Adding, Subtracting, Multiplying, and Factoring Polynomials Using Algebra Tiles

This Lesson is Designed for 8\textsuperscript{th} Grade

The Lesson will last 5 days and you will need:
A Class Set of Algebra Tiles,
Algebra Tiles for the Overhead,
An Overhead
Basic operations with polynomials.

Objectives: When the students have completed this unit they should be able to:

NYS Standards:
1. Use physical models to perform operations with polynomials. (NYS 8.A.5)
2. Multiply and divide monomials (NYS 8.A.6)
3. Add and subtract polynomials with integer coefficients. (NYS 8.A.7)
4. Multiply a binomial by a monomial or a binomial (NYS 8.A.8)

Factor a trinomial in the form $ax^2 + bx + c; a=1$. (NYS 8.A.11.)

NCTM Standards:
1. create and use representations to organize, record, and communicate mathematical ideas;
2. select, apply, and translate among mathematical representations to solve problems;
3. recognize and use connections among mathematical ideas;
4. understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
5. communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
6. model and solve contextualized problems using various representations, such as graphs, tables, and equations
An overview of the week.

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<td>Introduce the students to the algebra tiles, show them what each tile represents and then use the tiles to model addition.</td>
<td>Use the algebra tiles to model the subtract of polynomials</td>
<td>Use algebra tiles to model the multiplication of a binomial by a monomial</td>
<td>Use the algebra tiles to model the multiplication of a binomial by a binomial.</td>
<td>Use the algebra tiles to factor trinomials.</td>
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Objectives: After the lesson the students should be able to represent polynomials with the algebra tiles and be able to add polynomials with and without the algebra tiles.

Standards met:

- Use physical models to perform operations with polynomials. (NYS 8.A.5)
- Add and subtract polynomials with integer coefficients. (NYS 8.A.7)

The beginning of the lesson will be devoted to passing out and explaining the symbolism of the algebra tiles. The students must understand what each tile represents so I will explain that:

The blue square tile has lengths that measure x units long so its area is $x^2$. So, it represents $x^2$.

The green rectangle has lengths x units by 1 unit long so its area is $x$. So, it represents $x$.

The small yellow square tile has lengths 1 unit by 1 unit so its area is $1$. So, it represents 1.

Note, the red sides of the tiles represent negative values, my students already know this because we used the small squares to add and subtract integers. They also know about zero pairs from adding and subtracting integers.
• After I am done explaining the meaning of each tile I will have the students represent some different polynomials at their desk.

I will have the students represent the following polynomials with their algebra tiles:

1. \( x + 2 \)

\[
\begin{align*}
\text{ans.} & \quad \includegraphics[width=0.5\textwidth]{x+2.png}
\end{align*}
\]

2. \( x^2 + 2x + 3 \)

\[
\begin{align*}
\text{ans.} & \quad \includegraphics[width=0.5\textwidth]{x2+2x+3.png}
\end{align*}
\]

3. \( 2x^2 + 2 \)

\[
\begin{align*}
\text{Ans.} & \quad \includegraphics[width=0.5\textwidth]{2x2+2.png}
\end{align*}
\]

4. \( x^2 + 2x - 2 \)

\[
\begin{align*}
\text{ans.} & \quad \includegraphics[width=0.5\textwidth]{x2+2x-2.png}
\end{align*}
\]
Now, after I believe all of the students know how to represent a polynomial using the algebra tiles I will them try some addition problems using what they have just learned.

I will have the students simplify each of the following expression using the algebra tiles.

1. $(2x+2) + (3x + 1)$

   Ans. 
   ![Diagram of algebra tiles](image)

   (Now combine every thing that is the same together)

   So, $(2x+2) + (3x+1) = 5x + 3$

2. $(x^2+2x+2) + (x^2+2x+1)$

   Ans. 
   ![Diagram of algebra tiles](image)

   Combine everything that’s the same

   So, $(x^2+2x+2) + (x^2+2x+1) = 2x^2+4x+3$
3. \((2x^2+4x+1) + (x^2 -1x +1)\)

Ans. \[
\begin{array}{cccccccc}
\text{Blue} & \text{Blue} & \text{Green} & \text{Green} & \text{Green} & \text{Yellow} \\
\text{Blue} & \text{Red} & \text{Yellow}
\end{array}
\]

Now, combine everything that is the same and cancel out if you can.

So, \((2x^2+4x+1) + (x^2 -1x +1) = 3x^2+3x+2\)

- Now, I will start asking the students to think of a rule that we can come up with so that we don’t have to always use algebra tiles when we are adding polynomials. Hopefully they will notice that if you add the number in front of the variable with all of the of the other variables that are the same you will get the correct answer. After this rule I will tell them that the number in front of the variable is called the coefficient so our new rule will be to add the coefficients of the same terms or variables.
Day 1 Homework

For the following problems, draw a representation of each problem and then simplify the answer.

1. \((2x + 1) + (2x + 2)\)

Model: __________________________________________________________________________

Answer: ___________________________________________________________________

2. \((x^2 + 2x + 1) + (x^2 + x + 1)\)

Model: __________________________________________________________________________

Answer: ___________________________________________________________________

3. \((x^2 + 3x + 2) + (2x + 1)\)

Model: __________________________________________________________________________

Answer: ___________________________________________________________________

4. \((3x + 2) + (2x - 1)\)

Model: __________________________________________________________________________

Answer: ___________________________________________________________________
For the following problems, draw a representation of each problem and then simplify the answer.

5. \((2x + 1) + (2x +2)\)
   
   Model: 
   
   Answer: \(4x + 3\)
   
6. \((x^2 +2x+1) + (x^2+x+1)\)
   
   Model: 
   
   Answer: \(2x^2 +3x+2\)
   
7. \((x^2+3x+2) + (2x+1)\)
   
   Model: 
   
   Answer: \(x^2+5x+3\)
8. \((3x+2) + (2x+ -1)\)

Model: 

Answer: ___5x+3
Lesson 2, Day 2  Subtracting Polynomials

Objectives: At the end of this lesson the students should be able to subtract polynomials with and without the use of algebra tiles.

Standards Covered:
- Use physical models to perform operations with polynomials. (NYS 8.A.5)
- Add and subtract polynomials with integer coefficients. (NYS 8.A.7)

At the beginning of class I will choose 5 students to display their homework on the front board. We will discuss their answers as class and reestablish the rule we came up with yesterday about adding the coefficients of the like terms. We will then have a discussion about if they think the same rules will apply for subtraction.

I will ask the students to make a guess on the answer to the problem \((3x + 2) - (2x + 2)\) and try to support their answer using the algebra tiles, reminding the students that the red side is the negative side.

- We will now show the terms cancel each other to give us an answer.

\[(3x + 2) - (2x + 2)\]

Now, after canceling what terms are left?
Ans. Is \(x\)
So, \((3x + 2) – (2x + 2) = x\)

Now, I will have the students try these next questions on their own using the algebra tiles and having them thinking about a rule for subtraction polynomials.

1. \((3x^2 + 4x + 3) – (x^2 + 2x + 2)\)

So, \((3x^2 + 4x + 3) – (x^2 + 2x + 2) = x^2 + 2x + 1\)

2. \((x^2 + 2x - 2) – (3x^2 + 2x + 2)\)

So, \((x^2 + 2x - 2) – (3x^2 + 2x + 2) = -2x^2 - 4\)
3. $(2x^2-2x+4) - (2x^2+2x+4)$

So, $(2x^2-2x+4) - (2x^2+2x+4) = -4x$

- After these 4 examples we will come together as a class and see if we can find a rule that will apply when subtracting polynomials and hopefully the students will see that all you have to do is subtract the coefficients in front of the like terms.
Day 2 Homework

For homework, draw a representation of each problem then simplify each expression by using the model you drew.

1. \((2x^2+x+2) - (x^2+2x+4)\)

Model: ________________________________________________________________

Ans: ___________________

2. \((3x^2+x-2) - (x^2-2x+2)\)

Model: ________________________________________________________________

Ans: __________________

3. \((x^2-2x+5) - (3x^2+2x+1)\)

Model: ________________________________________________________________

Ans: __________________
Day 2 Homework Answers

For homework, draw a representation of each problem then simplify each expression by using the model you drew.

1. \((2x^2+x+2) - (x^2+2x+4)\)

Model: 

\[ \text{Ans: } -x^2 - x - 2 \]

2. \((3x^2+x-2) - (x^2-2x+2)\)

Model: 

\[ \text{Ans: } 2x^2 + 3x - 4 \]
3. \((x^2 - 2x + 5) - (3x^2 + 2x + 1)\)

Model: 

Ans: -2x^2 - 4x + 4
Day 3, Lesson 3

Objective: By the end of this the students should be able to multiply a binomial by a monomial.

Standards Covered:
• Use physical models to perform operations with polynomials.(NYS 8.A.5)
• Multiply a binomial by a monomial or a binomial (NYS 8.A.8)

• Today we will start the lesson by having 3 students come to the board and put their answers to the homework on the board. We will then go over the two rules we have made for adding and subtracting polynomials. Now we will introduce multiplying binomials by giving the students the worksheets below with the “L” bracket on it so that they can begin to model the multiplication.
• After the students have the sheets above we will work on the following problem together and then the students will try some examples with a partner.

Example: Multiply $3x \cdot (2x + 3)$

• First use your algebra tiles to model the two parts of the expression, one on the top of the “L” bracket and one the side. After you have modeled the two parts of the expression we will need to find what algebra tiles will we can place inside the bracket to form a rectangle with sides that are the same length as the two parts of our expression. This will give us our new expression.

So, $3x \cdot (2x + 3)$ is equal to $6x^2 + 9x$
• Now I will have the students try these problems with a partner.

1. Multiply $2*(2x+3)$

   So, the answer is $4x + 9$.

2. Multiply $4x*(x-2)$

   So, the answer is $4x^2 - 8x$.
So, \(4x \times (x - 2) = 4x^2 - 8x - 21\).

2. Multiply \(5 \times (x + 2)\).

So, \(5 \times (x + 2) = 5x + 10\).

So, \(5 \times (x + 2) = 5x + 10\).
3. Multiply $2x \cdot (x + 2)$

So, $2x \cdot (x + 2) = 2x^2 + 4x$.

- After we have done these examples I will ask the students if they have noticed any patterns or anything that we can say may be a rule to use when multiplying polynomials. Hopefully the students will notice the use of the distributive property and see how the coefficients are being multiplied together and see that the variables are being multiplied together, using the rule for multiplying numbers with the same base which is adding the exponents.
Day 3 Homework

Use the brackets below to represent the multiplication you are performing and then find the answer.

1. \(2 \times (x + 3)\)

Ans: __________________________

2. \(3x \times (2x - 1)\)

Ans: __________________________
3. \(-x \times (2x - 2)\)

Ans: ________________________________

4. \(4 \times (2x + 2)\)
Use the brackets below to represent the multiplication you are performing and then find the answer.

1. \(2 \times (x + 3)\)
   
   Ans: \(2x + 6\)

2. \(3x \times (2x - 1)\)
   
   Ans: \(4x^2 - 2x\)
3. \(-x \cdot (2x - 2)\)

\[\text{Ans: } -2x^2 + 4x\]

4. \(4 \cdot (2x + 2)\)
Objective: At the end of this lesson, the students will be able to use what they have learned yesterday about distributing to multiply two binomials together.

Standards covered:
- Use physical models to perform operations with polynomials.(NYS 8.A.5)
- Multiply a binomial by a monomial or a binomial (NYS 8.A.8)
- At the beginning of the class I will have four students come to the board and write their homework answers on the board. We will then discuss their answers and go back over the distributive law and multiplying variables and coefficients. We then see if we can use these same concepts to multiply two binomials together. I will start with this example and then have the students work in pairs to solve some more examples.

Example: \((2x + 2) * (x - 1)\)

Combine all like terms and cancel anything out that can be cancelled and your answer should be \(2x^2 - 2x + 2\)
With a partner and using the algebra try to multiply the following binomials. Also, while you are working try to think of a rule that applies for all of the examples.

1. \((3x + 2) \times (x + 2)\)

So, \((3x + 2) \times (x + 2) = 3x^2 + 8x + 4\)

2. \((x - 2) \times (2x - 3)\)

So, \((x - 2) \times (2x - 3) = 2x^2 - 7x + 6\)
3. \((x+2) \times (x-2)\)

So, \((x+2) \times (x-2) = x^2 - 4\)

4. \((-2x + 2) \times (2x + 3)\)
So, (-2x + 2) * (2x + 3) = -4x^2 -2x +6

- After the students have completed this activity we will discuss their outcomes and see if any of them have noticed any patterns that may lead to a rule for multiplying to binomials. Hopefully one of the students will notice that we are using the same procedure as yesterday but we are distributing twice through and then combing all of the like terms.
Day 4 Homework

Using the brackets below draw a model to represent the multiplication of the binomials and then use your model to multiply.

1. \((x + 3) \times (x + 3)\)

Ans: _______________________

2. \((2x + 1) \times (2x + 4)\)
3. $(2x - 2) * (-x - 3)$

4. $(-2x + 1) * (-3x + 1)$
Using the brackets below draw a model to represent the multiplication of the binomials and then use your model to multiply.

1. \((x + 3) \times (x + 3)\)

\[
\begin{array}{c}
\text{Green: } x^2 \\
\text{Blue: } 6x \\
\text{Yellow: } 9
\end{array}
\]

Ans: \(x^2 + 6x + 9\)

2. \((2x + 1) \times (2x + 4)\)

\[
\begin{array}{c}
\text{Green: } 4x^2 \\
\text{Blue: } 6x \\
\text{Yellow: } 4
\end{array}
\]

Ans: \(4x^2 + 6x + 4\)
3. \((2x - 2) \times (-x - 3)\)

Answer: 

\(-2x^2 - 4x + 6\)

4. \((-2x + 1) \times (-3x + 1)\)

Answer: 

\(-6x^2 + 7x - 1\)
Objectives: At the end of this lesson the students will be able find the factors of a trinomial.

Standards Met:
- Use physical models to perform operations with polynomials. (NYS 8.A.5)
- Factor a trinomial in the form $ax^2 + bx + c; a=1$. (NYS 8.A.11.)
- At the beginning of the lesson we will go over the home work and discuss the rules used to answer the questions from the night before. I will then place a trinomial on the board and see if the students can guess what two binomials were multiplied together to get the trinomial. Then I will form the square, using the algebra tiles, which the trinomials make and see if that is easier for the students to find the two factors.

Example: Find the factors of the trinomial $x^2 + 2x + 1$.
See if the students can guess what the two factors are and then form the square or rectangle on the “L” bracket and see they can find the factors now.

Now, after the students make their own square at their table, ask them if they can find the top and the side that what form this square. They should come up with this.

So, the two factors of $x^2 + 2x + 1$ are $(x+1)^2$. 
Factors of the trinomials below.

1. \(x^2 + 3x + 2\)

So, the factors of \(x^2 + 3x + 2\) are \((x+2)\) and \((x+1)\)
So, the factors of $x^2 + 3x$ are $x$ and $x + 3$. 

Ans: 

So, the factors of $x^2 + 3x$ are $x$ and $x + 3$.
3. \( x^2 - 5x + 6 \)

Ans:

So, the factors of \( x^2 - 5x + 6 \) are \( x - 2 \) and \( x - 3 \).
• Now, after these examples I will ask if they notice any patterns or common things happening in each example. After today they will probably only notice minor details and tomorrow they may be able to form some strategies to find factors of polynomials. But, today hopefully they will notice that the last term in the trinomial is the product of the last two terms in the factors, and that both of the factors will start with x. For homework I will have the students draw the trinomials and find their factors and the next day of class we will discuss different strategies each student used to find the factors of the trinomials.
Homework Day 5

Draw the rectangle or square for each trinomial and then use that to find the two factors of the trinomial.

1. \( x^2 + 4x + 4 \)

Ans: __________________________

2. \( x^2 - 2x + 1 \)

Ans: __________________________
3. $x^2 + 5x + 6$

Ans: ______________________________

4. $x^2 - x - 6$

Ans: ______________________________
Homework Day 5: Answers

Draw the rectangle or square for each trinomial and then use that to find the two factors of the trinomial.

1. \( x^2 + 4x + 4 \)
   
   Ans: \( (x+2) \times (x+2) \)

2. \( x^2 - 2x + 1 \)
   
   Ans: \( (x - 1) \times (x - 1) \)
3. \( x^2 + 5x + 6 \)

\[
\begin{array}{cccc}
\text{Green} & \text{Blue} & \text{Green} & \text{Yellow} \\
\text{Green} & \text{Blue} & \text{Green} & \text{Yellow} \\
\text{Green} & \text{Blue} & \text{Green} & \text{Yellow} \\
\text{Green} & \text{Blue} & \text{Green} & \text{Yellow} \\
\end{array}
\]

Ans: \( (x+3)(x+2) \)

4. \( x^2 - x - 6 \)

\[
\begin{array}{cccc}
\text{Green} & \text{Red} & \text{Red} & \text{Red} \\
\text{Green} & \text{Red} & \text{Red} & \text{Red} \\
\text{Green} & \text{Red} & \text{Red} & \text{Red} \\
\text{Green} & \text{Red} & \text{Red} & \text{Red} \\
\end{array}
\]

Ans: \( (x-3)(x+2) \)