Multiplying and Factoring Polynomials with Algebra Tiles

A self-discovery approach to problem solving for algebra in terms of area.

A 5 Day Lesson Plan
For 9th Grade Algebra

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1^2T^2 2006
NCTM Standards Addressed: (Based on Chapter 6 of NCTM’s Principles and Standards for School Mathematics © 2000)
1: Numbers and Operations
2: Algebra
3: Geometry
4: Measurement
5: Problem Solving
6: Reasoning and Proof
7: Communication
8: Connections
9: Representation

New York State Standards Addressed:
Key Idea 1: Mathematical Reasoning
Key Idea 2: Number and Numeration
Key Idea 3: Operations
Key Idea 4: Modeling and Multiple Representation
Key Idea 5: Measurement
Key Idea 7: Patterns and Functions
Objectives
When the students have completed this unit they should be able to:

1. Use physical models to perform operations with polynomials. (NYS 8.A.5)
2. Multiply and divide monomials (NYS 8.A.6)
3. Add and subtract polynomials with integer coefficients. (NYS 8.A.7)
4. Multiply a binomial by a monomial or a binomial (NYS 8.A.8)
5. Factor a trinomial in the form ax^2 + bx + c; a=1. (NYS 8.A.11.)

Materials Required
1. An overhead set of Algebra tiles with an overhead projector.
2. Student sets of algebra tiles so that each student has a set
3. In the absence of sufficient student sets, a substitute for the tiles can be made from colored paper on an accu-cut machine, or even from colored construction paper that the students can cut by hand in advance of the activity. Three colors and sizes should be used: green denoting positive, red denoting negative, and blue representing +X^2.

This represents +1

This represents -1

This represents +X

This represents -X
This represents $+x^2$

This represents $-x^2$

If you are making these from construction paper, cut at least 10 of the + and − X’s and $X^2$s. Make 30 or more of the red and green 1’s. This is per each student set.
Representing $5 \times 4 = 20$ will take 29 of the greens 1’s as will be shown later.
4. It is also helpful to have a $8\frac{1}{2} \times 11$ paper for students, and an overhead transparency lined as below. When learning to use the tiles, this will separate the factors from the product.
The activities below will coordinate with Glencoe Algebra 1 2005 Edition. They are specially designed to go with the first two parts of Unit 3 “Polynomials and Nonlinear functions”

8-1 Multiplying Monomials
8-4 Adding and Subtracting Polynomials
8-5 Adding and Subtracting Polynomials
8-6 Multiplying a Polynomial by a Monomial
8-7 Multiplying Polynomials
8-8 Special Products
9-2 Factoring using the Distributive Property
9-3 Factoring Trinomials $x^2 + bx + c$
9-4 Factoring Trinomials $ax^2 + bx + c$

Additional activities can be easily created to go with
8-2 Dividing Monomials
9-5 Factoring Differences of Squares
9-6 Perfect Squares and factoring

At present, I do not see the use of Algebra Tiles with
8-3 Scientific Notation
9-1 Factors and Greatest Common Factor

or with the last part of unit 3, Chapter 10, which is the graphing of quadratic functions.

Put another way, Algebra Tiles can help a student understand the representation of $x^2 + bx + c$. However, this is not the same as $f(x) = x^2 + bx + c$. In addition to working with Algebra Tiles, the student must be able to transfer the concept of factoring to placing coordinates, specifically x- intercepts, on the Cartesian plane.
**Brief Overview by Day**

Day 1
Introduce the students to the Algebra Tiles. Some students will have seen them before and some will have not. Using the overhead and the translucent tiles, show them what each tile represents, introduce the concept of a “zero pair” and then use the tiles to model addition and subtraction.

Day 2
Use the Algebra Tiles to model multiplication of the ones and the addition and subtraction of polynomials.

Day 3
Use Algebra Tiles to model the multiplication of a binomial by a monomial.

Day 4
Use the Algebra Tiles to model the multiplication of a binomial by a binomial.

Day 5
Use the Algebra Tiles to factor trinomials.

Day 1 Adding and understanding the tiles.
Although addition seems axiomatic, many students count and recount the tiles to see that 3 and 4 does indeed equal 7. This activity is repeated several times with different numbers to customize the students to recognize that 3 and 4 are not just numerals, but numbers that represent objects.

Subtraction of the greens is also introduced, so that the answer is green (positive).
Many 9th grade students have difficulty with negative numbers, both with adding and subtracting negatives, as well as the concept of a negative. So we introduce the concept of a “zero pair”

\[ \text{+} \ \ \begin{array}{ccc} & & \end{array} \begin{array}{ccc} & & \end{array} = 0 \]

Positive and Negative 1. Usually we reserve the term \textit{canceling} for division. Since these cancel each other out, these zero pairs can be added to or subtracted from this type of problem.

In the above problem we can find 3 zero pairs we can take out leaving

\[ \begin{array}{ccc} & & \end{array} \begin{array}{ccc} & & \end{array} \]

as our answer.
The rule is “When you are subtracting, change the color and add”.
Now we have

Again remove the zero pairs and -4 is left.

Repeating the exercise with subtracting a negative reinforces the system of working with the tiles, and the concepts of adding and subtracting.

Change the lower row to green and add. Remove the zero pairs and you have + 2.
Objectives: After the lesson the students should be able to represent integers with the algebra tiles and be able to add or subtract integers with and without the algebra tiles. The students should have a comprehension of how subtracting a positive equals adding a negative, and how subtracting a negative equals adding a positive.

If the students are catching on quickly, you can continue the operations with the rectangle for X and the square for X². And show how the rules continue to hold true for adding and subtracting X’s.

Day 2 Multiplication of the ones and adding and subtracting polynomials.
reinforce the prior day’s learning by adding and subtracting with all the shapes.

\[
\begin{array}{c}
\text{+ } \\
\end{array}
\]

The above addition problem results in \(2x^2 + 4x - 5\)

As we move on to multiplication it is helpful to use the lines to separate the factors from the product.
Each one times one produces a 1.
In effect we simplify 3 x 4, to 1 x 1 repeated 12 times. As you can see this takes 19 tiles, so it is a good idea to have quite a few for the students’ use and to keep the numbers small.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Tiles required</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 1</td>
<td>4</td>
</tr>
<tr>
<td>2 x 2</td>
<td>8</td>
</tr>
<tr>
<td>3 x 3</td>
<td>15</td>
</tr>
<tr>
<td>4 x 4</td>
<td>24</td>
</tr>
<tr>
<td>5 x 5</td>
<td>35</td>
</tr>
<tr>
<td>6 x 6</td>
<td>48</td>
</tr>
<tr>
<td>7 x 7</td>
<td>63</td>
</tr>
<tr>
<td>8 x 8</td>
<td>80</td>
</tr>
</tbody>
</table>
To help students remember if the answer is positive or negative here is one idea to use. When the colors of the factors match, that’s good and you get a green answer. When the colors do not match, that’s not good and you get a red answer. Repeat several multiplications with different numbers and positive and negatives.

Now we move on to model a multiplication of a binomial by a monomial. Ask students what the factors are and what is the product.

\[ 3(2x + 2) = 6x + 6 \]

When the factors match the product is green.
\[-3(2x + 2) = -6x - 6\]
When the factors do not match the product is red.
\[-3(-2x - 2) = 6x + 6\]
When the factors match the product is green.
Use $-\text{ or } -$ for a negative and $+\text{ or } +$ for a positive

2 \( (3x-2) \)

What is the Shape of the product? _____________________

Homework Answer
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2(3x−2)

The Student should draw 6 of the positive x’s

+ 

and 4 of the negative one’s

−

What is the Shape of the product? The product when drawn correctly should form the shape of a rectangle.

On Day 4 we will multiply a binomial by a binomial. By now the students should comprehend why the product always forms a rectangle (length times width) and on special occasions it forms a square. This is key to understanding why x times x gives us $x^2$. 

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It is easiest to start with the positives.

\[(x + 1)(x + 2)\]

As with FOIL, our answer comes in 4 parts \(x^2 + x + 2x + 2\), simplified to the trinomial \(x^2 + 3x + 2\).

After several practices, you can explore the outcome of one or two negative terms.

\[(x-3) (x+2)\]
Our 4 terms are $x^2 - 3x + 2x - 6$ or by combining zero pairs, $x^2 - x - 6$

$(x-3)(x-2)$

As with FOIL we see we have $x^2 - 3x - 2x + 6$ or the trinomial $x^2 - 5x + 6$
We should also look at special products, such as the difference of 2 perfect squares.

\[(x-3) (x+3)\]
Here, we can push the tiles of the 4 products together to see that they do form a perfect square, and that the x terms are all zero pairs, or that they cancel each other. Our answer then, $x^2-9$, are also both perfect squares and can be expressed as “the difference between two perfect squares”.

Homework

Name____________________

Which of these are perfect squares?

**Answer only the questions that are perfect squares.**

1. $4+4 =$ ______________

2. $4 \times 4 =$ ______________

3. $x \times x =$ ______________

4. $2x + 2x =$ ______________

5. $3 \times 6 =$ ______________

6. $m \times m =$ ______________

7. $10 \times 10 =$ ______________
8. \(-5 \times 5 = \) ____________

9. \((x + 2)(x + 3) = \) ____________

10. \((x + 4)(x - 4) = \) ____________

Homework ANSWERS  Name __________________________

Which of these are perfect squares?

**Answer only the questions that are perfect squares.**

1. \(4 + 4 = \) ______________

2. \(4 \times 4 = ___16___ or \(4^2\)________

3. \(x \times x = ___x^2___\) ____________

4. \(2x + 2x = \) ______________

5. \(3 \times 6 = \) ______________

6. \(m \times m = ___m^2___\) ____________

7. \(10 \times 10 = ___100___ or ___10^2___\) ____________

8. \(-5 \times 5 = ___-25___\) ____________

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9. \((x+ 2)(x + 3) = \) ___________

10. \((x + 4)(x - 4) = \) ___\(x^2 - 16___\)

Day 5 Factoring
It is easiest to begin factoring by giving one of the factors and again working with the positive first.
What must we place on top to give us product shown when using the factor shown on the left?

Practice and explore negative factors.

\((-x + 2)(\text{some factor}) = -x^2 - 3x + 2x + 6\)
(x + 2) (some factor) = \( x^2 - 3x + 2x - 6 \)
What factor on top will give us this answer?
What color must the top factor be to give us a green answer?

Or to give us a red answer?

Are there situations with no answer? Why is that?
Homework
From the **Glencoe** textbook, page 493. Factoring Trinomials Numbers 17–27.

As many students have difficulty in factoring, this activity can continue into the following week, if you have time.
As the students learn to sketch trinomials, they can use those sketches to find the factors. This can help the student in a testing situation when they do not have access to the tiles.

Standards met:

A.PS.1 Use a variety of problem solving strategies to understand new mathematical content
A.PS.2 Recognize and understand equivalent representations of a problem situation or a mathematical concept

A.PS.5 Choose an effective approach to solve a problem from a variety of strategies (numeric, graphic, algebraic)
A.PS.6 Use a variety of strategies to extend solution methods to other problems
A.RP.1 Recognize that mathematical ideas can be supported by a variety of strategies
A.RP.6 Present correct mathematical arguments in a variety of forms
A.CM.2 Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs, Venn diagrams, and other diagrams
A.CM.3 Present organized mathematical ideas with the use of appropriate standard notations, including the use of symbols and other representations when sharing an idea in verbal and written form.
A.CM.4 Explain relationships among different representations of a problem
A.CM.13 Draw conclusions about mathematical ideas through decoding, comprehension, and interpretation of mathematical visuals, symbols, and technical writing
A.CN.1 Understand and make connections among multiple representations of the same mathematical idea
A.CN.2 Understand the corresponding procedures for similar problems or mathematical concepts
A.CN.3 Model situations mathematically, using representations to draw conclusions and formulate new situations
A.R.1 Use physical objects, diagrams, charts, tables, graphs, symbols, equations, and objects created using technology as representations of mathematical concepts
A.R.2 Recognize, compare, and use an array of representational forms
A.R.3 Use representation as a tool for exploring and understanding mathematical ideas
A.R.4 Select appropriate representations to solve problem situations
A.A.13 Add, subtract, and multiply monomials and polynomials

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A.A.14 Divide a polynomial by a monomial or binomial, where the quotient has no remainder
A.A.15 Find values of a variable for which an algebraic fraction is undefined
A.A.16 Simplify fractions with polynomials in the numerator and denominator by factoring both and renaming them to lowest terms
A.A.17 Add or subtract fractional expressions with monomial or like binomial denominators
A.A.18 Multiply and divide algebraic fractions and express the product or quotient in simplest form
A.A.19 Identify and factor the difference of two perfect squares
A.G.8 Find the roots of a parabolic function graphically Note: Only quadratic equations with integral solutions

NYS Algebra Standards March 2005
Investigate/Explore – Students will be given situations in which they will be asked to look for patterns or relationships between elements within the setting.
Discover – Students will make note of possible patterns and generalizations that result from investigation/exploration.
Conjecture – Students will make an overall statement, thought to be true, about the new discovery.
Reasoning – Students will engage in a process that leads to knowing something to be true or false.
Argument – Students will communicate, in verbal or written form, the reasoning process that leads to a conclusion. A valid argument is the end result of the conjecture/reasoning process.
Justify/Explain – Students will provide an argument for a mathematical conjecture. It may be an intuitive argument or a set of examples that support the conjecture. The argument may include, but is not limited to, a written paragraph, measurement using appropriate tools, the use of dynamic software, or a written proof.
Proof – Students will present a valid argument, expressed in written form, justified by axioms, definitions, and theorems.
Apply – Students will use a theorem or concept to solve an algebraic or numerical problem.