What do perimeters and areas measure?

I²T² PROJECT
MELISSA AMES

Grade level:
- Ninth Grade

Time Span:
- Five day unit plan

Tools:
- Geometer’s Sketchpad
- Geoboards & Rubberbands
- Pentominoes
- ETA Cuisenaire Alphashapes
- Cardboard cutouts of quadrilaterals
Unit Objectives

Students should be able to:
- calculate perimeters given appropriate lengths
- calculate side lengths of equilateral polygons given perimeters
- calculate areas of squares, rectangles, triangles, trapezoids, and parallelograms given appropriate side lengths
- calculate side lengths of equilateral polygons given areas
- differentiate between perimeter and area
- have a better understanding of the area formulas used (especially with triangles, trapezoids, and parallelograms)
- know when to apply area and perimeter formulas

Standards & Key Ideas

NCTM Content Standards:
- algebra
- geometry
- measurement

NCTM Process Standards:
- problem solving
- communication
- connections
- representation

New York State Standard 3 Key Ideas:
- 3A – use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions
- 3C – recognize and identify symmetry and transformations on figures
- 4A – represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs
- 5A – apply formulas to find measures such as length, area, volume, weight, time, and angle in real-world contexts
- 5B – choose and apply appropriate units and tools in measurement situations
- 5I – use geometric relationships in relevant measurement problems involving geometric concepts
Resources & Materials

Resources:

- Geometer’s Sketchpad Software
- I2T2 notes and worksheets

Materials:

**Day One**
- Notesheet created with Geometer’s Sketchpad
- Meter stick
- Geoboards and rubberbands

**Day Two**
- Notesheet created with Geometer’s Sketchpad
- Geoboards and rubberbands

**Day Three**
- Notesheet
- Pentominoes
- Graph paper

**Day Four**
- Notesheet created with Geometer’s Sketchpad
- ETA Cuisenaire Alphashapes

**Day Five**
- Notesheet created with Geometer’s Sketchpad
- Cardboard cutouts of trapezoids, triangles, & parallelograms
- Computer lab to use Geometer’s Sketchpad
Unit Overview

Day One
- Perimeter definition
- Perimeter formulas
- Equilateral polygon definition
- Perimeter calculations
- Geoboard activity regarding perimeters

Day Two
- Area definition
- Rectangular area formula
- Area calculations
- Geoboard activity regarding rectangular areas

Day Three
- Perimeter and area questionnaire
- Pentominoes activity for area and perimeter comparison

Day Four
- Right triangle area formula
- Triangular area formula
- Altitude definition
- Triangle area calculations
- Alphashapes activity regarding the relationship between rectangular areas and triangular areas

Day Five
- Visualization with cutouts of the trapezoid and parallelogram area formulas
- Trapezoid area formula
- Parallelogram area formula
- Trapezoid and parallelogram area calculations
- GSP activity – students create trapezoids and parallelograms
Day One- Perimeter Formulas

Objective:
Students will gain an understanding of perimeter and methods of calculating perimeter. They will learn that addition, not multiplication, is used when calculating perimeter. Students will begin to understand why perimeter measurements are useful, and they will be able to provide examples of situations requiring perimeter measurements. Students will also learn that different shapes can have the same perimeter.

Opening Activity:
To begin class, students will be given the “Day One notesheet” and broken into eight groups. Four of the groups will remain in my room, which we will call “room A”. Four of the groups will head to another teacher’s room, which we will call “room B”. (Before class, I will speak with another teacher who happens to have a free period at this time and ask permission for some of my students to come in and measure the perimeter of her room.) Each group will be given a meter stick and asked to measure the length of one wall in the classroom they are assigned to. Each group will measure a different wall to the nearest centimeter. Once the groups return to their seats, the measurements will be collected and students will write down the data on their notesheets. The perimeter will be calculated after the definition has been reviewed.

Notes:
As a class, we will go over the notesheet, filling in the blanks as we go along. I will not give answers but will require students to give their ideas as to what words will correctly fill in the blanks. When we reach the end of the sheet, I will ask them to go back to the beginning and calculate the two classroom perimeters.

Main Activity:
Geoboards and rubberbands will be handed out. Following my lead on the overhead, I will have them create a 2x2 square. I will ask them to calculate the perimeter, and one student will explain to me how he or she arrived at the correct answer of eight. I’ll make sure students understand how to find the perimeter on the geoboards. Once everyone is ready to go, I will have them create shapes with varying perimeters. The students will come to realize that varying shapes can be created with the same perimeters.
**Homework:**

At home, students need to measure the perimeter of their bedroom to the nearest inch. A drawing should be made of the room on graph paper. Each unit on the graph paper will represent one inch.

Below are some GSP quadrilaterals that I will show on the overhead or on the television screen so that students can follow along with me as we progress through the notesheet.
**Day One – Notesheet**

Name ___________________  Date ___________________

**Classroom Measurements**

<table>
<thead>
<tr>
<th>ROOM A DIMENSIONS</th>
<th>ROOM B DIMENSIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>perimeter =</td>
<td>perimeter =</td>
</tr>
</tbody>
</table>

**Perimeter Formulas**

***The perimeter of a polygon is the __________ of the lengths of its __________.***

***You use __________, not __________, when finding perimeters.***
The perimeter of quadrilateral $ABCD = AB + BC + CD + DA$. Find the perimeter of $ABCD$ with the following measurements:

$AB = 5.12\text{cm}$
$BC = 8.62\text{cm}$
$CD = 6.49\text{cm}$
$DA = 8.11\text{cm}$

Perimeter =

***To find the perimeter of an equilateral polygon, you use the formula $p = ns$.\n
$p =$ perimeter
$n =$ _________ of _________ in the polygon
$s =$ _________ of the sides in the polygon

***What is an equilateral polygon?***

Find the side lengths of the square $EFGH$ if the perimeter = 108 inches.

***Don’t forget your units!!!***
**Day One – Notesheet Key**

Name ___________________  Date ___________________

**Classroom Measurements**

<table>
<thead>
<tr>
<th>ROOM A DIMENSIONS</th>
<th>ROOM B DIMENSIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>perimeter</strong></td>
<td><strong>perimeter</strong></td>
</tr>
</tbody>
</table>

*answers will depend on the classroom*

**Perimeter Formulas**

***The perimeter of a polygon is the sum of the lengths of its sides.***

***You use addition, not multiplication, when finding perimeters.***

![Diagram of quadrilateral ABCD](image)

The **perimeter** of quadrilateral \(ABCD = AB + BC + CD + DA\).
Find the perimeter of $ABCD$ with the following measurements:

$AB = 5.12\text{cm}$
$BC = 8.62\text{cm}$
$CD = 6.49\text{cm}$
$DA = 8.11\text{cm}$

Perimeter $= 5.12\text{cm} + 8.62\text{cm} + 6.49\text{cm} + 8.11\text{cm} = 28.34\text{cm}$

***To find the perimeter of an equilateral polygon, you use the formula $p=ns$.

\[ p = \text{perimeter} \]
\[ n = \text{number of sides in the polygon} \]
\[ s = \text{length of the sides in the polygon} \]

***What is an equilateral polygon?

An equilateral polygon is an n-sided figure in which all sides have equal length.

Find the side lengths of the square $EFGH$ if the perimeter $= 108\text{ inches}$.

\[ p = ns \]
\[ (108\text{in}) = (4)(s) \]
\[ (108\text{in})/(4) = s \]
\[ s = 27\text{in} \]

***Don’t forget your units!!!
Day Two – FUNdamental Properties of Area

Objective:
Students will learn what area measures and when/why it is used. Given side lengths of rectangles and squares, students can calculate areas. Once areas have been found, students can then begin to compare areas. Students will learn that varying shapes can have the same area, and they will see that each shape can only have one, unique area.

Opening Activity:
To begin class, students will be given the “Day Two notesheet” and asked to take out their “Day One notesheet”. The students should meet with the groups they were in yesterday and together come up with an answer to this question: Which room probably has a larger area? After all groups have an answer, we will have a class discussion on which room has the greater area and why.

Notes:
As a class, we will go over the notesheet, filling in the blanks as we go along. I will not give answers but will require students to give their ideas as to what words will correctly fill in the blanks. When we reach the end of the sheet, I will ask them to go back to the beginning and calculate the two classroom areas. Which room was bigger?

Main Activity:
Geoboards and rubberbands will be handed out. Following my lead on the overhead, I will have them create a 3x4 square. I will ask them to calculate the area, and one student will explain how to me how he or she arrived at the correct answer of twelve. I’ll make sure students understand how to find the area on the geoboards by simply counting the unit squares. Once everyone is ready to go, I will have them create shapes with varying areas. Once they find a figure with a given area, can they alter the shape while maintaining the same area? The students will come to realize that varying shapes can be created with the same areas and that each shape has only one, unique area.
**Homework:**

Students will be asked to find the area of their bedrooms. If a bedroom contains “nooks and crannies”, that student will need to measure those areas separately and add all the areas up to find the total area of his or her bedroom.

*Below are some GSP quadrilaterals that I will show on the overhead or on the television screen so that students can follow along with me as we progress through the notesheet.*
**Day Two – Notesheet**

Name ____________________ Date ____________________

**Classroom Measurements**

<table>
<thead>
<tr>
<th>ROOM A DIMENSIONS</th>
<th>ROOM B DIMENSIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Area = ____________________ Area = ____________________

**FUNdamental Properties of Area**

***Area is the number of nonoverlapping __________ __________ that can fit into a region.

***___________ is measured in square units!!!
(3) **Congruence Property**

Congruent figures have the __________ __________.

(4) **Additive Property**

The area of the union of two nonoverlapping regions is the __________ of the __________ of the regions.

Find the area of the following figures with the given side lengths:

For the parallelogram with side lengths:

- $AB = 7\text{in}$
- $BC = 15\text{in}$
- $CD = 7\text{in}$
- $DA = 15\text{in}$

Area =

For the square with side lengths:

- $JK = 12\text{m}$
- $KL = 12\text{m}$

Area =
Find the side lengths of the following square with the given area:

The area of square $WXYZ = 49\text{in}^2$. 
**Day Two – Notesheet Key**

Name ______________________  Date _____________________

**Classroom Measurements**

<table>
<thead>
<tr>
<th>ROOM A DIMENSIONS</th>
<th>ROOM B DIMENSIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*answers will depend on the classroom*

<table>
<thead>
<tr>
<th>Area =</th>
<th>Area =</th>
</tr>
</thead>
</table>

**FUNdamental Properties of Area**

***Area is the number of nonoverlapping square units that can fit into a region.***

***Area is measured in square units!!!***

***Four Fundamental Properties of Area:***

1. **Uniqueness Property:**
   
   Every polygon has a unique area.

2. **Rectangle Formula:**
   
   The area $A$ of a rectangle with dimensions $l$ and $w$ is $lw$.
   
   $A = lw$
(3) **Congruence Property**

Congruent figures have the same area.

(4) **Additive Property**

The area of the union of two nonoverlapping regions is the sum of the areas of the regions.

Find the area of the following figures with the given side lengths:

![Diagram of a rectangle with labeled sides](image)

\[ AB = 7\text{in} \]
\[ BC = 15\text{in} \]
\[ CD = 7\text{in} \]
\[ DA = 15\text{in} \]

Area = \[ lw = (15\text{in})(7\text{in}) = 105\text{in}^2 \]

![Diagram of a square with labeled sides](image)

\[ JK = 12\text{m} \]
\[ KL = 12\text{m} \]

Area = \[ lw = (12\text{m})(12\text{m}) = 144\text{m}^2 \]
Find the side lengths of the following square with the given area:

\[ A = lw = l^2 \text{ or } w^2 \text{ since } l = w \]
\[ (49 \text{ in}^2) = l^2 \text{ or } w^2 \]
\[ 7 \text{ in} = l \text{ or } w \]

The side lengths measure 7 in.

The area of square \( WXYZ = 49 \text{ in}^2 \).
**Day Three – Comparison of Perimeter and Area**

**Objective:**
Students will visually see the differences between perimeter and area using Pentominoes. Students will notice that a large area does not always imply a large perimeter and vice versa.

**Opening Activity:**
As they enter the classroom, students will be given the “Day Three notesheet” and asked to complete it before class begins. Once everyone has completed it, we will go over the answers. I’ll make sure everyone understands that an area measures the ground covered by a closed region, while perimeter measures the boundary of that closed region.

**Main Activity:**
Students will be broken into five or six groups of two or three students. Each group will be given two or three Pentominoes and a few sheets of graph paper. They will then fit the Pentominoes together to create a solid figure. The figure does not need to be a square or a rectangle. It only needs to be solid. (By solid, I mean that there are no “gaps” within the figure.) Once they have made their shape, they will draw it on a piece of graph paper. They will calculate the area and perimeter and write these values beneath their drawing. This will be their “key”. On a separate sheet of paper, students will rewrite the area and perimeter measurements. Once all of the groups have written down their measurements, I will take each set of Pentominoes with the corresponding measurements and rotate them around the room. When a group receives new measurements and Pentominoes, they will try to reconstruct the shape with the given area and perimeter. If the group believes they have the right figure, they can draw it on their graph paper and label it with the appropriate area and perimeter. Each rotation will last approximately five or six minutes.
Closing Activity:
Each group will need to write a statement about what they learned as a result of this activity. What do they now know about perimeter and area that they did not know when they walked into the classroom? Are there any questions they still have regarding these topics? A group member will read aloud his or her group’s statement, and other students can comment. Did the students come to realize that although each Pentomino piece had an area of five, the perimeters all varied? If this is not mentioned in the statements, I will prod students to come to this realization.

Homework:
None!
Day Three – Notesheet

Name ___________________  Date ___________________

Perimeter vs. Area

Would you measure the perimeter or area to find the answer??

Answer the following questions by writing “perimeter” or “area” in the space provided.

(1) On Black Friday, how many shoppers were in a line that stretched around Wal-mart? _________

(2) How many shoppers was Wal-mart able to hold? _________

(3) How many toys can Wal-mart’s toy department hold? _________

(4) On Black Friday, how many security guards were needed to surround the inside of Wal-mart? _________
Day Three – Notesheet Key

Name ___________________ Date ___________________

Perimeter vs. Area

Would you measure the perimeter or area to find the answer??

Answer the following questions by writing “perimeter” or “area” in the space provided.

(1) On Black Friday, how many shoppers were in a line that stretched around Wal-mart? perimeter

(2) How many shoppers was Wal-mart able to hold? area

(3) How many toys can Wal-mart’s toy department hold? area

(4) On Black Friday, how many security guards were needed to surround the inside of Wal-mart? perimeter
**Day Four – Areas of Triangles**

**Objective:**
Students will learn why the area of a triangle is \( \frac{1}{2}lh \). They will see that a square (or rectangle) cut in half diagonally yields two triangles, hence, the \( \frac{1}{2} \) in the area equation. Students will be able to calculate areas of triangles given relevant side lengths, and they will be able to apply this formula to real-world situations.

**Opening Activity:**
While entering the classroom, students will be given the “Day Four notesheet” and asked to complete the task found inside the box on the notesheet. Once everyone has finished, a quick discussion will cover the essential feature of a right triangle: the ninety degree angle.

**Notes:**
As a class, we will go over the notesheet, filling in the blanks as we go along. I will not give answers but will require students to give their ideas as to what words will correctly fill in the blanks.

**Main Activity:**
Students will be partnered with another classmate and given 8 Alphashapes (particularly two of I, Q, V, and W). Shapes I and V are right triangles. Shapes Q and W are the corresponding squares. The students’ only directions will be to play around with the shapes. What are they noticing? Are there any similarities between the pieces? I will walk around to ask questions, prod them if they are stuck, and clear up any confusion surrounding the assignment. I want them to see that triangle I is half of square Q, two triangle V’s equal one square W, the height and base of triangle I are equal to the height and base of square Q.

**Closing Activity:**
Before leaving for the day, students will need to write up a summary regarding their findings. Each student needs to write up his own paragraph and hand it in before the bell rings. If students are struggling to reach these conclusions, I will be sure to provide multiple hints and suggestions, perhaps give them some questions to answer that might lead them to a conclusion.
**Homework:**

Students need to draw three different triangles (similar to triangles $ABC$, $DEF$, and $GHI$ in the notesheet) all with the same area. Students can choose the area, but they must create and draw three different triangles on graph paper.

Below are some GSP triangles that I will show on the overhead or on the television screen so that students can follow along with me as we progress through the notesheet.

![Diagram of triangle JKL with measurements: $mKJ = 5.09$ cm, $mKL = 3.46$ cm.]

![Diagram of triangle MNO with measurements: $mOP = 2.80$ cm, $mNM = 5.24$ cm.]

Areas of Triangles

***Right Triangle Area Formula

The ________ of a right triangle is _________ the product of the lengths of its legs. ________

NOTE: $A = \frac{1}{2}bh$ is the same as $A = \frac{1}{2}lw$.

Find the area of a right triangle with a base of 6 and a height of 12.

***Triangle Area Formula (This formula can be used for any triangle!)

The ________ of a triangle is _________ the product of a side (__________) and the altitude (__________) to that side.

_______

***What is an altitude?
Draw the altitude in the following three pictures.

Find the areas of the following triangles.

$\angle JKL$ is a right angle.

$OP$ is the altitude.
Areas of Triangles

***Right Triangle Area Formula

The area of a right triangle is half the product of the lengths of its legs. \( A = \frac{1}{2}bh \)

**NOTE:** \( A = \frac{1}{2}bh \) is the same as \( A = \frac{1}{2}lw \).

Find the area of a right triangle with a base of 6 and a height of 12.

\[ A = \_hb = \left(\frac{1}{2}\right)(12)(6) = 36 \]

***Triangle Area Formula (This formula can be used for any triangle!)

The area of a triangle is half the product of a side (base) and the altitude (height) to that side. \( A = \frac{1}{2}hb \)

***What is an altitude?

An altitude is the perpendicular segment from a vertex to the line containing the opposite side.
Draw the altitude in the following three pictures. Altitudes drawn from A to B, from D to the line extending from EF, and from G to HI.

Find the areas of the following triangles.

<JKL is a right angle.

\[ A = \frac{1}{2}bh \]

\[ A = \frac{1}{2}(3.46\text{cm})(5.09\text{cm}) \]

\[ A = 8.8057\text{cm}^2 \]

OP is the altitude.

\[ A = \frac{1}{2}bh \]

\[ A = \frac{1}{2}(2.80\text{cm})(5.24\text{cm}) \]

\[ A = 7.336\text{cm}^2 \]
Day Five – Areas of Trapezoids

Objective:
Students really struggle to remember the area formulas for trapezoids and parallelograms because the shapes of these two figures are not “normal”. With this lesson, students will be able to physically see the area formulas for these shapes using Alphashapes and Geometer’s Sketchpad. Students will be able to calculate areas of trapezoids and parallelograms given relevant side lengths and vice versa. Students will also know which formula to apply to specific shapes to find the area.

Opening Activity:
We will have class in the computer lab, and to begin, students will be given the “Day Five notesheet” and asked to complete the task in the box on their notesheet. (I will freeze their computers until they actually begin doing work on them.)

Main Activity:
When everyone has finished, I will ask students to watch what I am doing on the Elmo (visual presenter). With a cardboard cutout of a trapezoid and two triangles (the base of each triangle will correspond to the two bases of the trapezoid), I will demonstrate how the area formula for a trapezoid is derived. I will give students the height and length of the two bases of my trapezoid, and together we will calculate the areas of two triangles found within the trapezoid. Writing out the formulas for our two triangles and adding them together to find the area of our trapezoid, I will prod the students to alter the formula. Are there any common terms between the two areas? (such as 1/2 and the height) Can we pull those two terms out front? In the end, we will arrive at the formula for the area of a trapezoid. If students are confused, I may have a student, who understands what I have done, come up and try to explain it again. Perhaps this would work better.

A parallelogram is really just a trapezoid with equal bases. If we apply the trapezoid area formula to the parallelogram, we will arrive at the parallelogram area formula. I will also show students (again with cutouts) that if you remove triangle $FHK$ from one end of parallelogram $EFGH$ and move it to the other end, a rectangle is created. Students already know the area of a rectangle.
Notes:
As a class, we will go over the notesheet, filling in the blanks as we go along. I will not give answers but will require students to give their ideas as to what words will correctly fill in the blanks.

Closing Activity:
Students (who will already be familiar with GSP) will open Geometer’s Sketchpad and create a trapezoid (one set of parallel sides). Their task will be to turn that trapezoid into a parallelogram by creating a congruent trapezoid, rotating it $180^0$, and lining up one of the legs on each trapezoid. I will not give them these specific directions but will provide hints throughout the period to guide them in the right direction. I will tell them that they need to duplicate their trapezoid in order to create the parallelogram. Their trapezoids need to be labeled. Altitudes need to be constructed. Side lengths need to be calculated. When they believe they have created a correctly labeled parallelogram, I will give it a “YAY” or a “NAY”. If they receive a “YAY”, then they can print out their parallelogram and help others still working. They are NOT to do the work for the other students but to, like me, provide some sort of guidance. Everyone will create different figures. These parallelogram sheets will be used during the next class period to show another method for deriving the trapezoid area formula. Students are to write their names on their sheets and bring them to our next class.

Homework:
None!
Areas of Trapezoids

***On trapezoid $ABCD$, label the height, base$_1$, and base$_2$. You may want to use a pencil in case you need to re-label the figure during the period.

HINT: Did you draw an altitude?

***Trapezoid Area Formula

The _________ of a trapezoid equals _________ the product of its _________ and the sum of the lengths of its _________.

Find the area of trapezoid $LMNO$. 
***Parallelogram Area Formula

The area of a __________ is the product of __________ of its bases and the __________ to that base. __________

Find the area of parallelogram $RSTU$ if $RS = 13m$ and the height $= 6m$.

***Don’t forget your units!!
**Day Five – Notesheet Key**

Name ___________________ Date ___________________

**Areas of Trapezoids**

***On trapezoid $ABCD$, label the height, base$_1$, and base$_2$. You may want to use a pencil in case you need to re-label the figure during the period.

Base$_1$ could be $AB$ or $CD$. Base$_2$ could be the opposite. The height is the altitude.

![Diagram of trapezoid ABCD]

**HINT:** Did you draw an altitude?

***Trapezoid Area Formula***

The area of a trapezoid equals half the product of its altitude and the sum of the lengths of its bases. $A = \_h(b_1+b_2)$

Find the area of trapezoid $LMNO$.

**HINT:** Did you draw an altitude?

$A = \_h(b_1+b_2)$

$A = (\_)(2\text{cm})(4\text{cm}+7\text{cm})$

$A = (1\text{cm})(11\text{cm}) = 11\text{cm}^2$
**Parallelogram Area Formula**

The area of a parallelogram is the product of one of its bases and the altitude to that base. \( A = hb \)

Find the area of parallelogram \( RSTU \) if \( RS = 13 \text{m} \) and the height = 6m.

\[
A = hb = (13\text{m})(6\text{m}) = 78\text{m}^2
\]

***Don’t forget your units!!***