TAKE TIME FOR ACTION

Assessing Geomet

Understanding Us

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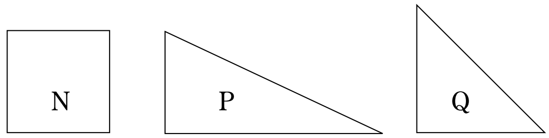
STUDYING GEOMETRY BENEFITS STUdents in a number of ways. Geometry enables students to represent and make sense of the world, analyze and solve problems, and represent abstract symbols pictorially to facilitate understanding (NCTM 2000). Similarly, measurement establishes important connections between school mathematics and everyday life. However, students often have very little understanding of geometry and measurement concepts (Martin and Strutchens, in press). More often than not, students are asked to memorize geometric properties rather than to experience geometry through nature walks or worthwhile tasks that involve hands-on explorations. Further, students learn measurement through memorizing formulas rather than exploring the underlying concepts.

As an experiment, administer to your students the three questions shown in **figure 1.** These questions appeared on the mathematics portion of the 1992 and 1996 National Assessment of Educational Progress (NAEP). Students were given manipulatives in the form of geometric shapes. You will need to make the shapes from lightweight card stock using the following dimensions: N is a 3.5 × 3.5 cm square; P is a right triangle with the same height as N, 3.5 cm, and a base of 6 cm; and Q is an isosceles right triangle with two legs that measure 4.5 cm. See **figure 2** for shapes in the correct sizes. Students should not be allowed to use rulers to complete the tasks. Consider the following questions: How did your students do on the items? What questions emerge for you as you consider their performance? Did any responses surprise you? How can you find out more about what students know and can do? Do they demonstrate a conceptual understanding or a procedural one? How did their performance relate to experiences that they may have had in class? How did they react to using the manipulatives? How might their responses to questions 2 and 3 have differed had they been permitted to use rulers?

These shapes were available for use as manipulatives for problems 1–3. Only shapes N and P were used for problem 3.



1. Laura was asked to choose 1 of the 3 shapes N, P, and Q that is different from the other 2. Laura chose shape N. Explain how shape N is different from shapes P and Q.

2. Which of the shapes N, P, and Q has the longest perimeter (distance around)? Shape with the longest perimeter:____________ Use words or pictures (or both) to explain why.

3. Bob, Carmen, and Tyler were comparing areas of N and P. Bob said that N and P have the same area. Carmen said that the area of N is larger. Tyler said that the area of P is larger. Who was correct? ________________ Use words or pictures (or both) to explain why.

Source: National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 1992 and 1996 Mathematics Assessments.

Fig. 1 Geometry questions

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Discussion

IN ADDITION TO SUMMARY DATA ON STUDENT performance, we were able to examine a convenience sample of student responses from the NAEP to problems 1–3 as stated in **figure 1.** The first problem was administered to fourth-, eighth-, and twelfth-grade students. Students had the opportunity to analyze properties of geometric figures. About three-fourths of the eighth-grade students in the NAEP sample answered problem 1 correctly. As would be expected, the number of fourth-grade students who answered the problem correctly was lower, 60 percent, and the number of twelfth graders who answered correctly was higher, 93 percent.

Most students' explanations in our convenience sample for problem 1 consisted of lists of differences in the shapes. For example, an eighth-grade student gave the following response: "N has 4 sides, P and Q only have 3, P's and Q's sides are different lengths, and N's are all equal." Other eighth-grade students classified the shapes, stating that N was a square and that the other two shapes were triangles.

These responses support the van Hiele model of student geometric thinking (Geddes and Fortunato 1993). The first level of the van Hiele model is the visual level, in which shapes are judged by their appearances. In the second level, the analysis level, components and properties of shapes are discovered. In answering problem 1, some students were able to reach the second level, whereas others remained at the first. For a more detailed explanation of the van Hiele model of geometric thinking, refer to Fuys, Geddes, and Tischler (1988).

Problem 2, given only to eighth- and twelfth-grade students, demonstrated how students' concepts of mathematics interacted with their understanding of geometry. For problem 2, students

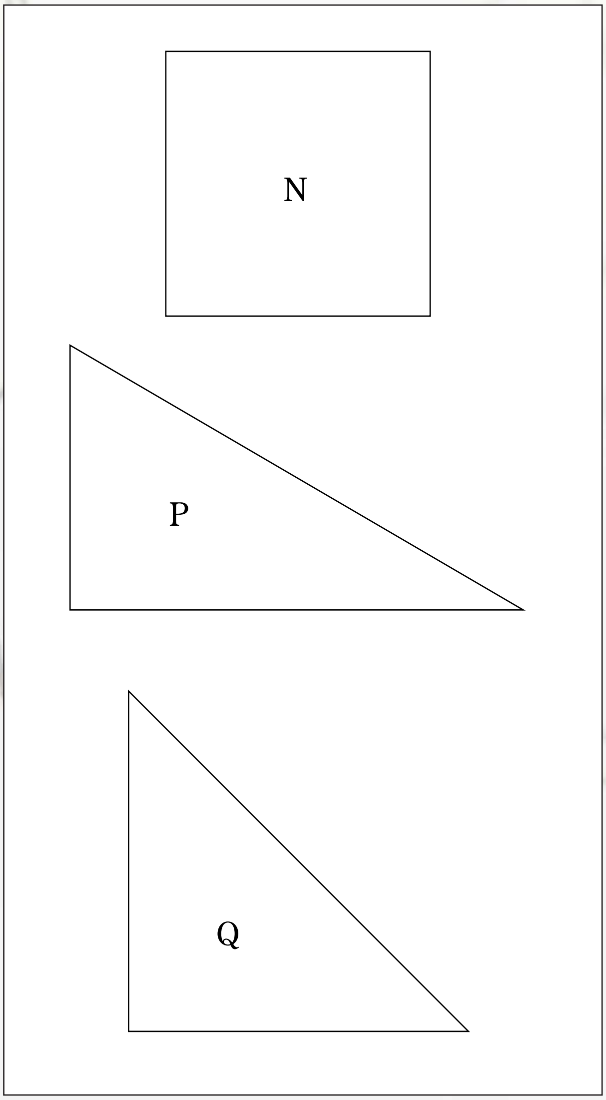

Fig. 2 Actual sizes of shapes

were given the same manipulatives as in problem 1 and were asked to determine the shape with the longest perimeter. They were told that perimeter is the distance around a shape, but they were not given rulers. According to NAEP's scoring guidelines, a correct response must identify which shape has the longest perimeter and must include an explanation of the answer. This problem was difficult for both eighth- and twelfth-grade stu-

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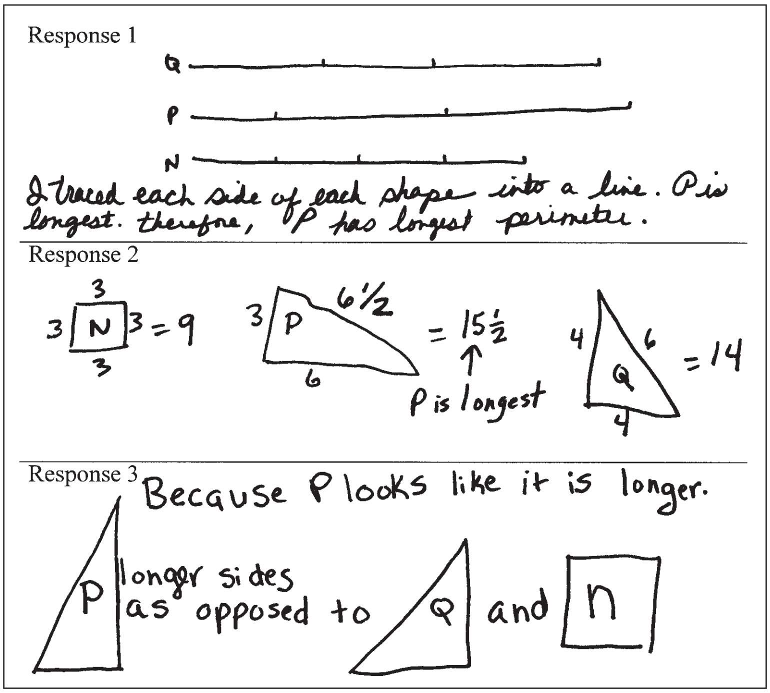

Fig. 3 Sample responses to problem 2

dents; only 6 percent of the eighth graders and 12 percent of the twelfth graders gave completely correct answers. Typical responses given by middle school students included empirical approaches to comparing the three shapes, as shown in **figure 3.**

The sample responses in **figure 3** show some of the correct responses for problem 2. In the first response, a student drew a line segment for each shape by tracing each side of the figures end to end, then comparing the combined lengths of the three resulting segments. In the second response, an eighth-grade student who may not yet have studied the Pythagorean theorem used a self-constructed measuring system to determine the relationship among the sides. Using the assumption that a side of N is 3 units, the student discovered that one leg of P is equal to a side of N, that the other leg of P is twice as long, and that the hypotenuse is a little more than 6 units. The two legs of Q are a little more than a side of N, approximately 4 units, and the hypotenuse of Q is the same length as the longer leg of P, 6 units. Once the lengths of all sides were known, although computing the perimeter of the square incorrectly, the student responded that P has the longest perimeter. Fifty-four percent of eighth-grade students produced the correct answer but were unable to explain why P has the longest perimeter.

For problem 3, students were given pairs of shapes, N and P, but again no rulers were provided. Of the three NAEP problems in **figure 1,** problem 3 produced the lowest results for eighth-grade students; only about one-fourth of these students gave a completely correct response to the question concerning the areas of shapes N and P. Performance at the other two NAEP grade levels was also relatively low for problem 3; 6 percent of fourth-grade students answered correctly, and 35 percent of twelfth graders answered correctly.

Student responses included solving the problem by comparing the lengths of the sides of the figures and using the area formulas (see **fig. 4**). This approach is illustrated in the first response. The student determined that the side of N is the same as the short leg of triangle P and that the length of one of the legs of P is twice that of its other leg. Other students solved the problem by manipulating the shapes. In the second response, an eighth-grade student used two drawings of each shape to demonstrate that two squares and two triangles have the same area.

Results from these NAEP problems illustrate that students across grade levels are relatively successful in uncomplicated tasks, such as problem 1. However, students are not as successful with less familiar situations, such as those that involve more than one complex figure, comparisons of complex figures, and perimeter and area concepts, particularly when asked to justify answers.

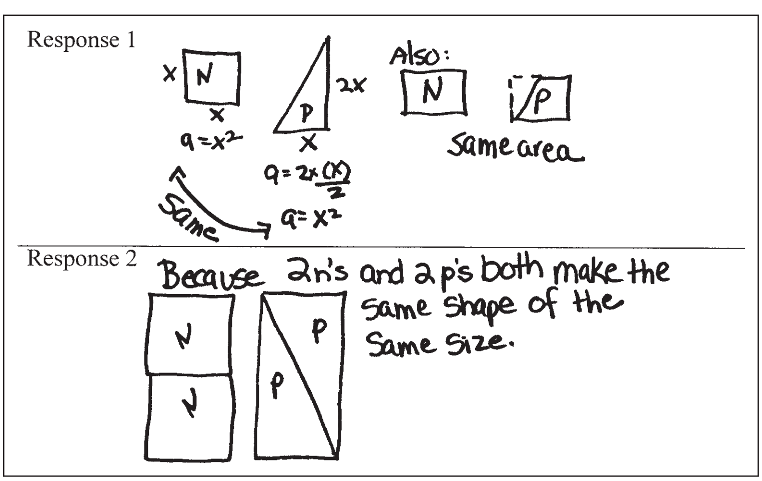

Fig. 4 Sample responses to problem 3

Implications for Teaching and Learning

THESE PROBLEMS ASSESS STUDENTS' UNDERSTANDING of several concepts and skills related to geometry. These skills include those that NCTM (2000) outlines for students in grades 6 through 8: (1) describe, classify, and compare polygons according to their main features; (2) analyze and understand geometric relationships among polygons; (3) compose and decompose polygons to solve a problem; and (4) develop and use formulas for the perimeter and area of polygons. Administering the NAEP problems may help you determine whether your students are meeting some of the objectives for geometry put forth in NCTM's *Principles and Standards for School Mathematics*.

Did your students responses to problem 1 show that they have a strictly visual view of geometric shapes and their properties? Results from the 1996 NAEP suggest that students need more experience with a variety of geometric figures in more complex settings. Students must be given opportunities to develop the ability to visualize geometric figures and their properties. This ability forms a basis for advanced geometric thinking (Wheatley 1990). Teachers should design activities that require students to sort shapes according to their geometric attributes, to identify shapes using a list of properties, and to determine the fewest properties needed to describe a particular shape (Van de Walle 1997). To ensure that students understand interrelationships among ideas, have them build family trees to determine relationships among properties of shapes. For example, students can build a family tree to show logical relationships among a quadrilateral, rhombus, square, rectangle, and trapezoid. You may also use the idea of concept cards that show examples and nonexamples of a concept to distinguish its characteristics. This approach allows students to compare and contrast different characteristics, to make and test conjectures, and to develop their own descriptions or definitions of the concept. See Geddes and Fortunato (1993, p. 208) for an activity that uses concept cards.

You may also want to consider whether your students were able to give correct responses and explanations for problems 2 and 3. NAEP data suggest that students need the opportunity to develop deeper conceptual understanding of perimeter and area. This goal can be achieved by designing activities in which students use string, measuring tape, or other appropriate measuring tools when learning about perimeter. To help foster a more meaningful understanding of area for your students, introduce the concept by asking students to cover regions of shapes with subregions. Then allow students to determine the area of the regions by counting or by using graph paper and geoboards and counting the units embedded in the figures (Van de Walle 1997). You may also give students a cutout parallelogram and ask them to cut it up and arrange the parts to determine the area easily (see **fig. 5**). The same exercise can be done for other geometric figures. You should allow students to develop the formulas for perimeter and area inductively, through counting, after using one of the methods mentioned above.

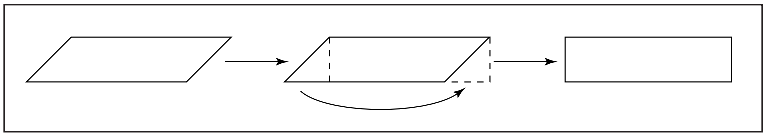

Fig. 5 Parallelogram to introduce students to the concept of area

We hope that this discussion has given you an impetus to assess your students' geometric thinking skills. Additionally, we hope that you will use the methods discussed here or develop other methods to improve your students' conceptual understanding of geometric properties and measurement.

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