A World of
Probability & Statistics:
A look at probability and statistics as it applies to situations in the real world.

I2T2 2005
Lauren Boniface
Grade 7
5 Day Unit Plan
Standards and Objectives

NCTM Standards Addressed:
- Data Analysis and Probability
- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

New York State Standards
- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

Objectives:
- Students will be able to develop and put into action a plan to test their predictions of probability and statistics.
- Students will be able to create a tally chart based on the data they obtain.
- Students will be able to create a graph based on the data they have obtained.
- Students will be able to identify the population size versus the sample size.
- Students will be able to obtain their own data.
- Students will be able to organize their data in a table and graphically.
- Students will be able to determine the range and mean of their data.
- The students will be able to determine a sample space.
- Students will be able to determine the probability of an event using “and” and “or”
- Students will be able to create a stem leaf plot based on the data they have previously obtained.
- Students will be able to determine the mean, median, and mode using the stem leaf plot.
- Students will be able to show the difference between finding the probability of two events with replacement and without replacement.
Tools and Resources

- Pennies
- TI 84 calculator (w/ Probability Simulators)
- TinkerPlots
- Computers
- Overhead
- Overhead dice
- Overhead cards
- Over head Spinner
- Spinners
- Dice
- Deck of Cards
- Colored chips
- Colored Marbles

Some statistical activities were from Dr. Buckholtz MAT311 class.
Unit Overview

Day 1:
Students will be working on making predictions and testing their predictions based on probability. The students will use pennies to find the probability of flipping a head or flipping a tail. In the end, students will summarize that the probability of a head is $\frac{1}{2}$ and the probability of a tails is $\frac{1}{2}$.

Day 2:
Students will be obtaining their own samples from a population of pennies. The students will record their data and then interpret this data using charts and graphs. At the end of the lesson, the students will put their data into TinkerPlots.

Day 3:
Students will be using manipulatives such as playing cards and dice to determine the probability of different events. Students will be working in groups. Each group will be determining different probabilities. Students will be working with “and” and “or”. At the end of the lesson, the students will present their work to the class with overhead cards and dice.

Day 4:
Students will be creating a stem leaf plot based on data they have obtained in day 2’s lesson. The students will use the dates on the pennies to create this stem leaf plot. After the students have constructed the stem leaf plot, they will determine the mean, median, and mode.

Day 5:
The students will work with the idea of replacement and non-replacement to determine the probability of a variety of events. The students will test their predictions about whether or not replacement and non-replacement actually affects the probability of consecutive events.
Lesson 1: An introduction to Probability and Statistics

Time: 50 minutes

Objectives:
- Students will be able to develop and put into action a plan to test their predictions of probability and statistics.
- Students will be able to create a tally chart based on the data they obtain.
- Students will be able to create a graph based on the data they have obtained.
- Students will be able to identify the population size versus the sample size.

Materials:
- Pennies
- TI-84 calculators with Probability Simulators
- Chart paper for word wall
- Student worksheets for homework

Classroom Setup:
The students will be situated in their regular classroom setup. There will be six clusters of four desks. Students are of mixed abilities at these tables.

Assessment:
The students will be assessed formatively at the beginning of the lesson when the teacher listens to the students’ ideas about the probability of getting a heads or a tails. The teacher is listening to their predictions and the reasons behind these predictions. During the lesson, the teacher will be walking around to observe how the students are obtaining and recording their data. The worksheet for homework will be used to assess the knowledge that the students have attained.

Opening:
The teacher will begin. “Let’s look at a regular everyday penny. If you flipped this penny a bunch of times, do you think you would get more heads or more tails? Why do you think this? (Students answers will vary.) Out of one hundred flips, how many heads and how many tails do you think you would get? Why?” (Students answers may vary.) The teacher will briefly let the students make their predictions and explain their thoughts.

“We are going to begin a unit on probability and statistics today! Flipping this penny can be looked at through probability and statistics.

Activities:
1. “What do you think the probability of getting a head is? What do you think the probability of getting a tail is?” At this point, the teacher will look to see if the students’ ideas of the probability of getting heads or tails matches up with their ideas of the number of heads and tails received from flipping a coin one hundred times.
2. “Can you think of a way we can test our ideas?” The students should be able to decide on testing their predictions. The students should try flipping a coin one hundred times or more.
3. Teacher Instructions: “Part of statistics is keeping track of how many times a particular EVENT occurs. In this case we can get heads or tails. To keep track of the occurrences of each EVENT, we will keep a TALLY chart. Example: 

```
  H | T
  |   |
```

Adding up the TALLIES gives you the FREQUENCY, that is, how many times an event occurs.

4. I will then break the students up into groups of two. Each group will receive a penny. The groups must flip the penny fifty times. They will record the outcomes in a TALLY chart. As part of differentiation, some students will have a chance to use the TI-84 calculators and the Probability Simulator program. This will allow them to flip a coin fifty times using the calculator. The students will also be able to see the graph that would be form from the outcome of each event.

5. Once the students have completed their trials we will analyze their data. I will ask each group what their POPULATION SIZE is. (The total number of events in the trial.) This would be fifty for each group. I will then ask the students to count the number of tallies for each event. For example: Tails= 25 Heads= 25 or Tails= 23 Heads= 27. Students will then make a graph of this information. Example:

```
20
22
24
26
28
```

```
H  T
|   |
```

“Are the numbers close to each other?”

6. We will then put the whole class’s data together. “What would our POPULATION SIZE be now?” This should be six hundred if there are twelve groups doing fifty trials each. Since each group represents only a portion of the entire population size, we call each group’s data a SAMPLE.

7. The data should now represent close to half the outcomes being heads and the other half being tails. “In this population, what is the probability of getting heads? (x/600) What is the probability of getting tails? (600-x/ 600) The x will vary depending on the data collected by the students.

8. The class will look at the data, analyze it, and decide how close their predictions were to the actual outcome.

Closing:

The class will end by summarizing that the probability of flipping a head is one half and the probability of flipping a tail is also one half. The class will also look at their sample data versus the population data. The teacher will ask the students whether they feel it would be more valid to go by the population or the sample data. The students must support their answers. The class will then create a Probability and Statistics word wall with the terms we have learned in this lesson, including event, tally, population size, sample, and frequency. This will be hung on the wall throughout the entire unit. Throughout the unit, we will add to this word wall.
**Homework:**

Each student will be given a spinner to take home with them. The student must begin by writing a prediction for the probability of each number on the spinner. The students must then spin the spinner at least one hundred times. The students must make a chart that includes the number on the spinner, tallies for each number, and the frequency of each number. From this information, the student will make a frequency histogram. The student will also conclude on the probability of spinning a number on the spinner.

Sample of quality student work:

Prediction: I think that the probability of spinning a 2 on the spinner would be around $\frac{1}{4}$ or 25% because there are four spaces on the spinner and getting one would be 1 out of 4.

<table>
<thead>
<tr>
<th>Numbers</th>
<th>Tallies</th>
<th>Frequency</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[Tallies image]</td>
<td>24</td>
<td>24/100 is about $\frac{1}{4}$</td>
</tr>
<tr>
<td>2</td>
<td>[Tallies image]</td>
<td>26</td>
<td>26/100 is about $\frac{1}{4}$</td>
</tr>
<tr>
<td>3</td>
<td>[Tallies image]</td>
<td>27</td>
<td>27/100 is about $\frac{1}{4}$</td>
</tr>
<tr>
<td>4</td>
<td>[Tallies image]</td>
<td>23</td>
<td>23/100 is about $\frac{1}{4}$</td>
</tr>
</tbody>
</table>
Lesson 2: Collecting and Organizing Data

Time: 50 minutes

Objectives:
- Students will be able to obtain their own data.
- Students will be able to organize their data in a table and graphically.
- Students will be able to determine the range and mean of their data.

Materials:
- Pennies (U.S. and Canadian (George VI and Elizabeth II) ranging from 1934-1999)
- Computers
- Tinker Plots
- Student worksheets for homework

Classroom Setup:
The students will be arranged in their normal classroom setup at the beginning of the lesson. There are six clusters of four desks. The students are of mixed abilities at these clusters. Students will later be broken into groups of three. These groups may find a quiet place to work within the room.

Assessment:
The students will be formatively assessed at the beginning of the lesson based on their prior knowledge of maximum, minimum, range, and mean. During the lesson, the teacher will walk around to observe the students collecting and analyzing the data. The teacher is looking for how the students go about these activities. The students will complete the homework sheet to assess their knowledge after the lesson.

Opening:
“Today we are going to continue to work on collecting data and organizing it. When we have a collection of data, we want to look at certain features. These features include minimum, maximum, range, and mean.” We will have a brief discussion of what each other these terms refer to. Students should be relatively familiar with these terms from their previous math classes.

Activities:
1. Students will be broken into groups of three. Each group will have a box of pennies. The pennies will be from 1934-1999. There will be U.S. and Canadian pennies. The Canadian pennies will have heads of either Elizabeth II or George VI.
2. Students will be asked to pick a SAMPLE of twenty-five pennies. The students must record the year, the country (U.S. or Ca-Canadian), and the head (L-Lincoln, E-Elizabeth, G-George) of each of the twenty-five pennies they pick.
3. Using the data the students have collected, students will find the maximum, minimum, range, and mean for both the year and age of each penny. I will explain to the students how to find the age of the penny. 2005-date on penny = age of penny.
4. The students will then organize the data they have collected. The students will create CLASSES. They will divide the dates into groups, such as 1934-1940, 1941-1947, etc. We will discuss that it may be appropriate to have eight different classes spanning ten years, or it may be more appropriate to have more or less classes depending on the students’ data. The students will then create a tally chart for the number of pennies that fit into each class. Students will then total the tallies to find the frequency of outcomes for each class. The students should have a chart like the following with their own data filled in:

<table>
<thead>
<tr>
<th>CLASSES</th>
<th>TALLIES</th>
<th>FREQUENCY</th>
</tr>
</thead>
</table>

All members of the group must have this information copied down.

5. Students will then take their data and enter it into TinkerPlots to make a card, chart and plot of their data. There is an example of what the students should do as part of this file.

Closing:
After the students have finished putting their data on TinkerPlots, we will have a class discussion about the students’ data. We will also add the words minimum, maximum, range, mean, and class to our word wall.

Homework:
The students will receive a worksheet that asks questions that are dependent on the data they have collected, along with other questions.
Lesson 2: Data collection
Answer the following questions based on the data you collected today.

1. In what year do you think George VI died?

2. What fraction of your data are U.S. pennies?

3. What fraction of your data are Canadian pennies?

4. During which era did you collect the most pennies?

In the following problem, base your answers on the data below.

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<td>E</td>
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</tbody>
</table>

5. What is the:
   - Minimum date
   - Maximum date
   - Minimum age
   - Maximum age

6. What is the range of the date and age?

7. What is the mean date?

8. What is the mean age?
Lesson 2: Data collection

Answer the following questions based on the data you collected today.

1. In what year do you think George VI died?
   He died around 1937 based on data.

2. What fraction of your data are U.S. pennies?
   Answers vary.

3. What fraction of your data are Canadian pennies?
   Answers vary.

4. During which era did you collect the most pennies?
   Answers vary.

In the following problem, base your answers on the data below.

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</tbody>
</table>

5. What is the:
   Minimum date- 1936
   Maximum date- 1997
   Minimum age- 8 years
   Maximum age- 69 years

6. What is the range of the date and age?
   1997- 1936= 61
   69- 8= 61

7. What is the mean date?

8. What is the mean age?
   \[41.1 \text{ years} = \frac{69+64+62+51+50+49+23+22+16+8}{10}\]
Lesson 3: Probability: What are the chances?

Time: 50 minutes

Objectives:
- The students will be able to determine a sample space.
- Students will be able to determine the probability of an event using “and” and “or”

Materials:
- Overhead dice
- Over head playing cards
- Dice
- Playing cards
- Spinners
- Overhead

Classroom Setup:
The students will be arranged in their normal classroom setup at the beginning of the lesson. Later in the lesson, students will be working in groups of three. These groups may find a quiet place to work anywhere in the classroom.

Assessment:
The students will be formatively assessed at the beginning of the lesson based on their ideas of the probability of rolling a number on a die once and then twice in a row. To assess the students during the lesson, the teacher will be walking around the room to observe the students are working. The teacher will be looking to see how the students are finding the probabilities of each event. The students will also present their findings to the class, which the teacher will be observing. The students will be formatively assessed at the end of the lesson based on their work on the homework sheet.

Opening:
When playing Yatzee, in order to get Yatzee, all six dice need to be the same number. What do you think my chances are of getting all the same number on the first roll of all six die? At this point, the teacher would allow the students to make predictions or guesses about the probability of the situation. The teacher will be looking for students to support or explain the reasoning behind their predictions.

Activities:
1. The teacher will begin by having the students look at a regular six sided die. “How many sides are there? (6) How many of each number are there? (1) What do you think the probability of getting a six is? Why? (1/6 because there are six numbers and only one is a six.) Would the probability be the same or different for the other numbers on the die? (It would be the same for all the numbers on a regular six-sided die.)

2. The teacher would then move on to the example of rolling a die two times in a row. What would the possible outcomes be? How can we show these outcomes? (LISTS or TREE DIAGRAMS) The possible outcomes would be $S = \{1, 1, 2, 1, 2, 1\}$. 
These outcomes are called the SAMPLE SPACE. There are thirty-six possible outcomes in the SAMPLE SPACE. The teacher will also draw a tree diagram of the outcomes for the students. “Does the first roll of the die affect the second role?” (no)

3. We will then review the ideas of “or” \( \cup \) and “and” \( \cap \).

4. Students will then be broken into groups of three. Each group will be given a deck of cards, dice, and spinners. Each group will have to figure out the number of possible outcomes for each event and the probability of each event. Each group will have to find a variety of different probabilities. The students will be instructed to not include the jokers in the deck.

Closing:

After all groups have found the probability of each event, they will use the overhead manipulatives to show the other students what they have found.

**Homework:** Students will complete a worksheet.
Lesson 3: What are the chances?

Using what we talked about in today’s lesson, find the following probabilities.

Josie has a standard deck of cards. She takes out the jokers and any instruction cards. Josie picks one card. What is the probability that Josie picks

1. A number less than four and a heart?
2. A 5 or a spade?
3. A face card and a red card?
4. A black card and a red card?
5. A black card or a red card?
6. A number greater than 3 or a club?
Lesson 3: What are the chances?

Using what we talked about in today’s lesson, find the following probabilities.

Josie has a standard deck of cards. She takes out the jokers and any instruction cards. Josie picks one card. What is the probability that Josie picks

1. A number less than four and a heart?
   \[ P(\text{number} < 4 \text{ and a heart}) = \frac{3}{52} \]

2. A 5 or a spade?
   \[ P(5 \text{ or spade}) = \frac{16}{52} \]

3. A face card and a red card?
   \[ P(\text{face card and red card}) = \frac{6}{52} \]

4. A black card and a red card?
   \[ P(\text{black and red}) = 0 \]

5. A black or a red?
   \[ P(\text{black or red}) = 1 \]

6. A number greater than 3 and club?
   \[ P(\text{number} > 3 \text{ or club}) = \frac{43}{52} \]
Lesson 4: Stem Leaf Plots

Time: 50 minutes

Objectives:
- Students will be able to create a stem leaf plot based on the data they have previously obtained.
- Students will be able to determine the mean, median, and mode using the stem leaf plot.

Materials:
- Previous data collected on day 2’s lesson.
- TinkerPlots

Classroom Setup:
Students will be allowed to work in groups, at their desks or at the computers.

Assessment:
During the lesson, the teacher will walk around to observe the students creating a stem leaf plot. The teacher will be looking for how the students set up the stem leaf plot and how they organize their data. The teacher will also look for how the students use the chart to determine the mean, median, and mode. For homework, the students will receive a worksheet to complete. This will assess the students overall knowledge of the days activities.

Opening:
The teacher will begin by explaining to the students that there are a variety of ways to display data. There are a variety of graphs. One in particular, is the stem leaf plot. The teacher will then instruct the students that they will be using the data they previously obtained in lesson 2.

Activities:
1. The teacher will demonstrate how to construct a stem leaf plot using simple data. There will also be a brief review of how to find the median and mode of data. This should be a review of what the students have learned in previous grades.
2. Students will be placed back into the groups they were in on day two’s lessons. The students will go to TinkerPlots to retrieve the data they collected with the pennies.
3. The students will create a stem leaf plot based on the dates of the pennies they collected.
4. The students will use their stem leaf plots to find the mean, median, and mode of their data.

Closing:
Students may experiment with TinkerPlots to create a stem leaf plot. The teacher will go over the information with each group individually.

Homework: Students will complete a worksheet.
Base your answers to the following questions on the data given below.

Denise was doing a survey at the mall for a school project. As part of her survey, she took down each participant’s age. Denise surveyed 35 people. There ages are as follows: 16, 21, 32, 41, 50, 62, 71, 16, 23, 32, 41, 50, 65, 73, 17, 17, 79, 52, 55, 47, 48, 49, 28, 18, 19, 23, 25, 18, 33, 35, 18, 41, 33, 51, 42, 68, 26, 49, 47, 38, 35, 26, 51, 35, 67, 44, 44, 44, 54, 53, 26, 35, 47, 65, 39, 43, 45, 54, 58, 57.

Using the data above, create a stem leaf plot and find the mean, median, and mode of the ages.

Here is a quality sample student response.

<table>
<thead>
<tr>
<th></th>
<th>1 6 6 7 7 8 8 8 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1 3 3 5 6 6 6 8</td>
</tr>
<tr>
<td>3</td>
<td>2 2 3 3 5 5 5 5 8 9</td>
</tr>
<tr>
<td>4</td>
<td>1 1 1 2 3 4 4 4 5 7 7 8 9 9</td>
</tr>
<tr>
<td>5</td>
<td>0 0 1 1 2 3 4 4 5 7 8</td>
</tr>
<tr>
<td>6</td>
<td>2 5 5 7 8</td>
</tr>
<tr>
<td>7</td>
<td>1 3 9</td>
</tr>
</tbody>
</table>

Mean: 41.5 (The students may write out how they added all the numbers and then divided by 60, but they may do this work on the calculator.)

Median: 42.5

Mode: 35
Lesson 5: Replacing and Non-Replacing

**Time:** 50 minutes

**Objectives:**
- Students will be able to show the difference between finding the probability of two events with replacement and without replacement.

**Materials:**
- Marbles (red, blue, and yellow)
- Deck of cards

**Classroom Setup:**
The students will be situated in groups of two throughout the lesson

**Assessment:**
Students will be assessed at the beginning of the lesson, by their responses to the opening questions. The teacher is listening to hear the students' ideas and reasoning about replacement and non-replacement. During the lesson, the teacher will walk around the room to observe the students finding the probabilities of several events using non-replacement. The students will be given a worksheet for homework. This will assess their overall understanding of the day's lesson.

**Opening:**
The teacher will begin by asking the students the probability of picking out a blue marble from a sack that contains 3 blue marbles, 4 red marbles, and 5 yellow marbles. The probability would be 3/12. The teacher would then ask the students to decide if the probability of picking another blue would change if he or she put the marble back in the sack. Would it change if he or she did NOT put the marble back in? The teacher will allow the students to express their ideas. The students must support their predictions.

**Activities:**
1. The teacher will show the students that replacing and not replacing makes a difference in the probability of an event.
2. The teacher will have the students work with a partner to decide how the probability changes when the marble is not replaced.
3. Students will work with a partner to complete several probability questions relating to replacement and non-replacement. Students will be free to use the manipulatives such as the marbles.

**Homework:** The students will complete a worksheet based on the day's lesson.
Lesson 5: Replacement and Non-replacement

Answer the following questions based on the scenarios given.

1. Patrick and Sylvia are playing a game. The game includes a paper bag that contains five blue chips and two red chips. The object of the game is to pull at least one red chip from the bag. Each player gets two turns. Do Sylvia and Patrick have a better chance of getting a red chip on the second chance if the first chip is replaced or not replaced? Show all work to support your answer.

2. Sam wants a chocolate chip cookie. The cookie jar has five oatmeal cookies, three peanut butter cookies, and 4 chocolate chip cookies. She pulls out one cookie, it’s not what she wanted, so she put it back in the jar. In the meantime, her brother Roger ate one of the cookies. If her brother took a chocolate chip cookie, what is the probability that she will pick a chocolate chip cookie next? If Roger did not take a chocolate chip cookie, what is the probability that Sam will not pick a chocolate chip cookie next?
Lesson 5: Replacement and Non-replacement

Answer the following questions based on the scenarios given.

1. Patrick and Sylvia are playing a game. The game includes a paper bag that contains five blue chips and two red chips. The object of the game is to pull at least one red chip from the bag. Each player gets two turns. Do Sylvia and Patrick have a better chance of getting a red chip on the second chance if the first chip is replaced or not replaced? Show all work to support your answer.

   If the chip is replaced, the children have $\frac{2}{5}$ of a chance of picking a red chip.
   If the chip is not replaced, the children have a $\frac{2}{4}$ or $\frac{1}{2}$ the chance of picking a red chip. If the chip is not replaced, the students have a better chance of picking a red chip.

2. Sam wants a chocolate chip cookie. The cookie jar has five oatmeal cookies, three peanut butter cookies, and four chocolate chip cookies. She pulls out one cookie, it's not what she wanted, so she put it back in the jar. In the meantime, her brother Roger ate one of the cookies. If her brother took a chocolate chip cookie, what is the probability that she will pick a chocolate chip cookie next? If Roger did not take a chocolate chip cookie, what is the probability that Sam will not pick a chocolate chip cookie next?

   If Roger eats a chocolate chip cookie, the probability of Sam picking a chocolate chip cookie is $\frac{3}{11}$.
   If Roger does not eat a chocolate chip cookie, the probability of Sam not picking a chocolate chip cookie is $\frac{7}{11}$. 