5-day Lesson Plan on Parabolas

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I2T2 Final Project
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Objective:

Students will be introduced to:
~ what a parabola is
~ the equation of a parabola
~ various terms that will later be used to describe a parabola.

Students will also be able to:
~ determine whether a parabola opens upward or downward
~ identify the axis of symmetry
~ graph a parabola on a graphing calculator

Introduction-

Quickly review a linear function, its equation and what it looks like when graphed. Then introduce a quadratic function and explain how this differs from a linear function.

Lesson-

Today we will complete the guided notes (first one is blank second one is filled in). This will introduce the topic of parabolas to the students and identify key terms that we will be using throughout this section.

After completing the guided notes, students will use their graphing calculator to complete the Investigating Parabolas Lab Worksheet
21-8 Graphs of Parabolas

The graph of a first-degree polynomial function (a linear function) is a _______________.

For example $y = 2x + 5$ is a straight line.

The graph of a second-degree polynomial function (________________________),

is not a straight line and requires a larger number of points to draw its graph.

The graph of a quadratic function of the form $y = ax^2 + bx + c$,

where $a$, $b$, and $c$ are real numbers and $a \neq 0$, is a ________________.

An easy method for graphing parabolas involves preparing a chart.

Of course, the graphing calculator can also be used.

Example:

Graph the parabola $y = x^2 - 4x$

on the interval from $x = -1$ to $x = 5$.

Plot the points generated in the table.

<table>
<thead>
<tr>
<th>x</th>
<th>$x^2 - 4x$</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
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<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Draw a smooth curve through the points.

The points where the graph crosses the x-axis

are called the _________________.

The parabola crosses the x-axis at _______ and _______.

The _________________ is a vertical line passing through the turning point of a parabola.

In this example the turning point is _______. The equation of the axis of symmetry is ____________.

**Parabolas are of the form:** $y = ax^2 + bx + c$

If $a$ is ____________, the parabola opens upward and has a minimum point.

The axis of symmetry is $x = (-b)/2a$

If $a$ is ____________, the parabola opens downward and has a maximum point.

The axis of symmetry is $x = (-b)/2a$. 
Using the Graphing Calculator to Investigate Parabolas

Lab Sheet - Investigating Parabolas

1) Graph the following equations using the graphing calculator.

2) Use the STANDARD window (ZOOM 6).

3) Answer the question associated with each problem.

1. $Y1 = x^2$
   $Y2 = x^2 + 2$
   $Y3 = x^2 + 4$
   What happens to the graph when a number is added to $x^2$?

2. $Y1 = x^2$
   $Y2 = x^2 - 5$
   $Y3 = x^2 - 2.5$
   What happens to the graph when a number is subtracted from $x^2$?

3. $Y1 = x^2$
   $Y2 = 2x^2$
   $Y3 = 6x^2$
   What happens to the graph when $x^2$ is multiplied by a number greater than 1?
4. \[ Y_1 = x^2 \]
   \[ Y_2 = 0.5x^2 \]
   \[ Y_3 = 0.2x^2 \]

What happens to the graph when \( x^2 \) is multiplied by a number between 0 and 1?

5. \[ Y_1 = x^2 \]
   \[ Y_2 = -x^2 \]
   \[ Y_3 = -2x^2 \]

(Extra caution to use the negation key and not the subtraction key)

What happens to the graph when the coefficient of \( x^2 \) is negative?

6. \[ Y_1 = 2x^2 + 3 \]
   \[ Y_2 = -2x^2 + 3 \]
   \[ Y_3 = x^2 \]

Compare these two graphs with the third. What observations can be made?

7. \[ Y_1 = 0.5x^2 - 2 \]
   \[ Y_2 = -0.5x^2 + 2 \]

Compare these two graphs. What observations can be made?
The graph of a first-degree polynomial function (a linear function) is a straight line. For example, \( y = 2x + 5 \) is a straight line.

The graph of a second-degree polynomial function (a quadratic function), is not a straight line and requires a larger number of points to draw its graph.

The graph of a quadratic function of the form \( y = ax^2 + bx + c \), where \( a \), \( b \) and \( c \) are real numbers and \( a \neq 0 \), is a parabola.

An easy method for graphing parabolas involves preparing a chart.
Of course, the graphing calculator can also be used.

Example:

Graph the parabola \( y = x^2 - 4x \)
on the interval from \( x = -1 \) to \( x = 5 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( x^2 - 4x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>((-1)^2 - 4(-1))</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>((0)^2 - 4(0))</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>((1)^2 - 4(1))</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>((2)^2 - 4(2))</td>
<td>-4</td>
</tr>
<tr>
<td>3</td>
<td>((3)^2 - 4(3))</td>
<td>-3</td>
</tr>
<tr>
<td>4</td>
<td>((4)^2 - 4(4))</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>((5)^2 - 4(5))</td>
<td>5</td>
</tr>
</tbody>
</table>

Plot the points generated in the table.
Draw a smooth curve through the points.

The points where the graph crosses the x-axis are called the roots.
The parabola crosses the x-axis at (0,0) and (4,0).

The axis of symmetry is a vertical line passing through the turning point of a parabola.

In this example the turning point is \( (2, -4) \). The equation of the axis of symmetry \( x = 2 \).

**Parabolas are of the form:** \( y = ax^2 + bx + c \)

If \( a \) is positive, the parabola opens upward and has a minimum point.
The axis of symmetry is \( x = (-b)/2a \)

If \( a \) is negative, the parabola opens downward and has a maximum point.
The axis of symmetry is \( x = (-b)/2a \).
<table>
<thead>
<tr>
<th></th>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>What happens to the graph when a number is added to $x^2$?</td>
<td>The graph is shifted upward since the $+c$ values were positive. The graph moved up to the corresponding value on the y-axis.</td>
</tr>
<tr>
<td>2</td>
<td>What happens to the graph when a number is subtracted from $x^2$?</td>
<td>The graph is shifted downward since the equation had $-c$ values. The graph moved down to the corresponding value on the y-axis.</td>
</tr>
<tr>
<td>3</td>
<td>What happens to the graph when $x^2$ is multiplied by a number greater than 1?</td>
<td>The span (&quot;width&quot;) of the graph is becoming more narrow or thinner. The graph is becoming steeper at a faster rate.</td>
</tr>
<tr>
<td>4</td>
<td>What happens to the graph when $x^2$ is multiplied by a number between 0 and 1?</td>
<td>The span (&quot;width&quot;) of the graph is becoming wider or fatter. The graph is becoming steeper at a slower rate.</td>
</tr>
<tr>
<td>5</td>
<td>What happens to the graph when the coefficient of $x^2$ is negative?</td>
<td>The graph opens downward. Negative $x^2$ makes a frown.</td>
</tr>
<tr>
<td>6</td>
<td>Compare these two graphs with the third. What observations can be made?</td>
<td>The shift of $+3$ moved both graphs vertically up 3 units. The negative coefficient makes the graph open downward.</td>
</tr>
<tr>
<td>7</td>
<td>Compare these two graphs. What observations can be made?</td>
<td>The $+2$ shifted the graph vertically up two units and the $-2$ shifted the graph vertically down two units. The negative coefficient makes the graph open downward.</td>
</tr>
</tbody>
</table>
Objective:

Students will be able to:
~ determine the equation of a parabola using the vertex form of a parabola

Materials:

Computer with the Green Globs and Graphing Equations Program on it

Introduction-

Quickly review what the students discovered while completing the exploring parabolas lab sheet.

Lesson-

Today we will learn how to write the equation of a graphed parabola using the vertex form of a parabola.

The vertex form of a parabola is:

\[ Y = a(x - h)^2 + k \]

Where: 
- \( y \) and \( x \) do not change
- \( a \) – makes the parabola narrower or wider
- \( h \) – is the x-coordinate of the vertex
- \( k \) – is the y-coordinate of the vertex

Students will explore writing the equation of a parabola while using the Green Globs and Graphing Equation program.

After opening the Green Globs and Graphing Equation program:
~ go to programs on the menu bar
~ select Linear and Quadratic graphs
~ go to Linear and Quadratic graphs on the menu bar
~ select parabolas.

I suggest to start with easy problems and work up to harder problems.

Explain to students that a parabola will come up on the screen, they are to guess what the equation of the parabola might be by entering the vertex form of the parabola in the space.

Once they enter and equation – hit the **GRAPH EQUATION** button.
If they cannot determine the equation after a number of tries, hit the **SEE ANSWER** button and the answer will appear.
Objective:

Students will explore parabolas using:
~ CBR
~ Graphing Calculator
~ a ball

Students will be able to:
~ Complete Activity 8: The Bouncing Ball from the Modeling Motion: High School Math Activities with a CBR Book
~ capture one parabola on their graphing calculator
~ write the equation of their parabola

Materials:

CBR
Graphing Calculator
Ball
Activity 8 worksheet: The Bouncing Ball

Introduction-

Quickly review what we have learned so far about graphing Parabolas

Lesson-

Today we will complete Activity 8: The Bouncing Ball.
This is a real life example of creating parabolas.
After completing each part of the activity, students will answer the questions associated with the activity
DAY 5 – PARABOLAS CONTINUED

Objective:

Students will
~ Complete Quiz on what they learned this week

~ After completing the quiz, student will play a novice game of Green Globs

Materials:

Quiz (sample quiz attached)

Computer with the Green Globs and Graphing Equations Program on it
1) Which equation represents the axis of symmetry of the graph of the equation \( y = 2x^2 + 7x - 5 \)?

   a) \( x = -\frac{5}{4} \)  
   b) \( x = \frac{5}{4} \)  
   c) \( x = \frac{7}{4} \)  
   d) \( x = -\frac{7}{4} \)

2) If \( x = 3 \) is the equation of the axis of symmetry of the graph \( y = x^2 - 6x + 10 \), what is the y-coordinate of the turning point?

3) On the following graph, draw the graph of the equation \( y = x^2 - 8x + 2 \), including all values of \( x \) in the interval \( 0 \leq x \leq 8 \). Draw the axis of symmetry.

4) What is the equation of a parabola that joins the following points (3,4), (4,-1), (5,-4), (6,-5), (7, -4), (8,-1), (9,4)?
5) Which equation represents the axis of symmetry of the graph of the equation \( y = 2x^2 + 7x - 5 \)?

\[
a) \quad x = -\frac{5}{4} \quad \quad \quad b) \quad x = \frac{5}{4} \quad \quad \quad c) \quad x = \frac{7}{4} \quad \quad \quad d) \quad x = -\frac{7}{4}
\]

6) If \( x = 3 \) is the equation of the axis of symmetry of the graph \( y = x^2 - 6x + 10 \), what is the \( y \)-coordinate of the turning point? \((3,1)\)

7) On the following graph, draw the graph of the equation \( y = x^2 - 8x + 2 \), including all values of \( x \) in the interval \( 0 \leq x \leq 8 \). Graph the axis of symmetry.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = x^2 - 8x + 2 )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>((0)^2 - 8(0) + 2)</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>((1)^2 - 8(1) + 2)</td>
<td>-5</td>
</tr>
<tr>
<td>2</td>
<td>((2)^2 - 8(2) + 2)</td>
<td>-10</td>
</tr>
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<td>((3)^2 - 8(3) + 2)</td>
<td>-13</td>
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<td>((6)^2 - 8(6) + 2)</td>
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</tr>
<tr>
<td>7</td>
<td>((7)^2 - 8(7) + 2)</td>
<td>-5</td>
</tr>
<tr>
<td>8</td>
<td>((8)^2 - 8(8) + 2)</td>
<td>2</td>
</tr>
</tbody>
</table>

8) What is the equation of a parabola that joins the following points \((3,4), (4,-1), (5,-4), (6,-5), (7,-4), (8,-1), (9,4)\)?

\[ Y = (x - 6)^2 + 5 \]

Or

\[ Y = x^2 - 12x + 41 \]