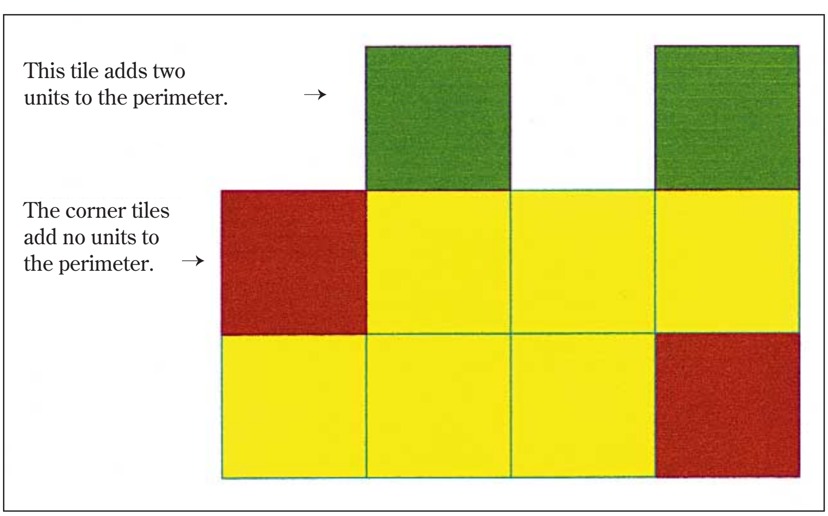

Fig. 5 Misha's solution; many solutions are possible

ter. Through a group discussion facilitated by the teacher, students were able to extend their thinking to the next level. They worked together to find solutions, learning in the process that different ways could be used to solve the problem and that many solutions were possible. Terrence helped Jarianne see that a perimeter of 16 was possible. Misha showed Terrence that he could have added just two tiles to get 16 because adding tiles to the corners did not change the perimeter. Jarianne was drawn into the problem and started asking questions about the smallest and largest numbers of tiles that could be used and still yield a perimeter of 16. The students became animated as they started to add the tiles and compare the area and perimeter. They found that the smallest number of tiles that they could add to arrive at a perimeter of 16 was two. They could see that each new row of tiles that they added to a rectangular shape, regardless of the size, added just 2 units to the perimeter. Using this discovery, the students found that the largest number of tiles that could be added to get a perimeter of 16 was ten. Additionally, they could see that the maximized area shown in **figure 7** is a square.

When the students were asked to summarize what they had learned, it became clear that even though they were at different van Hiele levels, they could all solve the problem, contribute to the group's thinking, and challenge one another. Terrence started to think about the classification of the placement, Jarianne changed the question to one that required informal deduction, and Misha was able to think about what happens with perimeters of 12 through 20. The growth that these students experienced in this investigation could not have been achieved without (1) the teacher's understanding of how the van Hiele research can foster growth in geometric understanding; (2) a worthwhile mathematical task that required the students to do more than just measure perimeter and area; and (3) the students' social interaction, which allowed them to use language as a tool for growth.

An additional bonus of this type of instruction is that students who do not know their number facts and do not have the confidence to attempt routine problems that involve only computations are able to achieve success in these problems. This success can be transformed into confidence at learning arithmetic facts and other mathematics.

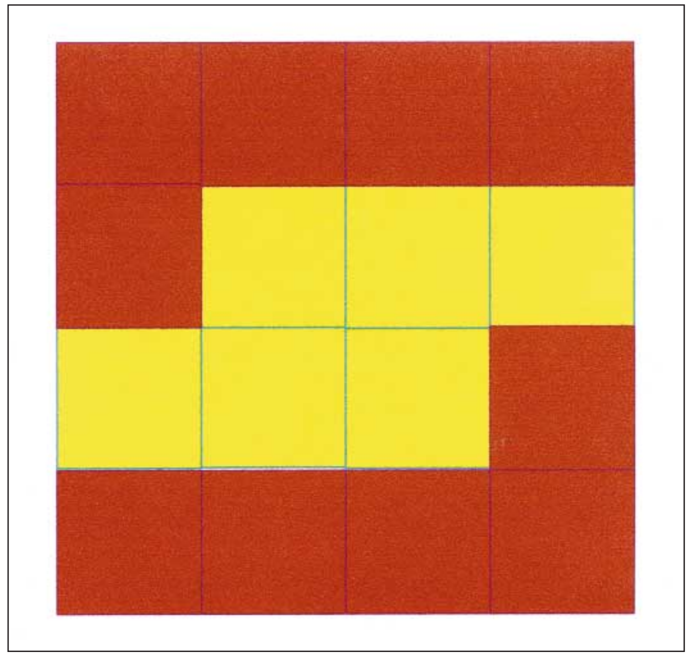

Fig. 7 Figure with maximum area for a perimeter of 16

References

Battista, Michael T. "The Mathematical Miseducation of America's Youth: Ignoring Research and Scientific Study in Education." *Phi Delta Kappan* 80 (February 1999): 424–33.

Fuys, David, Dorothy Geddes, and Rosamond Tischler. "The van Hiele Model of Thinking in Geometry among Adolescents." *Journal for Research in Mathematics Education* Monograph Series No. 3. Reston, Va.: National Council of Teachers of Mathematics, 1998.

O'Brien, Thomas C. "Parrot Math." *Phi Delta Kappan* 80 (February 1999): 434–38.

O'Daffer, Phares G., and S. R. Clemens. *Geometry: An Investigative Approach.* 2nd ed. New York: Addison-Wesley Publishing Co., 1991.

Pugalee, David K., and Carol E. Malloy. "Teachers' Action in Community Problem Solving." *Mathematics Teaching in the Middle School* 4 (February 1999): 296–300.