We begin our investigation of finance with the general form of an exponential equation \( f(x) = ab^x \) where \( f(x) \) is the value of the growth function, \( a \) is the initial amount, \( b \) is the growth (or decay) factor, and \( x \) is the number of growth periods.

The half-life of a certain radioactive element is 40 days, if a scientist had 10 grams of the substance present initially how much would be present in 90 days?

Identify the variables
\[
\begin{align*}
a &= \text{the initial amount of radioactive element} \\
b &= \text{the half-life of the radioactive substance} \\
x &= \text{the number of times the substance grows (or decays)}
\end{align*}
\]

Substitute the values for the variables and evaluate the function for the given quantities.

\[ F(x) = \]

**Compound Interest**
The formula used to compute compound interest is given by the exponential equation

\[ A(t) = P \left(1 + \frac{r}{n}\right)^{nt} \]  
where \( A(t) \) is the amount in the account at time \( t \) years after the initial investment \( P \), at the annual percentage rate (APR) \( r \) compounded \( n \) times per year.

Find the amount in an account after 4 years of an initial investment of $1000 at 3% APR compounded yearly, monthly, weekly, and daily.

<table>
<thead>
<tr>
<th>Principle P</th>
<th>APR r</th>
<th># of compounds per year n</th>
<th>Time t</th>
<th>Amount A(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

So the more times the investment is compounded the _______ the return.

Greater / Lesser
Continuous Compound Interest
What would happen if the bank would compound the interest an infinite number of times.

Then the compound interest formula would be \( A(t) = \lim_{n \to \infty} \left( P + \frac{r}{n} \right)^{nt} \) and in one undergraduate class we proved that \( \lim_{n \to \infty} 1 + \frac{1}{n} = e \), so the formula for compounding interest infinitely many times per year is \( A(t) = Pe^{rt} \).

What would be the result of the investment from the last page if the interest were compounded continuously many times per year?

\[
A(t) = 1000 \cdot e^{0.03 \cdot 4} = \text{__________}.
\]

If you were to deposit $1000 on the day your child was born at 4.25% interest compounded infinitely, how much could you give them on their 18\text{th} birthday? , on their 25\text{th} birthday? , on their 40\text{th} birthday?

Annuities
An annuity is an investment made by depositing a set amount at a given interval over the course of time. The future value of an annuity is given by the formula

\[
F = R \cdot \frac{\left( 1 + \frac{r}{n} \right)^{nt} - 1}{\frac{r}{n}}
\]

F = future value of the annuity
R = the periodic payment
r = APR (as a decimal)
t = time in years
n = number of payments per year

Instead of investing $1000 on the day your child is born, how about investing $20 per week at the same rate of 4% over a period of 18 years? , over 25 years?

Which investment would you rather make to send your kid to college?
Mortgages and Loans

A loan is a type of annuity where the value of the annuity is decreased as you make the payments. The present value of an annuity formula is used to calculate the amount the annuity is worth after you have been making payments for \( t \) years. So the time is now how long ago you began investing.

\[
L = R \cdot \frac{1}{1 + \frac{r}{n}}^{nt}
\]

- \( L \) = present value of the annuity
- \( R \) = the periodic payment
- \( r \) = APR (as a decimal)
- \( t \) = time in years
- \( n \) = number of payments per year

If you solve this equation for \( R \), your have

\[
R = L \cdot \frac{1}{1 + \frac{r}{n}}^{nt}
\]

- \( L \) = loan amount
- \( R \) = the periodic payment
- \( r \) = APR (as a decimal)
- \( t \) = time in years
- \( n \) = number of payments per year

Be careful, if you put this into your calculator, you may need to put ( ) into the formula. This will work!!

\[
R = L \cdot \frac{r}{n} \left( \frac{1}{1 - \frac{1}{1 + \frac{r}{n}}} \right)^{-nt}
\]

One of the things that was transferred to your calculator was the LOAN program. The program computes the monthly payment of any loan or mortgage.

The Stock Market

The stock market has been the most sound and consistent long term investment device since the crash of 1929. In any 30 year span, the stock market has returned a minimum of 6% even in the worst of times. Some people believe that we are presently in a “bad” time but an analysis of the history of the Dow Jones Industrial average paints quite a different picture. Thanks to Paul Schoaff for helping me obtain the data. In just the past 30 years the stock market has returned an APR of over 9%. That would be an acceptable number to most long term investors. You can find the data on the web. From yahoo choose the finance link, then from this page choose the Dow link, from the dow page choose the historical prices link, finally choose the start and end date and have at it.

Taxes and Teachers

Anything you purchase while you are a teacher that you use in your classroom and home that is for the purpose of doing your job as a teacher is tax deductible. Period!!