$S_1 = \frac{n(n+1)}{2}$ $S_2 = \frac{n(n+1)(2n+1)}{2}$ $S_3 = \frac{n^2(n+1)^2}{4}$ Find S_4

$(x+1)^5 - 1 = 5(S_4) + 10(S_3) + 10(S_2) + 5(S_1) + x$

Why? See pgs. 62-67

$$12\left((x+1)^5 - 1\right) = \left(5(S_4) + 10\left(\frac{x^2(x+1)^2}{4}\right) + 10\left(\frac{x(x+1)(2x+1)}{6}\right) + \frac{5(x(x+1))}{2} + x\right) \times 12$$

Sub S_n ← multiply by 12 →

$$12x^5 + 60x^4 + 120x^3 + 120x^2 + 60x = 60(S_4) + 30x^4 + 100x^3 + 120x^2 + 62x$$

$$12x^5 + 30x^4 + 20x^3 + -2x = 60(S_4)$$

$$\frac{12x^5 + 30x^4 + 20x^3 + -2x}{60} = S_4$$ ← Acceptable Ans.

$$\frac{(2x)(x+1)(2x+1)(3x^2+3x-1)}{60} = S_4$$

$$\frac{x(x+1)}{2} \cdot \frac{(2x+1)}{3} \cdot \frac{3x^2+3x-1}{5} = S_4$$ For those of you looking for A pattern.

More detail on next sheet.

$5 \cdot 12 \cancel{60} (S_4) = 60(S_4)$

$$12\left[10 \frac{n^2(n+1)^2}{2}\right] = 30n^4 + 60n^3 + 30n^2$$

$$12\left[10\left(\frac{n(n+1)(2n+1)}{6}\right)\right] = 40n^3 + 60n^2 + 20n$$

$$12\left[5\left(\frac{n(n+1)}{2}\right)\right] = 30n^2 + 30n$$

$$12n = 12n$$

on other sheet where $n = x$

$$= 60(S_4) + 30x^4 + 100x^3 + 120x^2 + 62x$$