Pythagorean Theorem

Inquiry Based Unit Plan

By: Renee Carey

Grade: 8
Time: 5 days
Tools: Geoboards, Calculators, Computers (Geometer's Sketchpad), Overhead projector, Pythagorean squares and triangle manipulatives, string, centimeter rulers.
Objectives:
- Relate the area of a square to its side length
- Develop strategies for finding the distance between two points on a coordinate grid
- Understand and apply the Pythagorean Theorem
- Estimate the values of square roots of whole numbers

Standards:
- 7N15 Recognize and state the value of the square root of a perfect square
- 7N16 Determine the square root of a non-perfect square using a calculator
- 7N18 Identify the two consecutive whole numbers between which the square root of a non-perfect square whole number less than 225 lies (with and without use of a number line)
- 7G5 Identify the right angle, hypotenuse and legs of a right triangle
- 7G6 Explore the relationship between the lengths of the three sides of a right triangle to develop the Pythagorean Theorem
- 7G8 Use the Pythagorean Theorem to determine the unknown length of a side of a right triangle
- 7G9 Determine whether a given triangle is a right triangle by applying the Pythagorean Theorem and using a calculator
- 8R6 Use representations to explore problem situations
- 8R8 Use representation as a tool for exploring and understanding mathematical ideas

Materials:
- Connected Mathematics 2, Looking for Pythagoras: The Pythagorean Theorem
- Geoboards and rubber bands
- Overhead projector and markers
- Scientific or graphing calculators
- Computers, Geometer's Sketchpad program
- Pythagorean theorem models: triangle with three squares off sides
- Dot Paper
Overview of Unit:

<table>
<thead>
<tr>
<th>Day</th>
<th>Description of Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>In this lesson, students begin by finding areas of figures on a dot grid, using a geoboard. They will begin to see that for some figures, it is easy to find areas of the component parts and break the figure into squares, rectangles, or triangles and calculate the individual areas. For others it may be easier to find the area of a square or rectangle enclosing the figure and subtracting the outside figures.</td>
</tr>
<tr>
<td>2</td>
<td>In this problem, students draw squares of various sizes on 5 dot by 5 dot grids. Students will sketch the squares on paper and practice calculating the areas using the strategies from the previous day's lesson. Students will then use Geometer's Sketchpad to check their work with regards to correctly creating squares and correctly calculating the area. In the process, they begin to see how the area of a square relates to the length of its sides.</td>
</tr>
<tr>
<td>3</td>
<td>In this problem, the concept of square root is introduced in the context of the relationship between the area of a square and the length of its sides. Students are also made aware of perfect and non-perfect squares. They will get practice estimating the values of non perfect squares without a calculating. The students will need to complete problems for homework which involve selecting the two whole numbers between which a non perfect square lies.</td>
</tr>
<tr>
<td>4</td>
<td>In this problem, students collect data about the areas of squares on the side of a right triangle. They use patterns in their data to conjecture that the sum of the areas of the two smaller squares equals the area of the largest square. With aide from Geometer's Sketchpad, students discover the Pythagorean Theorem and make conjectures that the Pythagorean Theorem only works with right triangles.</td>
</tr>
<tr>
<td>5</td>
<td>In this lesson, students use their knowledge of the Pythagorean Theorem and find the distance between two points that are not on the same horizontal or vertical line. They will work in partners to calculate the distance between three sets of points by creating a right triangle moving to the right and up.</td>
</tr>
</tbody>
</table>
Day 1: Finding Areas

Objective:
Develop strategies for finding areas of irregular figures on a grid.

Launch:
Create a simple figure on an overhead geoboard. Ask students how they could find the area of the figure. Let students share their ideas. There are two main strategies students tend to use: subdividing the figure and finding the areas of the pieces; and enclosing the figure in rectangle and subtracting the areas of the pieces outside the figure from the area of the rectangle. Students will explore the problem in pairs, drawing the figures on dot paper or constructing them on geoboards.

Explore:
In their work, students will review how to find areas of rectangles and triangles. Look for students who are actively applying this knowledge; they can share their strategies in the summary. Some students may need help applying the rule for the area of a triangle \( A = \frac{1}{2} b h \). Help them see that a triangle is half of a rectangle.

Summarize:
As students share answers and strategies, help them generalize their methods for finding area.
  o We can find areas of some figures by subdividing them and adding the areas of the smaller figures. For which figures in this problem is using this method easier?
  o We can find areas of some figures by enclosing them in a rectangle and subtracting the areas of the unwanted parts from the rectangle’s area. For which figures in this problem is using this method easier?

Student will need to be able to apply these methods for their future work in this unit, so make sure everyone can use at least one of them and explain why it works.

Homework:
Page 13 #15-25

Homework Answers:
15. 3 units squared 20. 5 units squared
16. 4 units squared 21. 5 units squared
17. 2 units squared 22. 2.5 units squared
18. 2 units squared 23. 1 unit squared
19. 3.5 units squared 24. 5.5 units squared
25. 8.5 units squared
Day 1: Finding Areas

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10.
Day 1: Finding Areas

1. 2 units squared
2. 1.5 units squared
3. 2 units squared
4. 4 units squared
5. 3 units squared
6. 4 units squared
7. 3.5 units squared
8. 6.5 units squared
9. 8.5 units squared
10. 8.5 units squared
Day 2: Looking for Squares

Objective:
Draw squares on 5 x 5 dot grids and find their areas. Students should begin to discover the relationship between square areas and the length of their side.

Launch:
Use a 5 dot by 5 dot grid and draw a unit square on the grid. Label the area as 1 square unit.

- I have drawn a square with an area of 1 square unit. Can someone come up and draw a square with a different area?

Explain that students will search for all the different size areas of squares that will fit on a 5 dot by 5 dot grid.

Explore:
Students will use Geometer’s Sketchpad to construct different size squares with different areas on 5 dot on 5 dot grids. Once they construct the square, they should calculate the area and record the square on their classwork.

If students have difficulty identifying tilted squares, display one on the overhead. Start with a square with area of 2.

Remind students to check the areas of each square they draw to verify that all the areas are different.

Summarize:
Ask students to share the various squares they found as you draw them on the overhead projector. Continue until all 8 squares have been identified. Discuss the strategies students used to find the squares.

- Which squares were easy to find? Why?
- Which squares were not easy to find? Why?
- How did you determine that your figure was a square?

Homework:
Page 23 #6

Homework Answers:
6. Area = 45 units squared
   Side length = 6.708 units
Day 2: Area and Side Length
Day 2: Area and Side Length

KEY
Day 3:

Objective:
Introduce the concept of square root. Understand square root geometrically, as the side length of a square with known area.

Launch:
Discuss the side length of the square with an area of 4 square units.
- What is the length of each side? How do you know your answer is correct?

Introduce the concept of square root.
- What number multiplied by itself is 4? We say the square root of 4 is 2.
- A square root of a number us a number that when squared, or multiplied by itself, equals the number. 2 is a square root of 4 because 2 \times 2 = 4.
- Is there another number you can multiply by itself to get 4?

Introduce square root notation. Write $\sqrt{4} = 2$ and $-\sqrt{4} = -2$ on the board.

Draw a square with area of 2 square units on a dot grid.
- What is the length of this square? Is it greater that one? It is greater that two? Is 1.5 a good estimate for $\sqrt{2}$?
- Can you find a better estimate?

Explore:
Ask students how they know the answers to questions A and B are correct. Ask them how they could check their answers.

Students should find the negative square roots of 1, 9, 16, and 25 as well. Check their work to see if they are using the square root symbol correctly.

Summarize:
Talk about the side length of the square with an area of 2 square units.
- How can you prove that the area of this square is 2 square units?
- What is the exact length of a side of this square?
- You estimated $\sqrt{2}$ by measuring a side of the square. What did you get? Is this the exact value of $\sqrt{2}$?
- You also found $\sqrt{2}$ by using the square root key on your calculator. What value did your calculator give? Enter this number into your calculator and square it. Is the result exactly equal to 2?

Emphasize that the results found by measuring and with a calculator are only approximate values for $\sqrt{2}$.

Ask students for decimal approximations for $\sqrt{5}$. As a class, use a calculator to square each approximation to check whether the result is 5.
### Homework Answers:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>11.</td>
<td>12</td>
<td>18.</td>
</tr>
<tr>
<td>12.</td>
<td>0.6</td>
<td>19.</td>
</tr>
<tr>
<td>13.</td>
<td>31</td>
<td>20.</td>
</tr>
<tr>
<td>14.</td>
<td>5 and 6</td>
<td>21.</td>
</tr>
<tr>
<td>15.</td>
<td>31 and 32</td>
<td>22.</td>
</tr>
<tr>
<td>16.</td>
<td>true</td>
<td>23.</td>
</tr>
<tr>
<td>17.</td>
<td>true</td>
<td>24.</td>
</tr>
</tbody>
</table>
PROBLEM 2.2: Square Roots

In this problem, use your calculator only when the question directs you to.

A.

1. Find the side lengths of squares with areas of 1, 9, 16, and 25 square units.

<table>
<thead>
<tr>
<th>Area</th>
<th>Side Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>

2. Find the values of

\[
\sqrt{1} = \qquad \quad
\sqrt{9} = \qquad \quad
\sqrt{16} = \qquad \quad
\sqrt{25} = \qquad \quad
\]

B.

1. What is the area of a square with a side length of 12 units?

What is the area of a square with a side length of 2.5 units?

2. Find the missing numbers:

\[
\sqrt{} = 12 \quad \quad \quad \quad \quad \sqrt{} = 2.5
\]

C. Refer to the square with an area of 2 square units you drew in Problem 2.1. The exact side length of this square is \(\sqrt{2}\) units.

1. Estimate \(\sqrt{2}\) by measuring a side of the square with a centimeter ruler.

2. Calculate the area of the square, using your measurement from part (1). Is the result exactly equal to 2?

3. Use the square root key on your calculator to estimate \(\sqrt{2}\).
4. How does your ruler estimate compare to your calculator estimate?
___________________________________________________________________
___________________________________________________________________

D.
1. Which two whole numbers is $\sqrt{5}$ between? Explain.
___________________________________________________________________
___________________________________________________________________

2. Which whole number is closer to $\sqrt{5}$? Explain.
___________________________________________________________________
___________________________________________________________________

3. Without using the square root key on your calculator, estimate the value of $\sqrt{5}$ to two decimal places.
___________________________________________________________________

E. Give the exact side length of each square you drew in Problem 2.1.

1. __________
2. __________
3. __________
4. __________
5. __________
6. __________
7. __________
8. __________
PROBLEM 2.2: Square Roots

In this problem, use your calculator only when the question directs you to.

F. 1. Find the side lengths of squares with areas of 1, 9, 16, and 25 square units.

<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
</tr>
<tr>
<td>25</td>
<td>5</td>
</tr>
</tbody>
</table>

2. Find the values of \( \sqrt{1} = ___+1,-1___ \)
\( \sqrt{9} = ___+3,-3___ \)
\( \sqrt{16} = ___+4,-4___ \)
\( \sqrt{25} = ___+5,-5___ \)

G. 1. What is the area of a square with a side length of 12 units?

\( _____ 144 \text{ units squared} \)

What is the area of a square with a side length of 2.5 units?

\( _____ 6.25 \text{ units squared} \)

2. Find the missing numbers:

\( \sqrt{144} = 12 \quad \sqrt{6.25} = 2.5 \)

H. Refer to the square with an area of 2 square units you drew in Problem 2.1. The exact side length of this square is \( \sqrt{2} \) units.

1. Estimate \( \sqrt{2} \) by measuring a side of the square with a centimeter ruler.

\( _____ \text{Approximately 1.4 cm} \)

2. Calculate the area of the square, using your measurement from part (1). Is the result exactly equal to 2?

\( 1.4 \times 1.4 = 1.96. \text{ No, the result is not exactly 2.} \)

3. Use the square root key on your calculator to estimate \( \sqrt{2} \).

\( 1.414 \)
4. How does your ruler estimate compare to your calculator estimate?

________The ruler estimate gives only the first few digits of the calculator estimate. In our case, the ruler estimate has only one decimal place. The calculator gives greater accuracy, but it is also just approximation.

I.

1. Which two whole numbers is \( \sqrt{5} \) between? Explain.

________2 and 3, 2 squared is 4 and 3 squared is 9. the value of the square root of 5 would be between 2 and 3.

2. Which whole number is closer to \( \sqrt{5} \)? Explain.

________2 would be closer to the square root of 5 because 5 is closer to 4 than it is to 9.

3. Without using the square root key on your calculator, estimate the value of \( \sqrt{5} \) to two decimal places.

________2.24

J. Give the exact side length of each square you drew in Problem 2.1.

1. ____1 unit_____
2. ____\( \sqrt{2} \) units_____
3. ____2 units_____
4. ____\( \sqrt{5} \) units_____
5. ____\( \sqrt{8} \) units_____
6. ____3 units_____
7. ____\( \sqrt{10} \) units_____
8. ____4 units_____

15
Day 4:

**Objective:**
Deduce the Pythagorean Theorem through exploration. Use the Pythagorean Theorem to find unknown side lengths of right triangles.

**Launch:**
Draw a tilted line segment on a dot grid at the board or overhead.
- How can we find the length of this line segment?
Using the original line segment as a hypotenuse, draw two line segments to make a right triangle.
- What kind of triangle have I drawn?
Explain that in a right triangle, the two sides that form the right angle are called the legs of the right triangle. The side opposite the right angle is called the hypotenuse.
- What are the lengths of the two legs of this triangle?
- What are the areas of the squares on the legs? What is the area of the square on the hypotenuse?

**Explore:**
Working in groups, the students will use the Geometer's Sketchpad program to adjust the lengths of the legs to find the value of the square on each side. After Sketchpad creates the diagram, students should record the right triangle and squares onto dot paper and complete the table.
Discuss the patterns in the table.
- What conjecture can you make about your results? This pattern is called the Pythagorean Theorem.
- Suppose a right triangle has legs of lengths a and b and a hypotenuse length c. Using these letters, can you state the Pythagorean Theorem in a general way?
- Do you think the Pythagorean Theorem will work for triangles that are not right triangles?
- Do you think the Pythagorean Theorem is true for all right triangles even if the sides are not whole numbers?

**Summarize:**
The theorem is true for all right triangles. Time permitting, complete ACE questions 13 and 14 as a class.
Choose one of the right triangles in the table, list the lengths of the three sides and ask students what the Pythagorean Theorem says about these lengths.

**Homework:**
Page 38 #1, 2, 5, and 6
Homework Answers:
1. a) $5^2 + 12^2 = 169 \text{ in}^2$
   b) 13 in.
2. $c^2 = 3^2 + 6^2 = 45$
   $c = 6.7 \text{ cm}$
5. $h^2 = 4^2 + 3^2 = 25$
   $h = 5 \text{ in}$
6. $k^2 = 3^2 + 8^2 = 73$
   $k = 8.5 \text{ cm}$

- red leg = 1 and blue leg = 1
- red leg = 1 and blue leg = 2
- red leg = 2 and blue leg = 2
- red leg = 1 and blue leg = 3
- red leg = 2 and blue leg = 3
- red leg = 3 and blue leg = 3
- red leg = 3 and blue leg = 4
- red leg = 4 and blue leg = 6
PROBLEM 3.1: The Pythagorean Theorem

A. Complete the table below. For each row of the table:

<table>
<thead>
<tr>
<th>Length of $\overline{AC}$</th>
<th>Length of $\overline{AB}$</th>
<th>Area red square</th>
<th>Area blue square</th>
<th>Area yellow square</th>
<th>Length of hypotenuse (in $\sqrt{_}$)</th>
<th>Length to the nearest hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>$\sqrt{1}$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>$\sqrt{4}$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>$\sqrt{4}$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td>$\sqrt{9}$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td>$\sqrt{9}$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td>$\sqrt{9}$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>$\sqrt{16}$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td>$\sqrt{36}$</td>
<td></td>
</tr>
</tbody>
</table>

B. Recall that a conjecture is your best guess about a mathematical relationship. It is usually a generalization about a pattern you think might be true, but that you do not yet know for sure is true.

For each triangle, look for a relationship among the areas of the three squares. Make a conjecture about the areas of the squares drawn on the sides of any right triangle.

_________________________________________________________________________
_________________________________________________________________________
_________________________________________________________________________

C. Draw a right triangle with side lengths that are different than those given in the table. Use your triangle to test your conjecture from Question B.
PROBLEM 3.1: The Pythagorean Theorem

D. Complete the table below. For each row of the table:

<table>
<thead>
<tr>
<th>Length of $\overline{AC}$</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>$\sqrt{2}$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>$\sqrt{5}$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>$\sqrt{8}$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>1</td>
<td>9</td>
<td>10</td>
<td>$\sqrt{10}$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>9</td>
<td>13</td>
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<td></td>
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<tr>
<td>3</td>
<td>3</td>
<td>9</td>
<td>9</td>
<td>18</td>
<td>$\sqrt{18}$</td>
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<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>$\sqrt{25}$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>16</td>
<td>36</td>
<td>52</td>
<td>$\sqrt{52}$</td>
<td></td>
</tr>
</tbody>
</table>

E. Recall that a conjecture is your best guess about a mathematical relationship. It is usually a generalization about a pattern you think might be true, but that you do not yet know for sure is true.

For each triangle, look for a relationship among the areas of the three squares. Make a conjecture about the areas of the squares drawn on the sides of any right triangle.

When the lengths of a triangle satisfy the Pythagorean Theorem, then the triangle is a right triangle.

F. Draw a right triangle with side lengths that are different than those given in the table. Use your triangle to test your conjecture from Question B.
Day 5: Finding Distances

**Objective:**
Use the Pythagorean Theorem to find the distance between two points on a grid. Relate areas of squares to the lengths of sides.

**Launch:**
Display Labsheet 3.3 and indicate points K and L.
- How can you find the distance between these two points?
- Draw segment KL and ask, How can we use the Pythagorean Theorem to find the length of this line segment? What right triangle has this hypotenuse?
- What are the lengths of the legs? How can you use this information to find the length of the hypotenuse? What is the distance between K and L?

**Explore:**
Students will work together in partners to calculate the distance between K and L, M and N and P and Q. They should use the strategy discussed in the Launch to create right triangles and use the Pythagorean Theorem to find the missing length.

**Summarize:**
Ask one or two students to describe their method. Students should be able to focus on the areas of the three squares on the sides of a right triangle and their relationship to the lengths of the sides.

**Homework:**
Page 42 #24

**Homework Answers:**
24. points A and B are 5 units apart. Points A and F are also 5 units apart.
\( m \overline{KL} = 5.39 \text{ cm} \)
\( m \overline{MN} = 4.99 \text{ cm} \)
\( m \overline{PQ} = 7.21 \text{ cm} \)