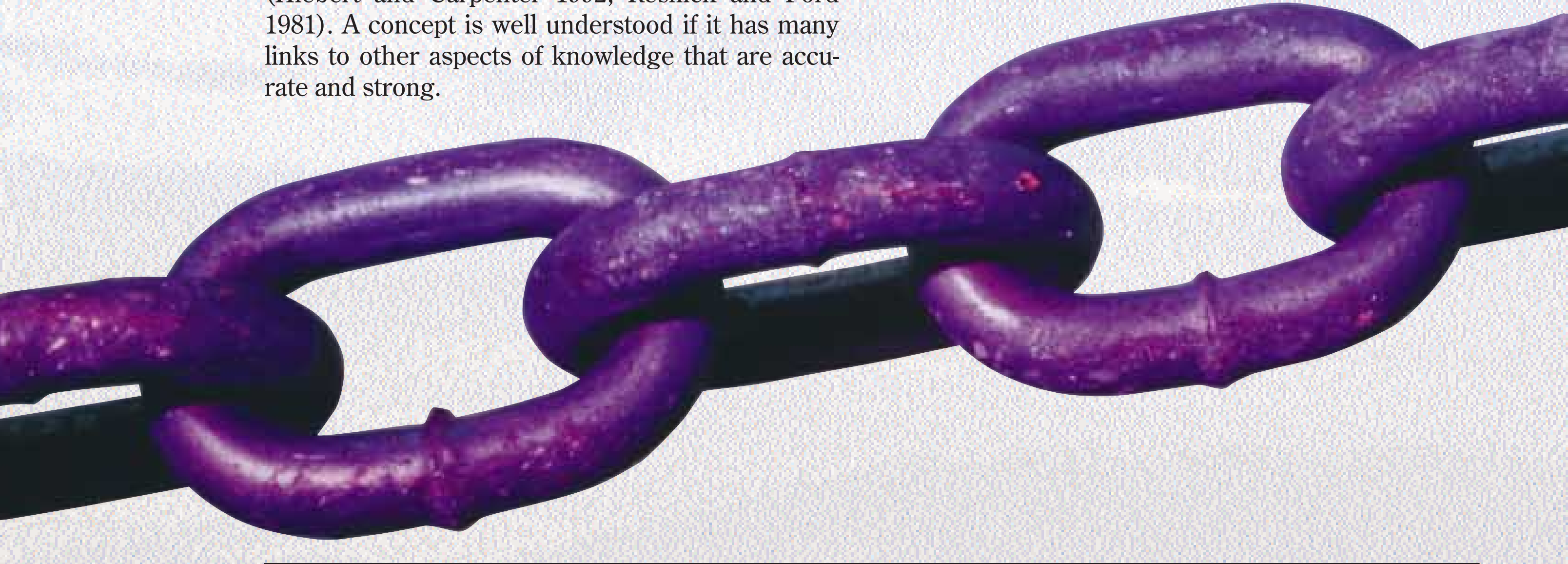
Using Concept Maps to Link Mathematical Ideas

ARTHUR J. BAROODY AND BOBBYE H. BARTELS

THE NATIONAL COUNCIL OF TEACHERS of Mathematics's standards documents (1989, 1991, 1995) emphasize making and assessing connections. Why are connections so important? Research suggests that understanding can be viewed as a connection between two pieces of information (Ginsburg 1977), and an understanding of elementary concepts is essential for mathematical power, for example, applying school mathematics to everyday tasks, inventing mathematical procedures, understanding and solving genuine problems, and comprehending more advanced mathematical ideas. The degree of a student's understanding is determined by the number, accuracy, and strength of connections (Hiebert and Carpenter 1992; Resnick and Ford 1981). A concept is well understood if it has many links to other aspects of knowledge that are accurate and strong.

Two examples of accurate connections follow: *(a)* addition and subtraction are inverse operations, for example, the addition of the number 3 can be undone by the subtraction of 3; and *(b)* addition facts can be used to figure out differences, for example, $5 - 3 = ?$ can be thought of as $3 + ? = 5$. An example of an inaccurate connection is that subtraction has the commutative property, for example, $5 - 3 = 3 - 5$. An example of a strong connection is consistently recognizing that all differences can be figured out by using a related addition combination. An example of a weak connection is sometimes recognizing that some subtraction combinations, such as $8 - 4$ and $12 - 6$, are related to addition doubles, such as $4 + 4 = 8$ and $6 + 6 = 12$.



ART BAROODY, *baroody@uiuc.edu, teaches at the University of Illinois, Champaign, IL 61820. His research focuses on the development of number and arithmetic concepts and skills by children, including those with learning disabilities.* BOBBYE BARTELS, *bbartels@pcs.cnu.edu, teaches mathematics and mathematics education courses at Christopher Newport University, Newport News, VA 23606. Her research interests are in mathematical cognition and understanding.*

What Is a Concept Map, and Why Is It Useful?

A CONCEPT MAP VISUALLY ILLUSTRATES MATHEMATICAL connections and describes them in writing. In other words, it is a drawing and a brief description of how someone or some group thinks that certain concepts are related. Consider, for example, activity 1, which was used as a means for helping students consider the relationships within the real-number system.

Activity 1: Number system (Grade level 6–8+). Use a concept map to diagram how the following concepts are related: *(a)* composite numbers, *(b)* counting numbers, *(c)* integers, *(d)* irrational numbers, *(e)* natural numbers, *(f)* negative integers, *(g)* common fractions that do not simplify to integers, *(h)* positive integers, *(i)* primes, *(j)* rational numbers, *(k)* real numbers, *(l)* whole numbers, and *(m)* zero. Include in your diagram the following examples:

$$\frac{1}{3},\ 1,\ 2,\ -3,\ \frac{\sqrt{3}}{2},\ 6,\ 13,\ \text{and } 15.$$

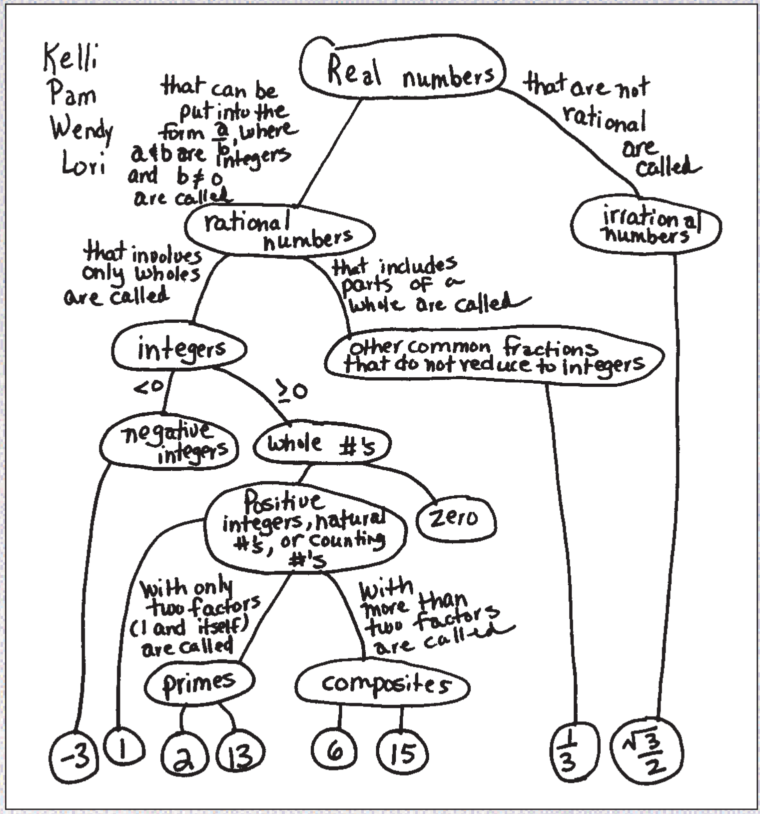
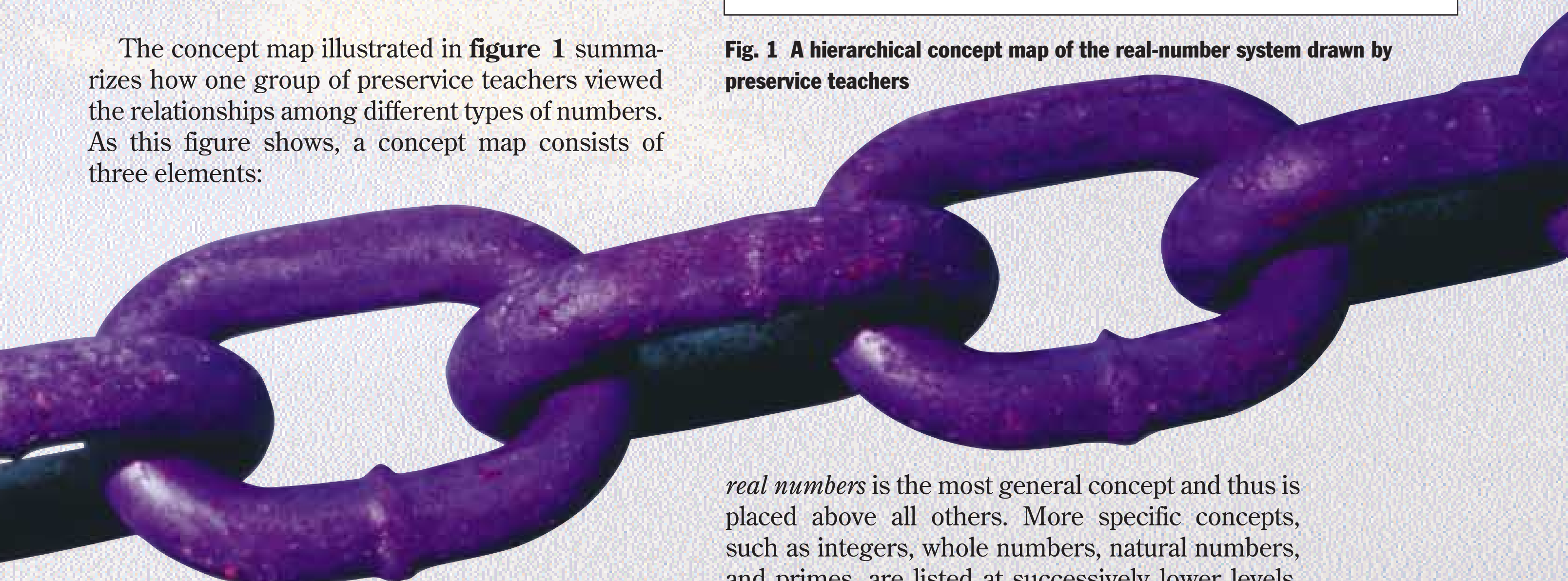


Fig. 1 A hierarchical concept map of the real-number system drawn by preservice teachers

The concept map illustrated in **figure 1** summarizes how one group of preservice teachers viewed the relationships among different types of numbers. As this figure shows, a concept map consists of three elements:



1. Concept names written inside loops, rectangles, or other shapes represent concepts.
2. Linking lines, as in webbing activities or flow charts; or arrows, as in arrow diagrams, show the connections between two concepts.
3. Linking phrases, which label linking lines, describe the relationships between concepts.

Concept maps can illustrate hierarchical relationships among ideas. In hierarchical maps, the most general concepts are placed at the top. In **figure 1,** *real numbers* is the most general concept and thus is placed above all others. More specific concepts, such as integers, whole numbers, natural numbers, and primes, are listed at successively lower levels. Such examples as 1/3, 1, 6, and 15, which are highly specific, are placed at the bottom of a hierarchical map.

Concept maps can also take the form of spider maps, which are used to illustrate webbed relationships, or chain maps, which are used to illustrate sequential relationships.

Because understanding involves "seeing" a connection and drawing a concept map requires explicitly defining connections, concept mapping can be an invaluable tool for fostering the meaningful learning of mathematics by students at any level. ☞

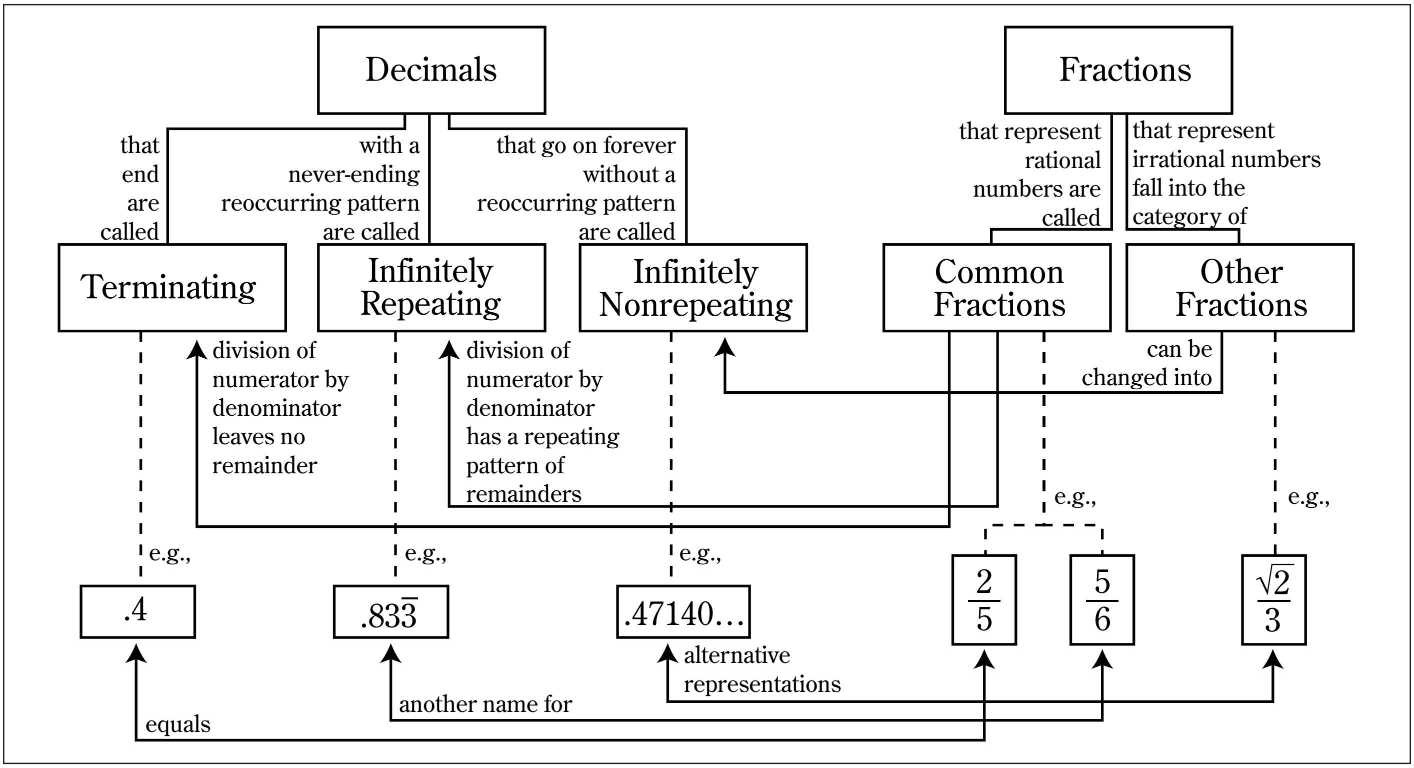


Fig. 2 A concept map about types of decimals and their connections to fractions drawn by a preservice class

What Roles Can Concept Maps Play in Meaningful Instruction?

CONCEPT MAPS CAN BE USED AS A TEACHING tool at the middle school level and beyond, including in preservice and in-service teacher education, in two different ways: *(a)* to transmit information about concepts to students or *(b)* to help students construct an understanding of concepts.

Transmitting information

A teacher could use a ready-made concept map, such as that shown in **figure 2,** as a class handout. The handout might serve as an advanced organizer, such as an overview of a lesson; a vehicle for discussing concepts and their relationships, such as a visual aid for developing a lesson; a means for tying together ideas, such as a visual aid for summarizing a lesson; or a study guide or review sheet (Bartels 1995b). In this capacity, concept maps will be beneficial in achieving a key goal of the *Curriculum and Evaluation Standards* (NCTM 1989)—helping students see connections among concepts.

Constructing understanding

More consistent with the inquiry-based approach envisioned by the National Council of Teachers of Mathematics (1989, 1991), a teacher could ask students to work in groups, develop their own concept maps, and then share their results with the class. To help you better appreciate this use of concept mapping, take a moment to complete activity 2.

Activity 2: Geometric concepts (Grade level 5–8+). Using **figures 1** and **2** as a guide, draw a concept map that includes the following concepts: geometry, parallelograms, plane geometry, polygons, quadrilaterals, rectangles, squares, trapezoids, and triangles. Hints: Identify the broadest concept, and place it at the top of your map. Then locate the next general concept, and place it under the first. Continue in this way until you have used all the concepts listed. To show how concepts are related, *(a)* draw in connecting lines and *(b)* label them with linking phrases that briefly explain how the concepts are related.

What Can Concept Mapping Achieve?

CONCEPT MAPPING CAN PROMOTE INQUIRY-based, meaningful learning in the following ways:

1. *Provides a vehicle for introducing new concepts and connecting them to one another and to known concepts.* Asking students to include a new or unfamiliar concept in a concept map creates a need to define the term. For example, activity 1 often prompts students to ask, “What is a trapezoid?” It also creates a need to consider how trapezoids may be related to other new or previously discussed concepts, such as quadrilaterals and parallelograms.

2. *Encourages active construction of concepts.* Constructing a concept map requires explicitly illustrating and describing connections, thinking clearly and precisely about the relationships among concepts (Bartels 1995b). In other words, concept mapping prompts reflection and thus actively involves students in learning (Novak 1987). Contemplating how to draw a concept map can give students an opportunity to consider *consciously* how concepts are linked. To correctly place trapezoids in a concept map, for example, students must recognize that a trapezoid is a four-sided shape and thus a type of quadrilateral. To accurately label the link between quadrilaterals and trapezoids on their concept maps, students must consider how trapezoids are a special kind of quadrilateral, including what characteristics distinguish them from other quadrilaterals.

3. *Fosters metacognitive knowledge and autonomy.* In attempting to draw a concept map, students often quickly recognize for themselves what concepts and connections are not clearly or completely understood. This self-evaluation may prompt them to reason out a connection, research the topic, or ask questions. For instance, students may recognize that they have either no idea or only a fuzzy idea of what a trapezoid is.

4. *Motivates conjecture making and testing.* Deciding how to draw and label a concept can involve students in making and evaluating educated guesses. For instance, in one group, several students offered different conjectures about the definition of trapezoids: "Aren't trapezoids those things with four different-sized sides?" "Based on the picture here in our math book, a trapezoid has two parallel sides and the other two sides have to be equal." To test the latter conjecture, one student drew two pictures (and) and asked the teacher if they were both trapezoids. To the group's surprise, the teacher indicated that they were. One child concluded, "Maybe you have to have parallel sides, but the four sides could be different lengths."

5. *Underscores personal interpretation.* Typically, no one correct way of doing a concept map exists. How a map turns out depends partly on its purpose and what concepts and relationships are viewed as important and chosen for inclusion. Moreover, concept maps may reflect different interpretations of a concept. For example, because it became clear that his elementary-level students did not know the definition of a trapezoid, a teacher encouraged them to look it up. The students found that the dictionary defined *trapezoid* as an enclosed four-sided figure with two parallel sides. Some students interpreted this definition to mean that trapezoids had *at least* one pair of parallel sides; others, that trapezoids had *only one* pair of parallel sides. These different interpretations were reflected by the groups using different linking phrases.

6. *Provides an opportunity to engage in logical reasoning.* In considering how concepts are related, students must engage in if-then, or deductive, reasoning. For example, some students reasoned that if trapezoids had at least one pair of parallel sides, then parallelograms had to be a special kind of trapezoid and thus be connected to, and located directly under, trapezoids on their map. Others reasoned that if trapezoids had only one pair of parallel sides, then parallelograms had to be a separate class of quadrilaterals and thus not be connected to, but rather located alongside, trapezoids. The concept map illustrated in **figure 3** is based on the second interpretation of trapezoids. Different placement of parallelograms on the concept maps offered further evidence that mathematics involves personal interpretations.

7. *Prompts problem solving.* Figuring out how to represent concepts and connections can involve students in geometric problem solving. For example, confronted with the complex task of representing how the four arithmetic operations and the properties of commutativity are interrelated, one preservice teacher proposed making a concentric hierarchical concept map by placing the broadest concept in the center and successively more specific

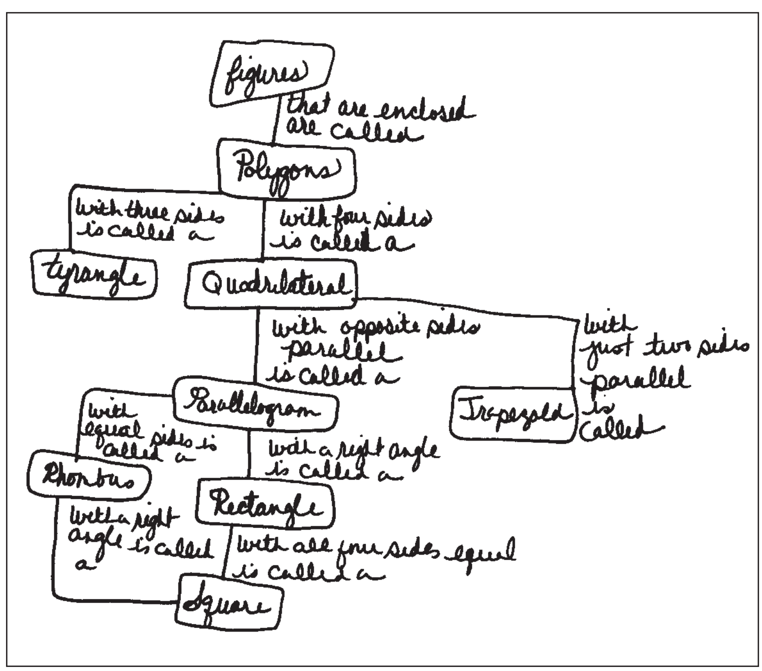


Fig. 3 A fourth-fifth grader's concept map of some geometric concepts

concepts in successive distant rings (see **fig. 4**). This method—called “Joy’s method” in honor of its creator, Joy Augustine—yielded a less confusing spatial display than the top-down hierarchical method introduced by the instructor.

8. *Kindles dialogue and a perception of the construction of mathematical knowledge as a social process.* Examining a concept map proposed by others can prompt questions, discussion, and debate. Students often differ on how a map should be constructed, and the resulting conflict can motivate map authors to defend or reevaluate their ideas (Baroody 1998). Comparing concept maps drawn for activity 1, for example, can reveal different interpretations of trapezoids and stir debate about which is correct. In this example, students may discover that either interpretation of a trapezoid given previously is workable (see, e.g., James and James [1949]) and that definitions are arbitrary, socially agreed-on conventions. Although different, multiple, or idiosyncratic interpretations sometimes make sense, at other times they are incompatible and require resolution. For example, although parallelograms might be considered a subclass of trapezoids, as some students proposed, trapezoids could not be considered a subclass of parallelograms, as others suggested. Concept mapping can provide a public forum for debating incompatible conjectures and reaching a consensus about mathematical concepts and their connections. In brief, the give-and-take prompted by sharing concept maps can underscore the fact that the construction of knowledge is very much a social process.

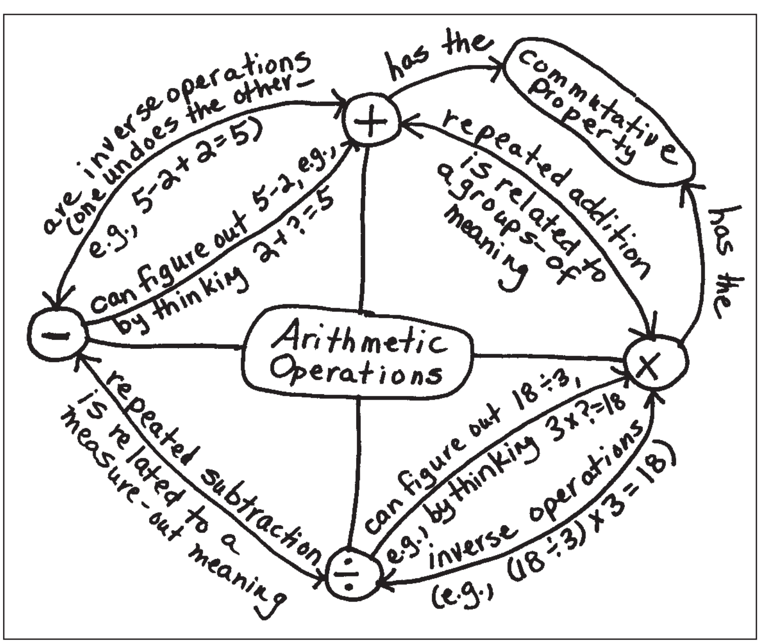


Fig. 4 Joy’s (a preservice teacher’s) concentric hierarchical map

9. *Promotes a view of knowledge as changeable.* As children’s understanding evolves, their concept maps will change. Comparing earlier and later maps on a topic can furnish concrete evidence that learning is an active, ongoing process.

10. *Creates a real need for introducing and practicing algebraic notation.* Because the space on a concept map is often limited, algebraic notation can be introduced as shorthand for writing perhaps extensive linking phrases. For instance, while representing the connections between addition and subtraction, a teacher can recommend using the connecting phrase “To do A – B = □, think B + □ = A” instead of a natural-language explanation of the complementary relationship, for example, “To do 5 – 3 ?, think what must be added to 3 to get 5.” Parenthetically, concept mapping can also provide purposeful practice of such language arts skills as spelling, vocabulary, and handwriting.

Teaching Tips on Getting Started

THE FOLLOWING TIPS MAY BE HELPFUL IN GETting students started with concept mapping.

- Begin with structured tasks and move gradually to less-structured tasks. Starting with an open-ended production task, which asks students to draw their own concept maps, may be too difficult for many students. Begin with a verification task, which asks students to evaluate the correctness of a completed concept map, or an add-on task, which involves adding a single concept or several concepts to an existing map. Fill-in tasks, in which students use a list of concepts and, perhaps, linking phrases to label an otherwise complete concept map; close-ended list tasks, in which students are given only a list of concepts and, perhaps, linking phrases; and then open-ended list tasks, in which students are given a list of concepts and, perhaps, linking phrases but are free to add other concepts or linking phrases, can serve as transition steps to the relatively difficult production task.

- Start with concept maps involving a few key concepts. Have students diagram just three or four main concepts in an area. Additional concepts, specific examples, and their linking phrases can be added gradually as a topic is explored at deeper levels.

- Focus first on mapping the concepts within a single topic and then on linking different maps or topics. This approach can make the construction of a relatively grand concept map manageable.

- Work together as a class to get students started on concept mapping. To introduce concept mapping, first lead a class through the evaluation, revision, or completion of a concept map several times. Students may need help learning the logic of concept maps, for example, how they can be used to represent different relations. Class discussions may also be helpful later when list tasks and other less-structured tasks are first introduced.

- Because concept mapping can be a challenging problem-solving task, many students feel more comfortable working on the maps as a team. Until they become accustomed to drawing concept maps, students will probably need supervision and feedback. With hierarchical maps, for example, students may need to be reminded about placing the most general items at the top and the most specific items, or examples, at the bottom. Remind students about labeling the connections.

- With less-structured tasks, encourage students to make a draft copy first, discuss and refine the draft, and then draw up a polished map. This process underscores the idea that concept mapping and learning itself are building processes. For example, students can see that making revisions is a natural process of any complex task. Moreover, knowing that they are working out their thoughts on a draft may be less frustrating for fastidious students.

- Encourage students to consider specific linking phrases. Concept maps are an ideal way of helping students reflect on and explicitly summarize the relationship between two concepts. However, novice concept mappers often rely too much on nondescriptive linking terms, such as *includes* or *are related* (Bartels 1995a). Although such linking terms may sometimes be correct or even ideal, they often do not spell out clearly the relationship between two concepts. For example, writing the linking phrase *with all sides equal* between rectangles and squares not only indicates an understanding that rectangles and squares are related but how they are related. If students use a nondescriptive phrase, a teacher should encourage them to be more specific.

Summary

IN BRIEF, BY ENCOURAGING STUDENTS TO consider explicitly how concepts are linked, concept mapping can be an invaluable tool in helping them build connections and, hence, understanding. Because concept mapping requires students to portray connections explicitly, it can prompt conscious reflection on concepts, conjecturing and reasoning about the meaning of concepts and their relationships, discussion and debate about mathematical ideas, and a creative search for ways to illustrate or describe concepts and their relationships. In brief, it can be a powerful tool for fostering meaningful learning and the skills central to mathematical inquiry.

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For an illustration of how concept mapping can serve as a basis for mathematical inquiry and meaningful learning, see vignette 4 on pages 3–34 of *Fostering Children's Mathematical Power: An Investigative Approach to K–8 Mathematics Instruction* (Baroody 1998) or send a request with a self-addressed, stamped envelope to Art Baroody, University of Illinois, Champaign, IL 61820.