A Week of Theoretical and Experimental Probability

Eighth Grade

One week of probability that could be used around St. Patrick’s Day for analyzing luck and probability.

By: Jeff Thorp
Overall NYS learning objectives:

8.CM.1 Provide a correct, complete, coherent, and clear rationale for thought process used in problem solving.

8.CM.9 Increase their use of mathematical vocabulary and language when communicating with others.

8.CN.4 Model situations mathematically, using representations to draw conclusions and formulate new situations.

8.R.1 Use physical objects, drawings, charts, tables, graphs, symbols, equations, or objects created using technology as representations.

8.R.2 Explain, describe, and defend mathematical ideas using representations.

8.R.6 Use representations to explore problem situations.

A.S.3 Determine when collected data or display of data may be biased.

A.S.20 Calculate the probability of an event and its complement

A.S.21 Determine empirical probabilities based on specific sample data

A.S.22 Determine, based on calculated probability of a set of events, if:
   o some or all are equally likely to occur
   o one is more likely to occur than another
   o whether or not an event is certain to happen or not to happen.

NCTM standards addressed are:
Data analysis and probability, communication, and representation.

Five Day Outline:
Day 1: Solve theoretical probabilities through situations and introduce notation as well as area models for probability.
Day 2: Experiment with the theoretical probabilities (some from day 1), comparing experimental data with theoretical data.
Day 3: Experiment with combined probability, and then combine class results for larger sample size. Predict the probability of the events.
Day 4: Calculate theoretical probability of combined events from day 3, using area models. Use these theoretical probabilities to compare with the experimental probabilities from day 3.
Day 5: Use the TI-84 to simulate unfair events with speed. The fairness of these events will be unknown to the students. The students will then analyze the idea of fair games or to have luck.
Day 1

Objectives: The students will be able to analyze familiar probability situations through area model representation and fractions.

Materials:
- Teacher- Class Copies of Handout
- Coins, Dice, Spinners, Bag, Color Cubes
- Students- Writing Utensils

Anticipatory Set:
Introduce the students to each of the objects they will be discussing today. Show the students how an experiment using each manipulative would work, knowing that many of these are not going to be new to the students. Show the students how to use an area model if they have not used them before.

Main Lesson:
Have the students break into groups of 2 or 3. Give the handout and have them work through the activities. Moderate the discussions in the groups while the students are working.

Guided Practice:
In review of the worksheet, make sure the students realize the similarities between spinner B and Bag #1.

Closing:
Make sure the students understand the use of the “P (x)” notation.
Go over the results of the worksheet with the class.

Homework:
Flip a coin 25 times. Record heads and tails on the homework worksheet.
Roll a die 25 times. Record the numbers on the homework worksheet.
Directions: Write the theoretical probability in fraction and percent (nearest hundredths) form for the events listed below. Draw an area model of that probability, shading the positive outcome.

1- You are flipping a coin.
   P (head) =
   P (tail) =

2- You are rolling a die.
   P (1) =
   P (1 or 6) =
   P (odd) =
   P (not 6) =
3- You are spinning spinner A.
   \[ P(\text{red}) = \]
   \[ P(\text{not red}) = \]

4- You are spinning spinner B.
   \[ P(\text{green}) = \]
   \[ P(\text{orange}) = \]
   \[ P(\text{blue, yellow or green}) = \]
   \[ P(\text{not red}) = \]
5- You are drawing a marble from Bag #1.

\[ P (B, Y \text{ or } G) = \]

\[ P (\text{not } R) = \]

6- You are drawing a marble from Bag #2.

\[ P (B) = \]

\[ P (R \text{ or } Y) = \]
P (not P or not G) =

Name_______________________    Day 1    Homework
Date_______

Flip a coin 25 times and record the results. Roll a die 25 times, record the results.

<table>
<thead>
<tr>
<th>Flip #</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Roll #</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>
Day 2

Objectives: Students will be able to understand the difference between experimental and theoretical probability through conducting trials and comparing the experimental and theoretical probability.

Materials:
Teacher- Class Copies of Handout
Bag, Color Cubes, Overhead Dice

Students- Writing Utensils, Bags, Color Cubes

Anticipatory Set:
Collect the results of the students’ homework and make a list of the results on the board. Have the students total the results and record them on their worksheet.

Main Lesson:
Have the students answer and then conduct the experiments on the worksheet in small groups. Provide them with a bag with one red, two blue, three yellow and 4 green cubes.

Guided Practice:
Stop the students when all of them have finished the coin and dice parts of the worksheet and have pulled the cubes from the bag 25 times. Have the students announce their findings. Ask questions to check their understanding of the difference in experimental and theoretical probabilities. Then allow the students to finish their worksheet.

Closing:
Have students describe the differences in experimental and theoretical probability in words using formal mathematical language.
Homework:

Have the students write the answers to the following questions in complete sentences, using correct mathematical language.

Suppose you flip a coin 1000 times. You record the results and find that you got 520 heads, and 480 tails. Using the vocabulary experimental and theoretical probability, answer the question “Is this a reasonable outcome and why?”

Suppose you flip a coin 1000 times. You record the results and find that you got 700 heads, and 300 tails. Using the vocabulary experimental and theoretical probability, answer the questions “Is this a reasonable outcome and why? And how could this result happen?”

Name______________________________   Day 2   Class Work   Date_________

Comparing the theoretical to our trials

With the results collected from the homework; find the fraction and the percents of these trials.

The Coins:

Total number of heads: _____   Total number of tails: _____

Total number of trials (flips): _____

What are the differences in percents of the theoretical heads and the experimental heads totals?

The Dice:

Total number of 1: _____   Total number of 2: _____   Total number of 3: _____

Total number of 4: _____   Total number of 5: _____   Total number of 6: _____

Total number of trials (rolls): _____
Write the percentage of each number rolled by our class.

Percent of 1: ____  Percent of 2: ____  Percent of 3: ____

Percent of 4: ____  Percent of 5: ____  Percent of 6: ____

What is the difference in the experimental and the theoretical probabilities? (Subtract the percentages using the experimental minus the theoretical totals.)

Difference of 1: ____  Difference of 2: ____  Difference of 3: ____

Difference of 4: ____  Difference of 5: ____  Difference of 6: ____

Now conduct a trial using your color cubes and bag.

Place one red, two blue, three yellow and four green cubes in your bag. Draw cubes, with replacement, 25 times with your partners. Record the results on the table below.

<table>
<thead>
<tr>
<th>Draw #</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>25</td>
</tr>
</tbody>
</table>

Now record the results from the whole class.

Class’s number of Red: _____

Class’s number of Blue: _____

Number of Red: _____

Number of Blue: _____

Number of Yellow: _____

Number of Green: _____

Now record the results from the whole class.
Class’s number of Yellow: ______  
Class’s number of Green: ______  
Number of total draws: ______  
Percent of Red: ______  
Percent of Blue: ______  
Percent of Yellow: ______  
Percent of Green: ______  

Write the theoretical results in Percents.

Theoretical P (R) = ______  
Theoretical P (B) = ______  
Theoretical P (Y) = ______  
Theoretical P (G) = ______  

Differences in percent (experimental - theoretical)

Red ______  Blue ______  
Yellow ______  Green ______  

Day 3

Objectives: Students will be able to experiment with experimental probability by conducting trials and estimating the theoretical probability.

Materials:
Teacher- Class Copies of Handout  
Bag, Color Cubes, Overhead Dice  
Students- Writing Utensils, Bags, Color Cubes, Dice, Coins

Anticipatory Set:
Have some students read their homework results. Help them to use proper language and fully explain ideas. Explain

Main Lesson:
Have the students answer and then conduct the experiments on the worksheet in small groups. Provide them with a bag, color cubes, dice, and coins.

Guided Practice:
Stop the students when all of them have finished the experiments. Collect the data and have the students record the results on the worksheet. Have the students carry out the rest of the worksheet.

Closing:
Have the students understand the difference between the probability of one event occurring, and the probability of combined events.

**Homework:**

Ask the student which is more likely to occur when rolling two dice; rolling a 7 or a 4, and why?

Name______________________________    Day 3    Class Work    Date_________

**Place one red, one blue and one yellow cube into a bag. Draw a cube and then flip a coin. Record the results on the table below and replace the cube back into the bag. Repeat this process 25 times.**

<table>
<thead>
<tr>
<th>Trial #</th>
<th>Coin Result</th>
<th>Cube Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

List all possible combinations for flipping a coin and drawing a cube below. How many times did you achieve each possible result?
Using your data and that collected as a class, what is the percent of each outcome?

As a group, discuss the theoretical probability for this trial. Write what you believe it is and why below.

**With the same bag and cubes used in the previous trial and a die, draw a cube and roll the die as you did before. Record the results in the table below. Repeat the trial 25 times.**

<table>
<thead>
<tr>
<th>Trial #</th>
<th>Cube Result</th>
<th>Dice Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

List all possible combinations for drawing a cube and rolling a die below. How many times did you achieve each possible result?
Using your data and that collected as a class, what is the percent of each outcome?

As a group, discuss the theoretical probability for this trial. Write what you believe it is and why below.

**Day 4**

**Objectives:** Students will be able to create area models of combined probability by analyzing the experiments of day 3 as well as the homework of day 3.

**Materials:**
- Teacher: Class Copies of Handout
- Students: Writing Utensils, Bags, Color Cubes, Dice, Coins

**Anticipatory Set:**
- Have some students read their homework results. Help them to use proper language and fully explain ideas. Explain

**Main Lesson:**
- Have the students answer and then conduct the experiments on the worksheet in small groups. Provide them with a bag, color cubes, dice, and coins.

**Guided Practice:**
Stop the students when all of them have finished the experiments. Collect the data and have the students record the results on the worksheet. Have the students carry out the rest of the worksheet.

**Closing:**
Have the students understand the difference between the probability of one event occurring, and the probability of combined events.

**Homework:**
Have the students write a response to the following questions:

Using vocabulary discussed this week, what does it mean to have a fair event? What does it mean to be unfair?

---

**As a class, create area models for the combined probability of:**

Two coins being flipped and getting two “heads” results.
What fraction is this?

Two coins being flipped and getting one “head” and one “tail”.
What fraction is this?

**As a group, make area models for the following events. Answer the questions that follow.**

Draw the area model for drawing cubes from a bag with one red, one blue and one yellow and
flipping a coin (same as yesterday’s first experiment).

\[ P (\text{red and heads}) = \]

\[ P (\text{blue and tails}) = \]

How do these results compare to the class’s experiment yesterday?

Draw the area model for drawing cubes from a bag with one red, one blue and one yellow and rolling a die (same as yesterday’s second experiment).

\[ P (\text{yellow and 6}) = \]

\[ P (\text{blue and 3}) = \]

How do these results compare to the class’s experiment yesterday?
Now to analyze yesterday’s homework question…

Create an area model for rolling two dice.

What fraction is each square?

What is the chance of rolling a 7?

What is a chance of rolling a 4?

Which event (rolling a 4 or a 7) is more likely?

Why, and how much more likely is it?

Day 5

Objectives: Students will be able to recognize unfair trials based on expectations through the use of the TI-84 “prob sim” application.

Materials:
Teacher- TI-84 Teacher Edition, worksheets
Students- Class set (or enough for small groups) of TI-84s (all with the “prob sim” application on them.)

Before the lesson— set the likely hood of certain outcomes for the student calculators under the “advanced” option in the “set” tab of the “toss coins”, “roll dice”, and “pick marbles” simulations so that the theoretical probability is not equal. Also, make sure the tables are set on frequency, not probability.
Suggested probability:
Coin: Heads: 0.75, tails: 0.25
Dice: 1: \(\frac{1}{12}\), 2: 0, 3: \(\frac{1}{6}\), 4: \(\frac{1}{4}\), 5: \(\frac{1}{6}\), 6: \(\frac{1}{3}\)
Marbles: A: 10, B: 25, C: 15, D: 20, E: 30
(make sure marbles total 100 for the worksheet)

**Anticipatory Set:**
Using the teacher version of the TI-84, show them how the probability simulation program works through demonstration of each three simulations.

**Main Lesson:**
Have the students answer and then conduct the experiments on the worksheet in small groups.

**Guided Practice:**
When the students have finished the worksheet, ask them:
Do you think each outcome had equal chance of occurring?
Is the [insert simulation name] fair for each result?

**Closing:**
Have the students understand the difference between the probability of one event occurring, and the probability of combined events.

**Homework:**
Answer the following questions with correct math language:
What is the difference between today’s experiments and earlier ones?
What does it mean to be a fair game?
What does it mean to have luck?

Name______________________________ Day 5 Class Work Date_________

**Using the TI-84 and the “prob sim” application, complete the following activities.**

Open the coin flip simulation. Conduct 100 simulated coin flips. (Make sure you use more than one flip at a time.) Sketch the table given as best you can.

About how many heads did you get?

About how many tails did you get?

Is this the result you expected to get? Why?
Open the dice rolling simulation. Conduct 600 simulated rolls. Sketch the table given.

About how many of each number did you get?
What estimated percent is each?
(Remember there were 600 rolls.)

1: _____  _____%
2: _____  _____%
3: _____  _____%
4: _____  _____%
5: _____  _____%
6: _____  _____%

Is this the result you expected to get? Why?

Open the marble drawing simulation. Conduct 500 simulated draws. Sketch the table given.

About how many of each letter marble did you get?
What estimated percent is each?
(Remember there were 500 draws.)

A: _____  _____%
B: _____  _____%
C: _____  _____%
D: _____  _____%
If there were 100 marbles in the bag, estimate how many of each letter are there?

Are the simulated experiments all fair? (Use the vocabulary theoretical and experimental probability in your explanation, as well as including what results you expected to see.)

**ANSWER KEY**

Since most of the answers will vary, the answer key will include the answers to all the questions that do not vary in response.

**Day 1**

1. You are flipping a coin.
   - \( P(\text{head}) = \frac{1}{2} \)
   - \( P(\text{tail}) = \frac{1}{2} \)

2. You are rolling a die.
   - \( P(1) = \frac{1}{6} \)

4. You are spinning spinner B.
   - \( P(\text{green}) = \frac{1}{6} \)
   - \( P(\text{orange}) = \frac{1}{6} \)
   - \( P(\text{blue, yellow or green}) = \frac{1}{2} \)
   - \( P(\text{not red}) = \)
Day 4

As a class, create area models for the combined probability of:

Two coins being flipped and getting two “heads” results. 
What fraction is this?
\[
\frac{1}{4}
\]

Two coins being flipped and getting one “head” and one “tail”. 
What fraction is this?
\[
\frac{1}{2}
\]

As a group, make area models for the following events. Answer the questions that follow.

Draw the area model for drawing cubes from a bag with one red, one blue, and one yellow and flipping a coin (same as yesterday’s first experiment).

\[
P(\text{red and heads}) = \frac{1}{6}
\]

\[
P(\text{blue and tails}) = \frac{1}{6}
\]

How do these results compare to the class’s experiment yesterday?
I did not use any text books, journals or lessons from the internet in this unit. I used my knowledge of various probability situations used to teach students probability as well as the information I acquired in attending I2T2.

Some of the images I did find online.

Dice are from: wilderdom.com/images/dice.gif

Penny is from: http://www.answers.com/topic/1959-penny-front-back-jpg

Probability bag is from:

I use Cabri Geometry and Microsoft paint to create the spinners.