THE STRING GAME

Materials

Prepare a chart showing all of the attributes to be used in the game and prepare a card for each attribute to form a "face-down" set. Prepare game pieces, one

0r each of the numbers that are listed below. Put magnetic material or loops of masking tape on the pieces and the string cards so that they will adhere to the board.

Game Pieces

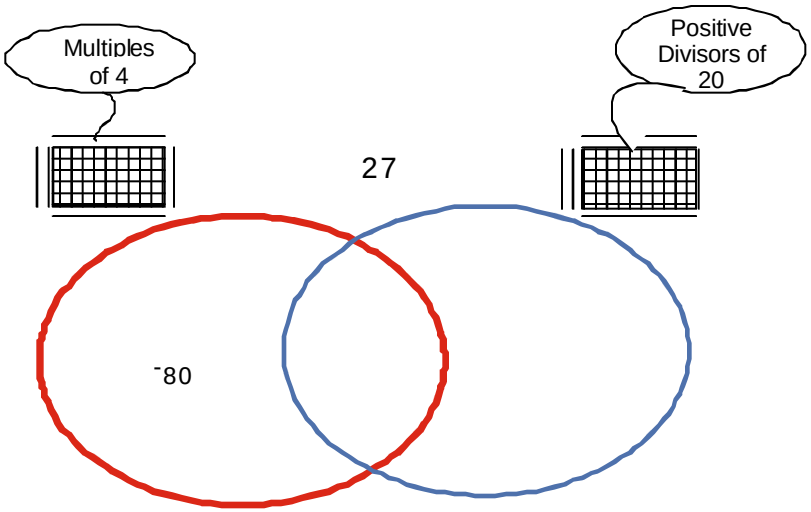
-100	-80	-55	-15	-10	-15
-1	0	1	2	3	4
5	6	7	8	9	10
12	18	20	24	27	40
45	50	60	99	100	105

Multiples of 2	Multiples of 3	Multiples of 4	Multiples of 5
Multiples of 10	Positive Divisors of 12	Positive Divisors of 18	Positive Divisors of 20
Positive Divisors of 24	Positive Divisors of 27	Larger than 50	Larger than -10
Smaller than 50	Smaller than -10	Odd Numbers	Positive Prime Numbers

Preparation

Draw two (or three) large overlapping strings on the board using two (or three) different colors. Next to each string attach a string card face-down. Place an even number (two or four) of the game pieces correctly in the string picture to provide a basis .(other than guessing) for plays early In the game. Divide the rest of the game, pieces evenly into two sets, one for each of the competing teams. The illustration below shows a sample set-up for a game; bubbles indicate the hidden labels.

Team A	Team B
40 1 12	2 99 6
45 -55	-100 -15
-10 24	-1 5
50 60 18	20 100
105 3 7	0 8 9
4	-15 10



Object of the game

Each team tries to place its game pieces correctly in the string picture according to the face-down string cards. The winning team is the one that identifies the facedown cards correctly after playing according to the rules.

Rules of the game

1) The students play the game in silence. Each student should have the opportunity to analyze the game alone. Infringement of this rule by anyone is penalized by the talker's team losing its next turn.

2) The teams alternate and the members take turns within each team. A player comes to the board and selects a. piece from the team's collection to place in one of the regions of the string picture.

3) You are the judge. If the piece is correctly placed, say "yes". The piece remains an the string picture and the player immediately has a second turn (no player may have more than two consecutive turns) . If the* piece is in correctly placed, say "no". The player returns the piece to the team's unplayed collection and play passes to the other team. As an aid in judging, prepare a crib-sheet showing the correct position of each game piece. if at any time you discover that you have made an error, say so immediately and rectify the mistake. Either move -an incorrectly placed piece whose position you had approved to its correct region or replace on the string picture a correctly placed piece that-you-had-rejected because you had disapproved its position...

4) When a team has correctly placed all of its pieces, the player who placed the last piece may thereupon attempt to Identify each of the string cards. There are two levels of acceptability dependent on the experience of the students In playing the game. In the first case, accept and verify a correct Identification. of a string and continue the game until the other, string or strings are identified. In the second stage, require all of the strings be identified correctly at once.

If a team has exhausted its stock, of game pieces and the strings have not been identified, then the team continues on its turn to attempt to identify the -strings, while the other team works to place its game pieces.

Analysis Sheets on next Page.

RED	BLUE		RED	BLUE
Multiples of 2	Multiples of 2		Multiples of 2	Multiples of 2
Multiples of 3	Multiples of 3		Multiples of 3	Multiples of 3
Multiples of 4	Multiples of 4		Multiples of 4	Multiples of 4
Multiples of 5	Multiples of 5		Multiples of 5	Multiples of 5
Multiples of 10	Multiples of 10		Multiples of 10	Multiples of 10
Odd Numbers	Odd Numbers		Odd Numbers	Odd Numbers
Positive Prime Numbers	Positive Prime Numbers		Positive Prime Numbers	Positive Prime Numbers
Larger than 50	Larger than 50		Larger than 50	Larger than 50
Smaller than 50	Smaller than 50		Smaller than 50	Smaller than 50
Larger than -10	Larger than -10		Larger than -10	Larger than -10
Smaller than -10	Smaller than -10		Smaller than -10	Smaller than -10
Positive Divisors of 12	Positive Divisors of 12		Positive Divisors of 12	Positive Divisors of 12
Positive Divisors of I8	Positive Divisors of I8		Positive Divisors of I8	Positive Divisors of I8
Positive Divisors of 20	Positive Divisors of 20		Positive Divisors of 20	Positive Divisors of 20
Positive Divisors of 24	Positive Divisors of 24		Positive Divisors of 24	Positive Divisors of 24
Positive Divisors of 27	Positive Divisors of 27		Positive Divisors of 27	Positive Divisors of 27

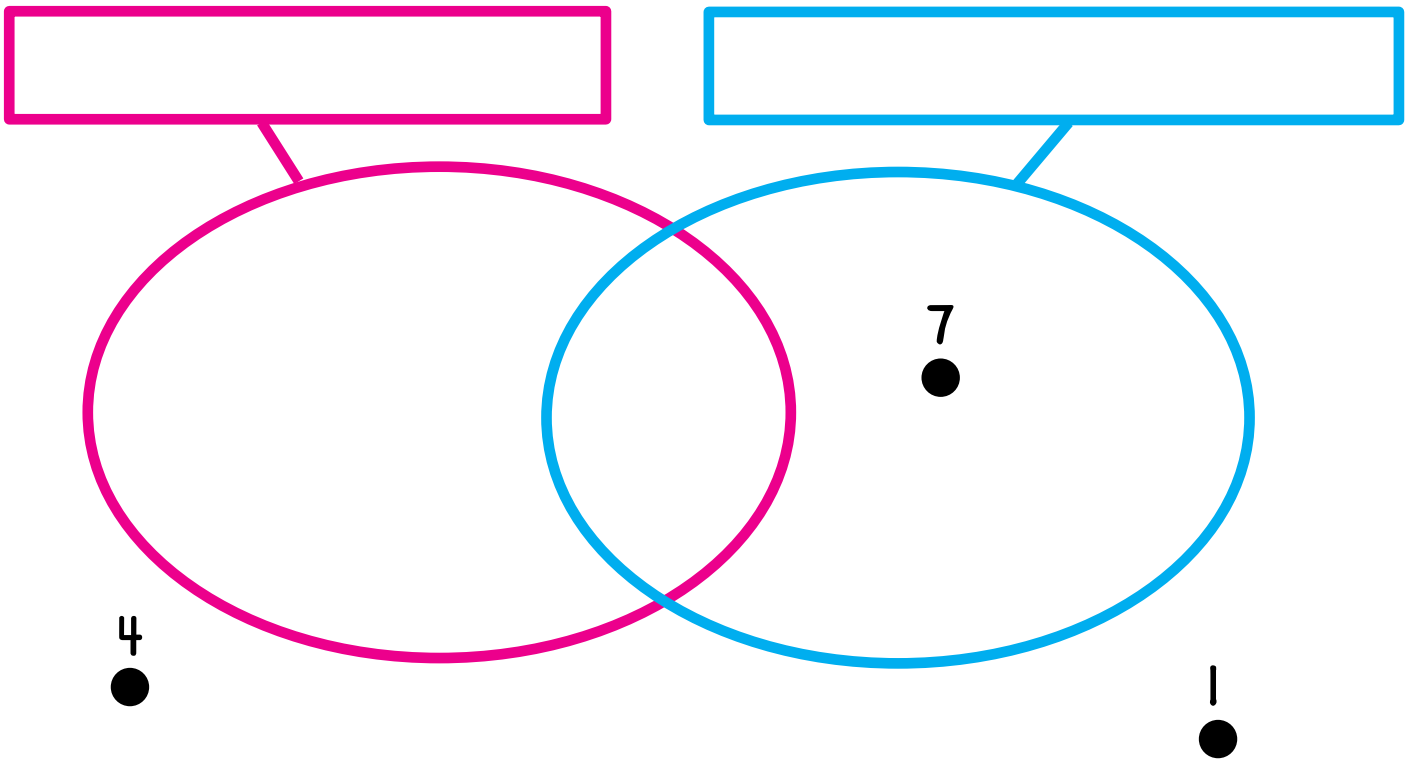
The red label is one of these:

Positive prime numbers
Positive divisors of 12
Multiples of 3
Greater than $\widehat{20}$
Less than 20
Odd numbers

The blue label is one of these:

Positive prime numbers
Positive divisors of 12
Multiples of 3
Greater than $\widehat{20}$
Less than 20
Odd numbers

Label the strings.



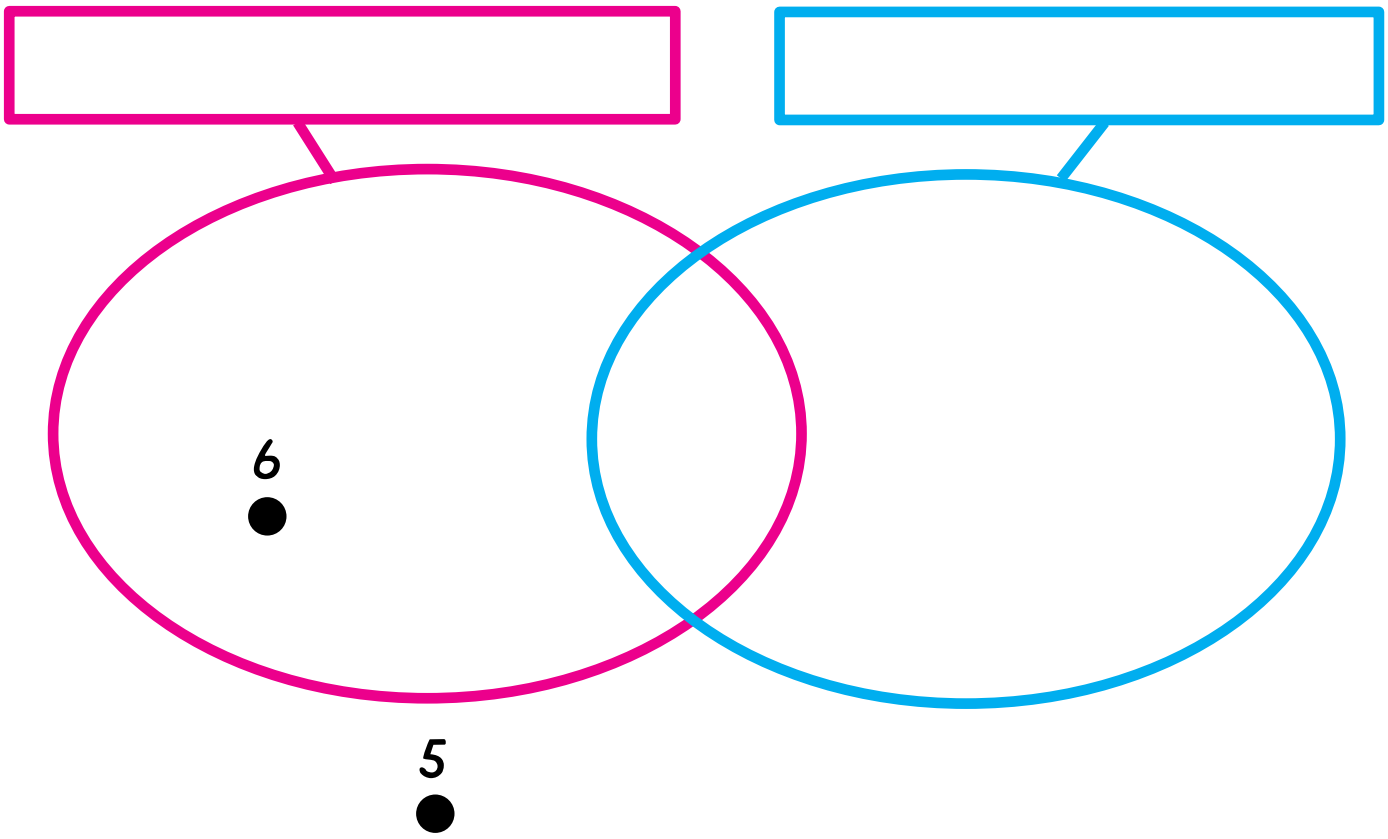
The red label is one of these:

Positive prime numbers
Positive divisors of 20
Multiples of 4
Greater than $\widehat{10}$
Less than 40
Positive divisors of 24

The blue label is one of these:

Positive prime numbers
Positive divisors of 20
Multiples of 4
Greater than $\widehat{10}$
Less than 40
Positive divisors of 24

Label the strings.



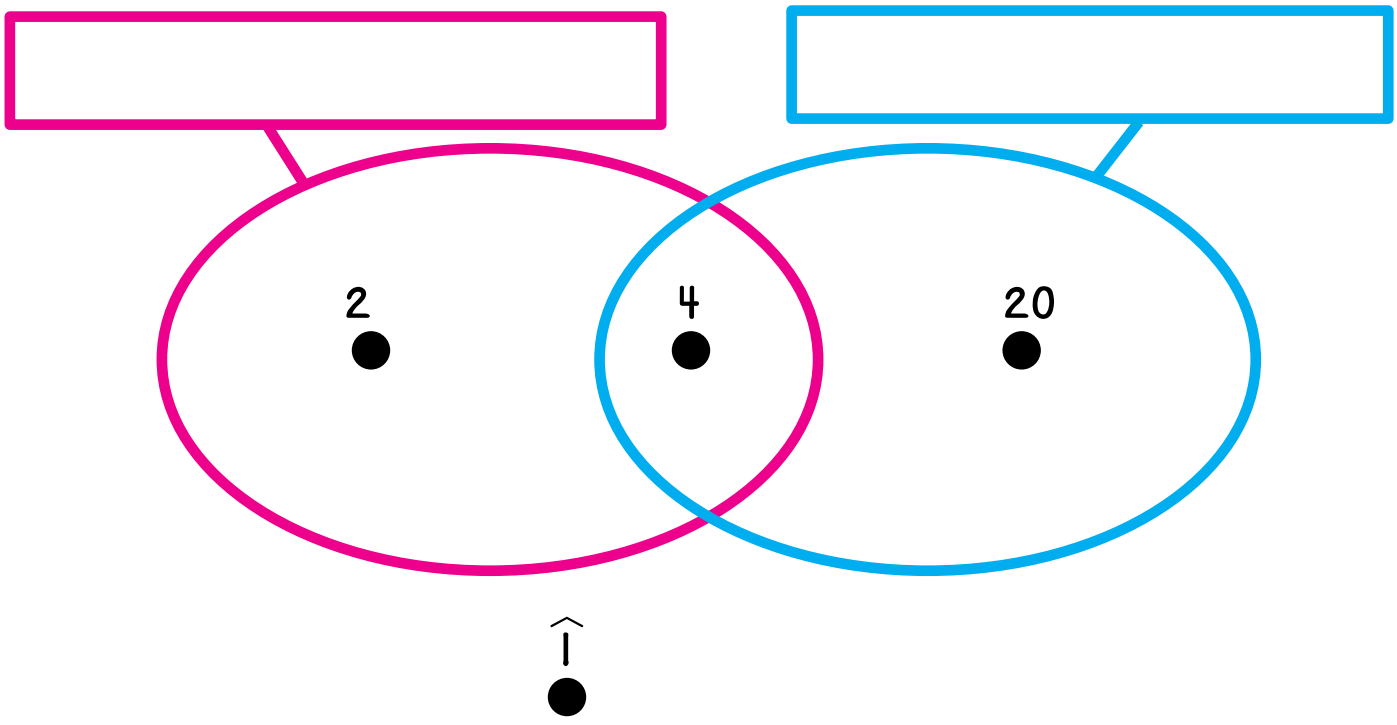
The red label is one of these:

Multiples of 2
Multiples of 4
Multiples of 5
Positive divisors of 24
Positive divisors of 20
Greater than $\widehat{10}$
Less than 10

The blue label is one of these:

Multiples of 2
Multiples of 4
Multiples of 5
Positive divisors of 24
Positive divisors of 20
Greater than $\widehat{10}$
Less than 10

Label the strings.



Capsule Lesson Summary

One at a time, correctly put some numbers (game pieces) in the string picture for a string game. Analyze the information obtained from each of these clues and determine the string labels. Play *The String Game* with numbers.

Materials

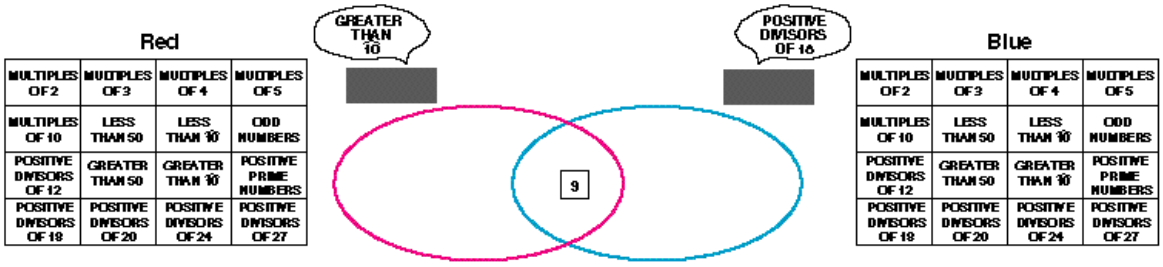
Teacher
- Numerical String Game kit
- Colored chalk
- Colored markers or crayons

Student
- String Game analysis sheet

Description of Lesson

Exercise 1

Before the lesson begins, prepare your board as illustrated below. Bubbles indicate what is on the hidden labels. Display two Numerical String Game posters (included in the Numerical String Game kit), one to the left of the red string and one to the right of the blue string.



T: ***We are going to play The String Game today, but first let's look at what information we get from some clues. This can help us be better players. Playing the game is like solving a detective story; we want to find out what labels the red and blue strings have, and each piece that is played is a clue. Let's play a game together.***

Our first clue is that 9 is in the center region; 9 is inside both strings. What information does this clue give us about the strings?

S: ***The red string cannot be for* MULTIPLES OF 2, *because 9 is not a multiple of 2.***

T: ***Could the blue string be for* MULTIPLES OF 2*?***

S: ***No, because 9 is also inside the blue string.***

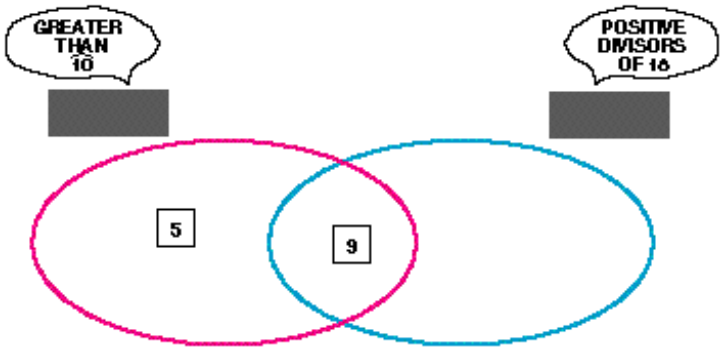
Encourage students to eliminate as many labels as they can from both lists (posters). Each time they suggest crossing out a label on one list because the corresponding string cannot have that label, they should see that the same label should be crossed out on the other list—a number in the center region gives the same information about both strings. Do not allow a label to be crossed out until an adequate explanation has been given. For example:

S: ***The red (blue) string cannot be for* POSITIVE DIVISORS OF 12, *because 9 is not a positive divisor of 12.***

L 7

When all of the information from this clue has been discussed, there should be only six remaining possibilities for each string.

T: ***Now, I'll give you a second clue. 5 is in the red string but not in the blue sting.*** (Place 5 correctly in the string picture.) ***What new information does this clue give us?***



S: ***The red string cannot be for*** **MULTIPLES OF 3**, ***because 5 is not a multiple of 3.***

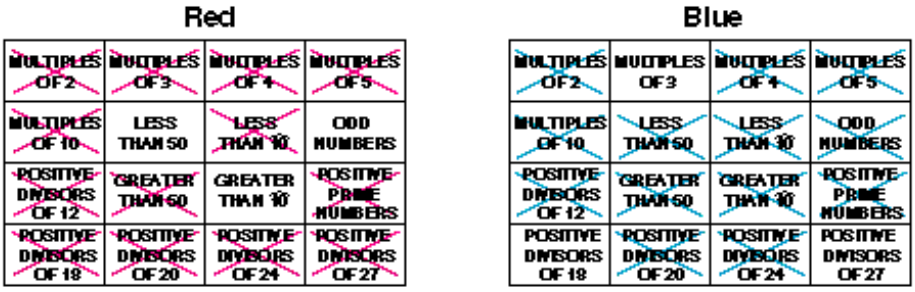
T: ***Could the blue string be for*** **MULTIPLES OF 3*?*** (Yes)

If it is not mentioned, lead the class to realize that with this clue whatever remaining label the red string cannot have, the blue string can have, and vice versa. For example:

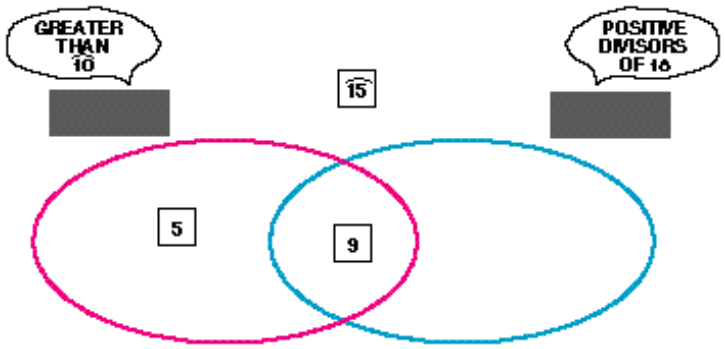
S: ***The blue string cannot be for*** **ODD NUMBERS**, ***because 5 is an odd number and is outside of the blue string.***

T: ***Could the red string be for*** **ODD NUMBERS*?*** (Yes)

When all of the information from this clue has been discussed, there should be only three remaining possibilities on each list.



T: ***My third clue is that $\widehat{15}$ is outside of both strings.*** (Place $\widehat{15}$ correctly in the string picture.) ***What new information does this clue give us?***



S: ***The red string cannot be for* ODD NUMBERS*, because $\widehat{15}$ is odd and it is outside of the red string.***

S: ***The blue string cannot be for* MULTIPLES OF 3*, because $\widehat{15}$ is a multiple of 3 and it is outside of the blue string.***

These three clues determine the red string, and there are two possibilities remaining for the blue string.

Red

~~MULTIPLES OF 2~~	~~MULTIPLES OF 3~~	~~MULTIPLES OF 4~~	~~MULTIPLES OF 5~~
~~MULTIPLES OF 10~~	~~LESS THAN 50~~	~~LESS THAN 10~~	~~ODD NUMBERS~~
~~POSITIVE DIVISORS OF 12~~	~~GREATER THAN 50~~	GREATER THAN 10	~~POSITIVE PRIME NUMBERS~~
~~POSITIVE DIVISORS OF 18~~	~~POSITIVE DIVISORS OF 20~~	~~POSITIVE DIVISORS OF 24~~	~~POSITIVE DIVISORS OF 27~~

Blue

~~MULTIPLES OF 2~~	~~MULTIPLES OF 3~~	~~MULTIPLES OF 4~~	~~MULTIPLES OF 5~~
~~MULTIPLES OF 10~~	~~LESS THAN 50~~	~~LESS THAN 10~~	~~ODD NUMBERS~~
~~POSITIVE DIVISORS OF 12~~	~~GREATER THAN 50~~	~~GREATER THAN 10~~	~~POSITIVE PRIME NUMBERS~~
POSITIVE DIVISORS OF 18	~~POSITIVE DIVISORS OF 20~~	~~POSITIVE DIVISORS OF 24~~	POSITIVE DIVISORS OF 27

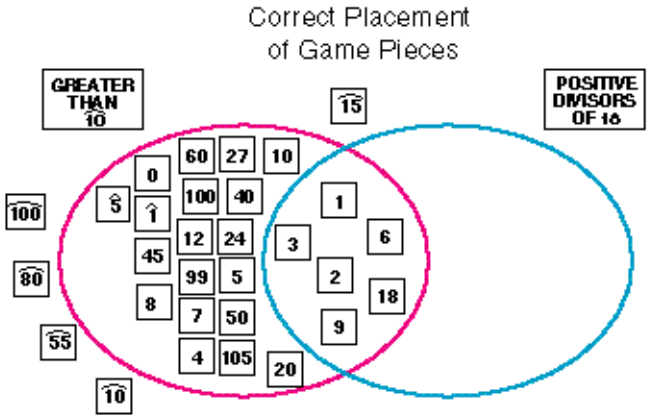
T: ***Now we know that the red string is for numbers* GREATER THAN $\widehat{10}$*. There are still two possibilities for the blue string. We need another clue. Can someone suggest a number to play? I'll put it in the correct place. Try to choose a number so that once we know where it belongs in the picture, we can determine the blue string.***

This problem is difficult; allow students to make several trials, if necessary. If a student chooses $\widehat{100}$, $\widehat{80}$, $\widehat{55}$, $\widehat{10}$, $\widehat{5}$, $\widehat{1}$, 0, 1, 3, 4, 7, 8, 10, 12, 20, 24, 40, 45, 50, 60, 99, 100, or 105, ask him or her to place it. Where these numbers belong in the picture is already determined, so playing any one of them gives no new information. If someone chooses 2, 6, 18, or 27, place the number correctly yourself. If, after several trials, the class does not see that 2, 6, 18, or 27 are the only numbers that can determine the blue string, suggest one of these numbers yourself.

L 7

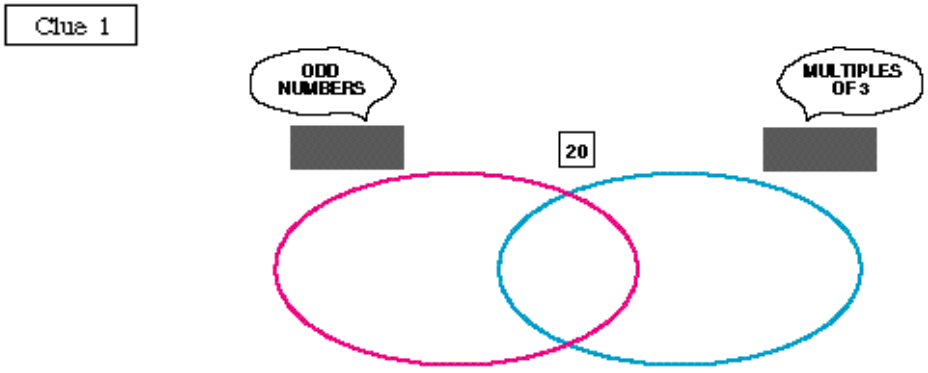
The following dialogue assumes that 27 is placed correctly in the string picture.

S: ***The blue string cannot be for*** **POSITIVE DIVISORS OF 27,** ***because 27 is outside of the blue string. So the blue string must be for*** **POSITIVE DIVISORS OF 18.**



Exercise 2[†]

Repeat Exercise 1 with the situation below. You can move more quickly by encouraging one student to eliminate several similar labels simultaneously.



These are the labels that can be crossed out after the first clue.

Red

~~MULTIPLES OF 2~~	~~MULTIPLES OF 3~~	~~MULTIPLES OF 4~~	~~MULTIPLES OF 5~~
~~MULTIPLES OF 10~~	~~LESS THAN 50~~	LESS THAN 10	ODD NUMBERS
POSITIVE DIVISORS OF 12	GREATER THAN 50	~~GREATER THAN 10~~	POSITIVE PRIME NUMBERS
POSITIVE DIVISORS OF 18	~~POSITIVE DIVISORS OF 20~~	POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27

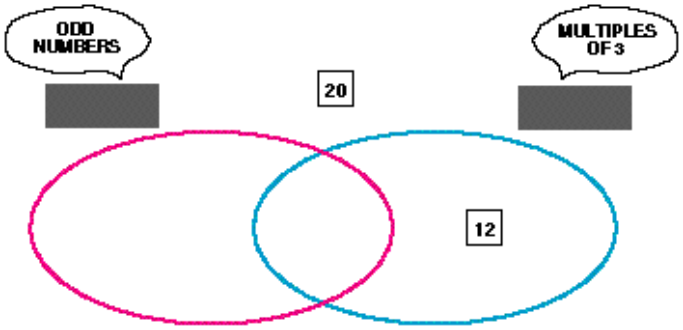
Blue

~~MULTIPLES OF 2~~	~~MULTIPLES OF 3~~	~~MULTIPLES OF 4~~	~~MULTIPLES OF 5~~
~~MULTIPLES OF 10~~	~~LESS THAN 50~~	LESS THAN 10	ODD NUMBERS
POSITIVE DIVISORS OF 12	GREATER THAN 50	~~GREATER THAN 10~~	POSITIVE PRIME NUMBERS
POSITIVE DIVISORS OF 18	~~POSITIVE DIVISORS OF 20~~	POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27

Note: The same labels are crossed out on both lists because this clue gives the same information about both strings; 20 is outside of both strings.

[†]If Exercise 1 took more than half of your class time, you may wish to skip this exercise and go on to Exercise 3, playing *The String Game* in the time that remains.

Clue 2



These are the labels that can be crossed out after the second clue.

Red

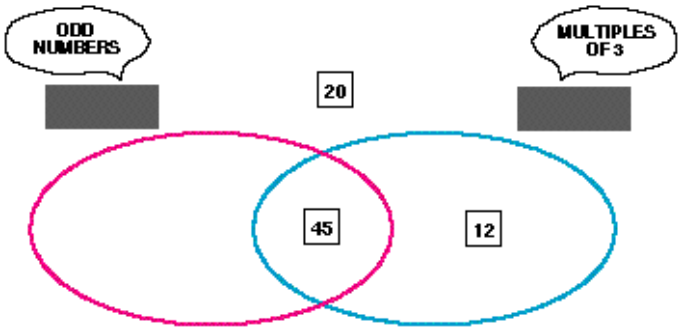
~~MULTIPLES OF 2~~	~~MULTIPLES OF 3~~	~~MULTIPLES OF 4~~	~~MULTIPLES OF 5~~
~~MULTIPLES OF 10~~	~~LESS THAN 50~~	LESS THAN 10	ODD NUMBERS
~~POSITIVE DIVISORS OF 12~~	GREATER THAN 50	~~GREATER THAN 10~~	POSITIVE PRIME NUMBERS
POSITIVE DIVISORS OF 18	~~POSITIVE DIVISORS OF 20~~	~~POSITIVE DIVISORS OF 24~~	POSITIVE DIVISORS OF 27

Blue

~~MULTIPLES OF 2~~	MULTIPLES OF 3	~~MULTIPLES OF 4~~	~~MULTIPLES OF 5~~
~~MULTIPLES OF 10~~	~~LESS THAN 50~~	~~LESS THAN 10~~	~~ODD NUMBERS~~
POSITIVE DIVISORS OF 12	GREATER THAN 50	GREATER THAN 10	~~POSITIVE PRIME NUMBERS~~
~~POSITIVE DIVISORS OF 18~~	~~POSITIVE DIVISORS OF 20~~	POSITIVE DIVISORS OF 24	~~POSITIVE DIVISORS OF 27~~

Note: Of the possibilities remaining after the first clue, those that get crossed out on one list do not get crossed out on the other; 12 is inside one string and outside of the other.

Clue 3



These three clues determine both the red string and the blue string.

Red

~~MULTIPLES OF 2~~	~~MULTIPLES OF 3~~	~~MULTIPLES OF 4~~	~~MULTIPLES OF 5~~
~~MULTIPLES OF 10~~	~~LESS THAN 50~~	~~LESS THAN 10~~	ODD NUMBERS
~~POSITIVE DIVISORS OF 12~~	~~GREATER THAN 50~~	~~GREATER THAN 10~~	~~POSITIVE PRIME NUMBERS~~
~~POSITIVE DIVISORS OF 18~~	~~POSITIVE DIVISORS OF 20~~	~~POSITIVE DIVISORS OF 24~~	~~POSITIVE DIVISORS OF 27~~

Blue

~~MULTIPLES OF 2~~	MULTIPLES OF 3	~~MULTIPLES OF 4~~	~~MULTIPLES OF 5~~
~~MULTIPLES OF 10~~	~~LESS THAN 50~~	~~LESS THAN 10~~	~~ODD NUMBERS~~
~~POSITIVE DIVISORS OF 12~~	~~GREATER THAN 50~~	~~GREATER THAN 10~~	~~POSITIVE PRIME NUMBERS~~
~~POSITIVE DIVISORS OF 18~~	~~POSITIVE DIVISORS OF 20~~	~~POSITIVE DIVISORS OF 24~~	~~POSITIVE DIVISORS OF 27~~

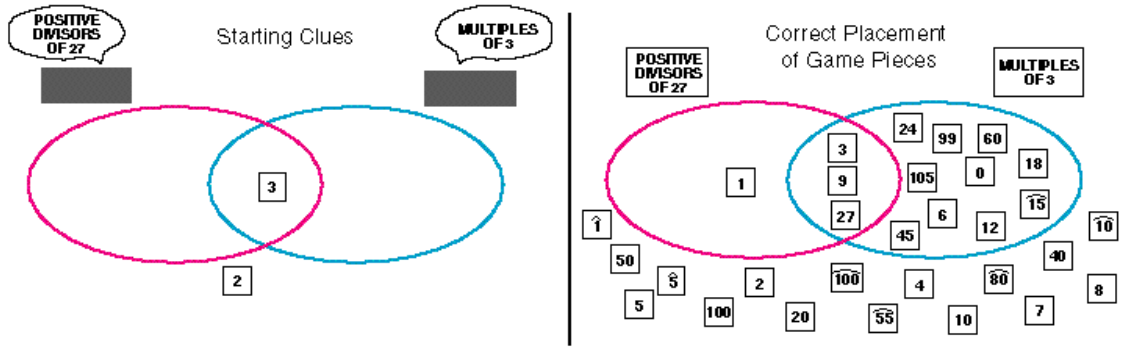
L 7

Exercise 3

Distribute String Game analysis sheets to students, and point out that now they have their own lists of possibilities for the red and blue strings. Suggest that students try to use these lists during the game to help discover the string labels.

Prepare to play *The String Game* in the usual way. The illustration below shows a possible game with two pieces placed correctly as starting clues.

In order to give students time to do some of their own analysis between plays, you may pause briefly between turns. Such pauses in the game should encourage students to use the clues to cross out labels on their individual lists.



Capsule Lesson Summary

Using correct plays in a string game as clues, see how quickly the class can identify the string labels in a cooperative game. Play *The String Game* with numbers.

Materials

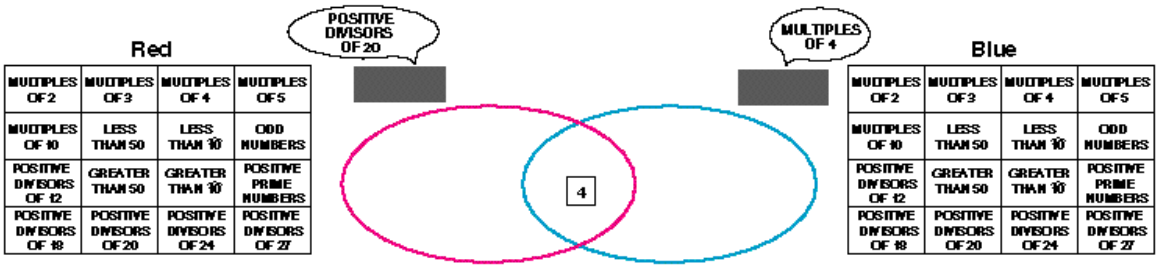
Teacher
- Numerical String Game kit
- Colored chalk
- Colored markers or crayons

Student
- String Game analysis sheet
- Worksheets L9*, **, ***, and ****

Description of Lesson

Exercise 1

Distribute String Game analysis sheets to students. Using two Numerical String Game posters from the Numerical String Game kit, prepare your board as illustrated below. Bubbles indicate what is on the hidden labels.

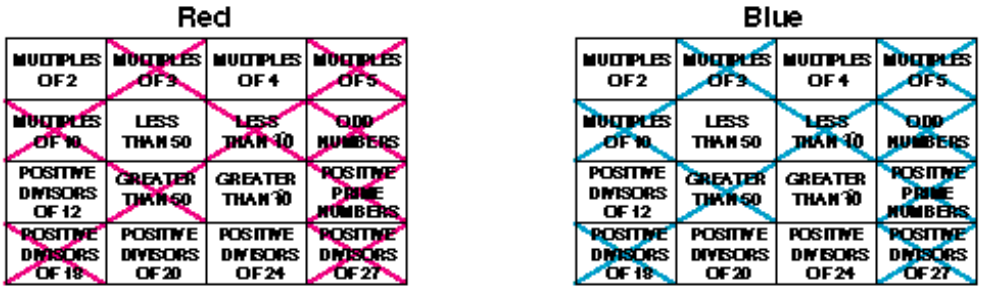


T: ***Today we are going to play The String Game. First, let's play together as a class and see how quickly we can discover the strings' labels. The first clue is that 4 is in the center region. What information does this give us about the strings?***

S: ***The red string cannot be for*** **MULTIPLES OF 3** ***because 4 is not a multiple of 3.***

S: ***Also, the blue string cannot be for*** **MULTIPLES OF 3.**

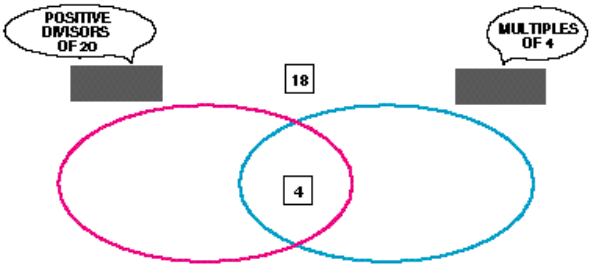
Cross out labels from the two lists as they are mentioned. Encourage students to recognize that the same labels are crossed out on both lists because 4 is inside both strings, thus giving the same information about both strings. This clue eliminates all but seven possibilities on each list.



L 9

Place 18 in the picture as you announce a second clue.

T: ***The second clue is that 18 is in the outside region. What new information does this clue give us about the strings?***



S: ***The red string cannot be for*** **MULTIPLES OF 2,** ***because 18 is a multiple of 2 and it is outside of the red string.***

S: ***Also, the blue string cannot be for*** **MULTIPLES OF 2.**

Again, cross out labels from the two lists as they are mentioned. Encourage students to recognize again that this clue gives the same information about both strings; 18 is outside of both strings. After using information from this clue, only four possibilities remain for each of the strings.

Red

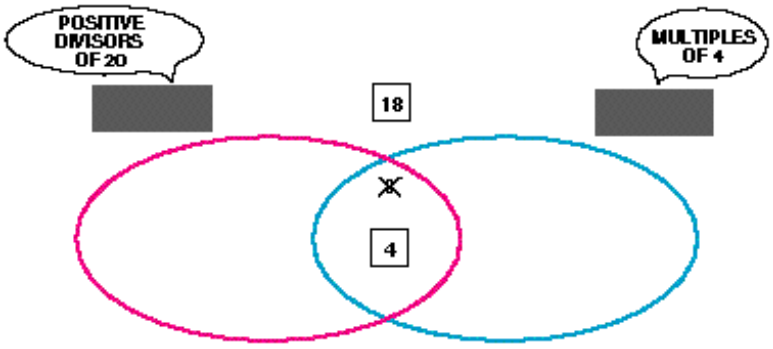
MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	LESS THAN 50	LESS THAN 10	ODD NUMBERS
POSITIVE DIVISORS OF 12	GREATER THAN 50	GREATER THAN 30	POSITIVE PRIME NUMBERS
POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20	POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27

Blue

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	LESS THAN 50	LESS THAN 10	ODD NUMBERS
POSITIVE DIVISORS OF 12	GREATER THAN 50	GREATER THAN 30	POSITIVE PRIME NUMBERS
POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20	POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27

T: ***Now, let's play the game together in the usual way and see now quickly we can discover the strings' labels. Who would like to make a first play?***

Call on students to make plays, keeping count of how many plays are made. Accept any play, answering "yes" if the piece is played correctly and "no" if it is played incorrectly. Whenever a piece is played correctly, stop and analyze what new information is obtained. Cross out the appropriate labels on the two lists. Whenever a piece is played incorrectly, record the information in the picture. For example, if 8 is played in the center, indicate on your picture that it was put in the wrong region (see picture below).

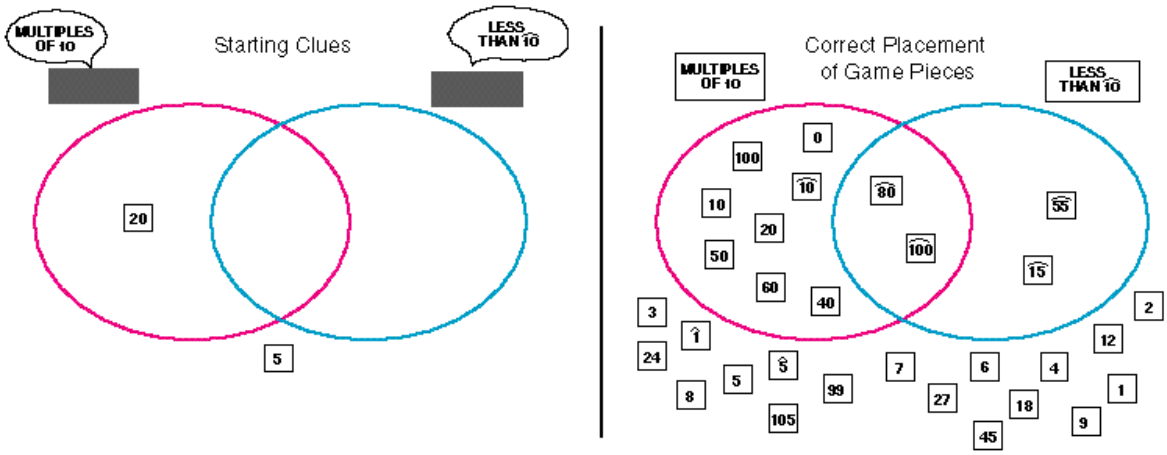


Caution: Students might suggest that this play tells them that the red string cannot, for example, be for **POSITIVE DIVISORS OF 24**. Remind the class that they still do not know where 8 belongs; it could still be inside the blue string, inside the red string, or outside of both strings.

Continue playing the game as a class until the strings are determined. You may wish to ask students to make plays even after the string labels are known just for the practice of recognizing positive divisors of 20 and multiples of 4.

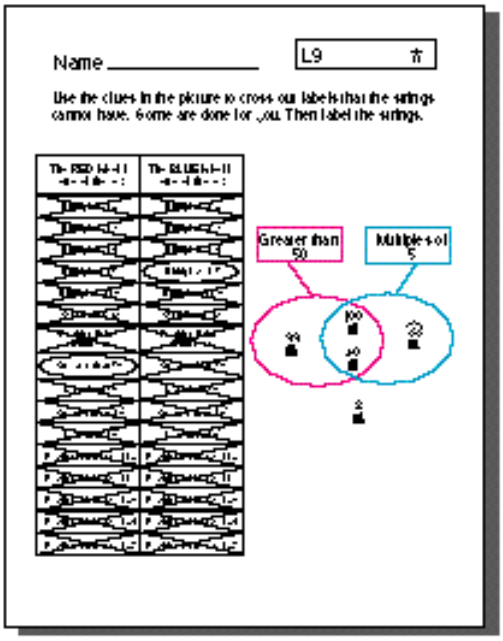
Exercise 2

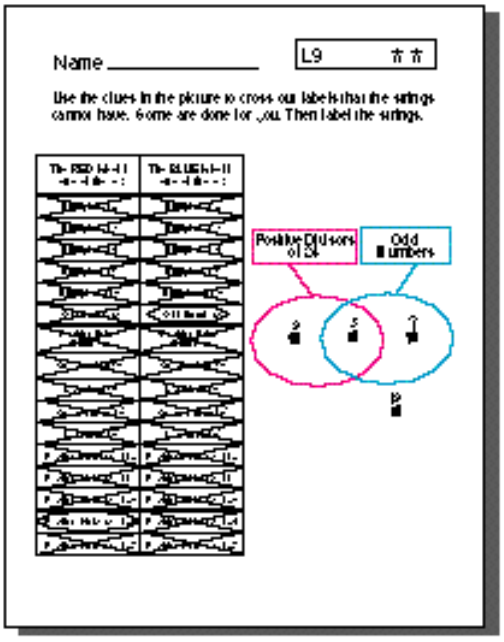
Play *The String Game* in the usual way. The illustration below shows a possible game with two starting clues. Encourage students to use their lists (String Game analysis sheets) to eliminate possibilities for string labels during the game. Suggest that when they are sure what the strings are for, they should circle the correct label on each list. This will help them avoid mistakes when playing.

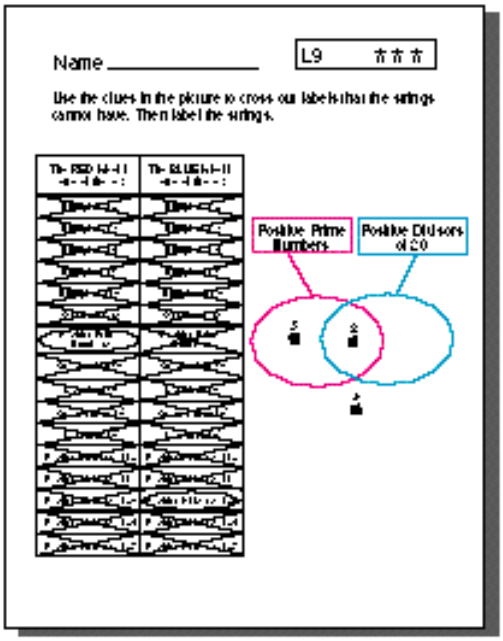


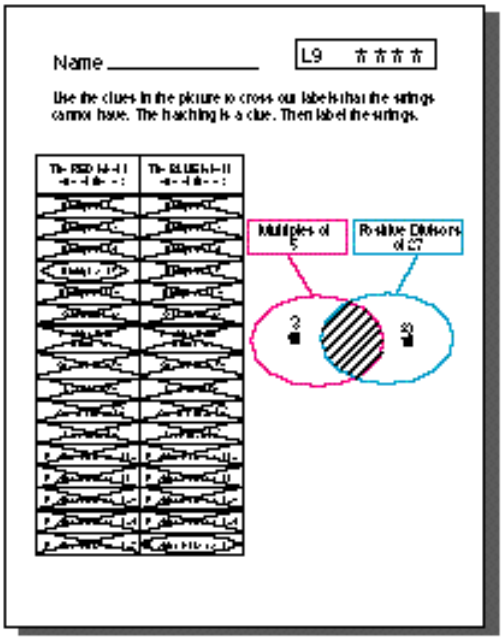
Worksheets L9*, **, ***, and **** are available for individual work.

L 9


Name
L9 ☆
Use the clues in the picture to cross out labels that the strings cannot have. Some are done for you. Then label the strings.
Greater than 50
Multiples of 5


Name
L9 ☆☆
Use the clues in the picture to cross out labels that the strings cannot have. Some are done for you. Then label the strings.
Positive Divisors of 24
Odd Numbers


Name
L9 ☆☆☆
Use the clues in the picture to cross out labels that the strings cannot have. Then label the strings.
Positive Prime Numbers
Positive Divisors of 20


Name
L9 ☆☆☆☆
Use the clues in the picture to cross out labels that the strings cannot have. The hatching is a clue. Then label the strings.
Multiples of 5
Positive Divisors of 27

Capsule Lesson Summary

Analyze a situation where the starting clues of a string game determine the strings. Begin to discuss the strategy of certain plays in a string game. Play *The String Game* with numbers.

Materials

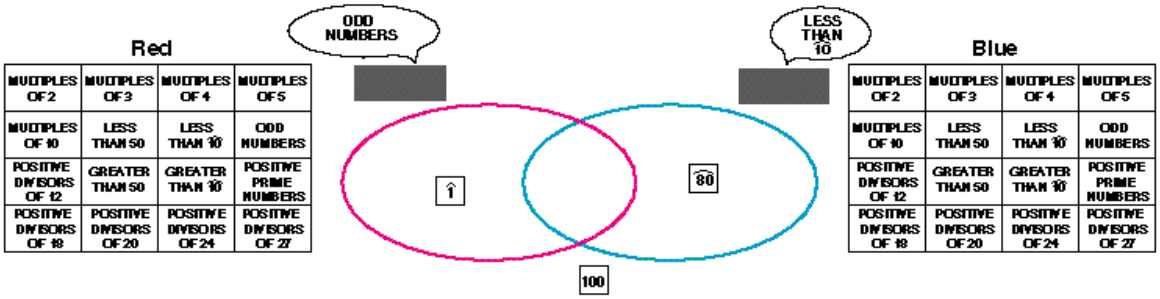
Teacher
- Numerical String Game kit
- Colored chalk
- Colored markers or crayons

Student
- String Game analysis sheet

Description of Lesson

Exercise 1

Using two Numerical String Game posters, prepare your board as illustrated below. Bubbles indicate what is on the hidden labels. In this situation the clues do, in fact, determine the string labels.



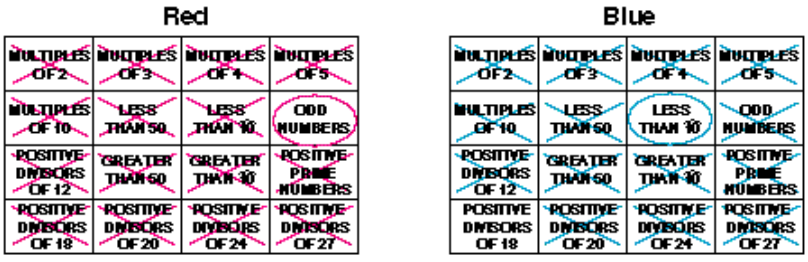
T: ***What information do these clues give us about the strings?***

Let students suggest labels to cross out on the lists. Each time, ask for an explanation as to why a label can be crossed out. For example:

S: ***The red string cannot be for numbers* LESS THAN 50 *because 80 is less than 50 and it is outside of the red string.***

S: ***The blue string cannot be for positive divisors of any number because 80 is negative.***

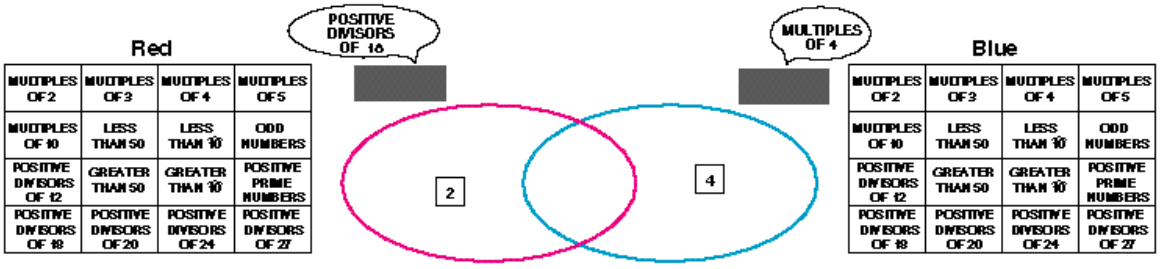
On the two lists, cross out the labels that the strings cannot have as verified by students. All labels except one on each list should be crossed out. That is, these clues determine the strings; the red string is for **ODD NUMBERS** and the blue string is for numbers **LESS THAN 10**.



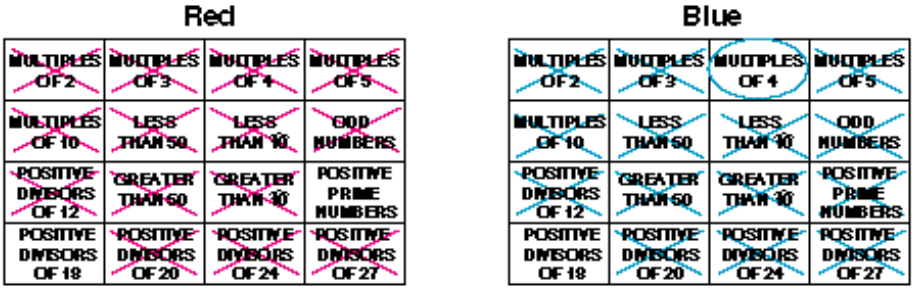
L 10

Exercise 2

Prepare your board for *The String Game*, again taping Numerical String Game posters near the strings. Distribute String Game analysis sheets to students. Start with two clues as shown here.

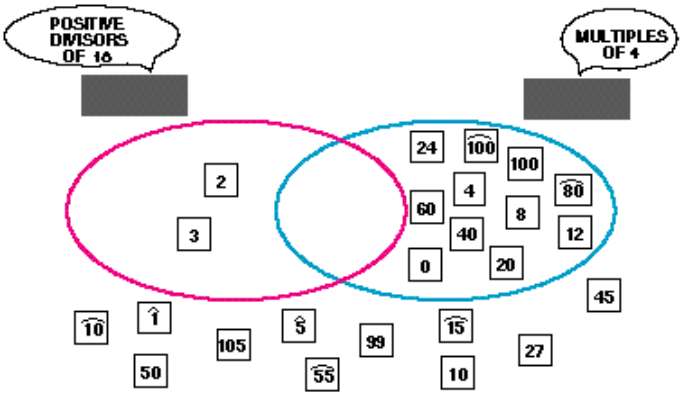


Direct students to cross out as many labels as they can on their lists. After a few minutes, collectively do the analysis with these clues. The class should find that there are two remaining possibilities for the red string and that the blue string is determined.



T: ***Now we know that the red string is for either* POSITIVE DIVISORS OF 18 *or for* POSITIVE PRIME NUMBERS. *We also know that the blue string is for* MULTIPLES OF 4. *Let's see which numbers we can put in the picture correctly even though we are not yet sure what label the red string has.***

Let students place as many of *The String Game* numbers as they can. This activity should allow everyone a chance to participate. Encourage discussion about why, for example, you are sure that 10 is in the outside region (10 is not a positive divisor of 18 and not a positive prime number, and 10 is not a multiple of 4); or why you are sure that 3 is inside the red string but outside the blue string (3 is a positive divisor of 18 and a positive prime number, but 3 is not a multiple of 4); and so on. Students should put all of the string game numbers except 1, 5, 6, 7, 9, and 18 in the picture.



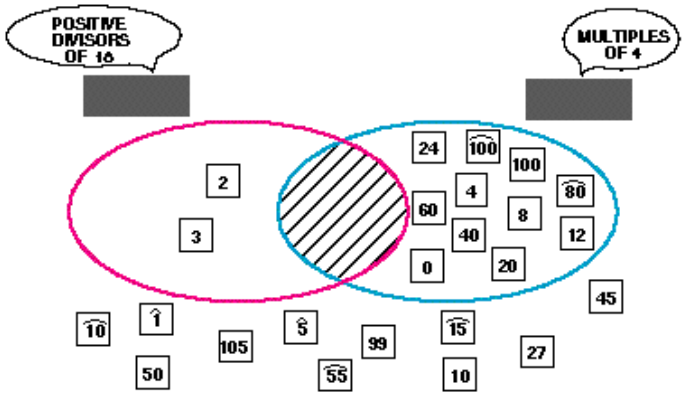
Discuss possibilities for the numbers 1, 5, 6, 7, 9, and 18. Each of these numbers could be in the outside region, or inside the red string and outside of the blue string.

T: ***Do you see a region that could be hatched because it is empty?***

S: ***The center region.***

T: ***Why?***

S: ***No number is both a positive divisor of 18 and a multiple of 4, and no number is both prime and a multiple of 4.***



Remove all of the game pieces except 2 and 4; that is, return to the original situation.

T: ***Suppose it is your turn to play in The String Game, and you would like to determine the red string as quickly as possible. Which piece would you play?***

Give students a few minutes to think about what play would be helpful. They should, after some trial and error, find that playing any of the numbers 1, 5, 6, 7, 9, or 18 would give them the necessary information (as long as they did not make a careless play). For example, if 5 were played in the outside region and given a "no" answer, then it would be certain that 5 belonged inside the red string. So the red string would be for **POSITIVE PRIME NUMBERS**. If 5 were played in the outside region and given a "yes" answer, then it would be certain that the red string was for the **POSITIVE DIVISORS OF 18**. A similar analysis applies to each of the numbers 1, 6, 7, 9, and 18.

Let students make one of these plays to determine the red string.

Students may be concerned about where 1 belongs in the picture. Ask someone to locate 1 (inside the red string but outside the blue string). Emphasize that 1 is a divisor of any number, but 1 is not a prime number.

S: ***3 could be in the box (only positive divisors are 1 and 3).***

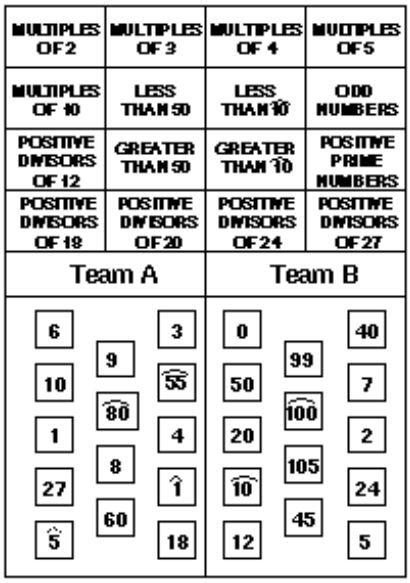
S: ***9 (only positive divisors are 1, 3, and 9).***

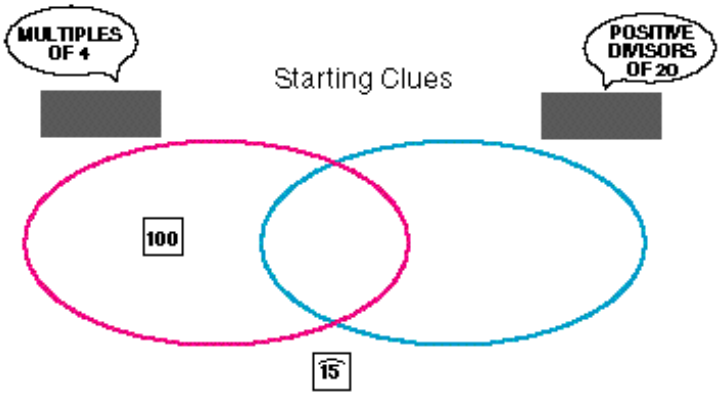
S: ***27 (only positive divisors are 1, 3, 9, and 27).***

Check suggested possibilities with the picture. Soon students should make other observations, such as an even number cannot be in the box (otherwise 2 would be a divisor and 2 is a prime number). They may observe a pattern in the list of possibilities, especially if it is presented in numerical order, i.e., start the list with 3 and multiply by 3 each time to find the next number (3, 9, 27, 81, …).

Exercise 3

Prepare to play *The String Game* with numbers by setting up a team board and taping the Numerical String Game poster above it. Divide the class into teams and distribute the game pieces on the team board. The illustration below shows a possible game with one piece from each side of the team board placed correctly as starting clues.

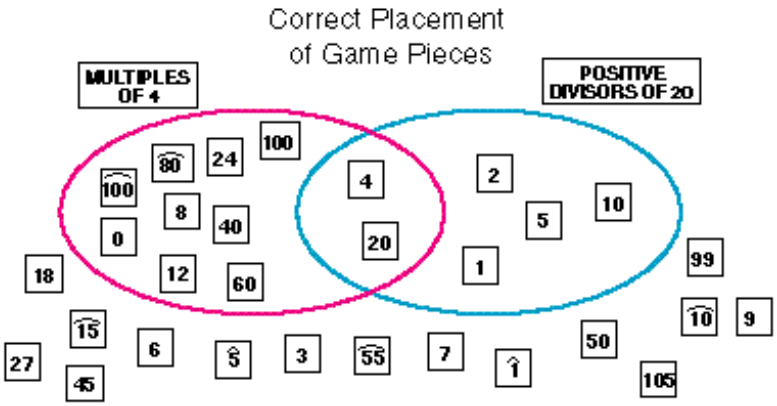




Note: If you decide to play the game with three or more teams, create a team board with sections for more teams.

L 1

The following illustration shows correct placement of all 30 numbers and may be used by you as a crib sheet during the play of the game.



Writing Activity

You may like students to take lesson notes on some, most, or even all their math lessons. The "Lesson Notes" section in the Notes to the Teacher gives suggestions and refers to forms in the Blacklines you may provide to students for this purpose. In this lesson, for example, students may note several facts and patterns about numbers with a specific number of divisors, about numbers that have a particular prime number as a divisor, or about numbers with particular odd numbers as divisors.

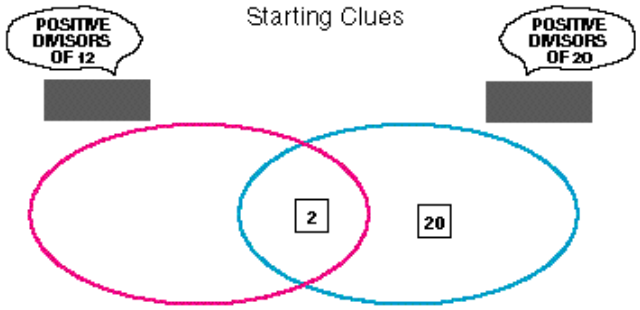
Home Activity

Suggest that parents work with their child to make a list of numbers that have exactly three or exactly six positive divisors.

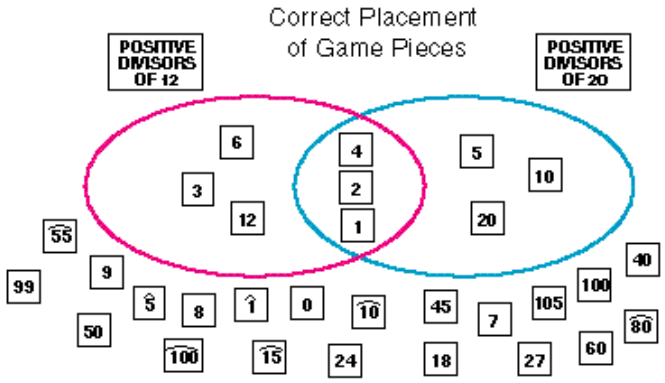
Exercise 3

Prepare to play *The String Game* with numbers. The illustration below shows a possible game with two pieces placed correctly as starting clues.

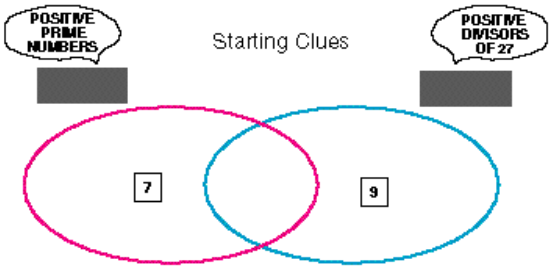
MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	LESS THAN 50	LESS THAN 10	ODD NUMBERS
POSITIVE DIVISORS OF 12	GREATER THAN 50	GREATER THAN 10	POSITIVE PRIME NUMBERS
POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20	POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27
Team A		Team B	
6, 10, 1, 27, 5, 9, 80, 8, 60, 3, 55, 4, 1, 18		0, 50, 20, 10, 12, 99, 100, 105, 45, 40, 7, 2, 24, 5	

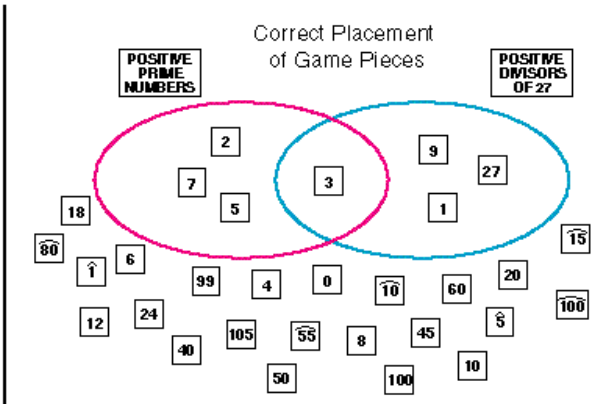


The following illustration shows correct placement of all 30 numbers and may be used by you as a crib sheet during the play of the game.



If time permits, play a second game such as the one illustrated below.





D

GAME PIECES AND STRING CARDS

One set of game pieces and string cards is needed for each version of the game. A poster list of the string cards should be posted above the team board—it is a constant reminder during the game of the possible labels for the strings.

Game Pieces

String Game with A-Blocks

String Cards

RED	YELLOW	GREEN	BLUE
NOT RED	NOT YELLOW	NOT GREEN	NOT BLUE
○	△	□	BIG
NOT ○	NOT △	NOT □	LITTLE

String Game with Numbers

100	80	55	15	10	5
1	0	1	2	3	4
5	6	7	8	9	10
12	18	20	24	27	40
45	50	60	99	100	105

MULTIPLES OF 2	MULTIPLES OF 3	MULTIPLES OF 4	MULTIPLES OF 5
MULTIPLES OF 10	LESS THAN 50	LESS THAN 10	ODD NUMBERS
POSITIVE DIVISORS OF 12	GREATER THAN 50	GREATER THAN 10	POSITIVE PRIME NUMBERS
POSITIVE DIVISORS OF 18	POSITIVE DIVISORS OF 20	POSITIVE DIVISORS OF 24	POSITIVE DIVISORS OF 27

Game pieces, string cards, and the poster of the string cards can be found in the corresponding *String Game* kit, (*A-Blocks* or *Numerical*).

a) Magnetic: You can magnetize the game pieces (A-blocks or number cards) by sticking a small piece of magnetic material to the back of each one. (Magnetic material is included in *The String Game* kits, or it is available in many stores, in the hobby or notions departments.) Similarly, you can magnetize string cards by sticking a small piece of magnetic material to the front of each card, taking care not to obscure what is written on it.

b) Non-magnetic: Game pieces can be attached to the team board using loops of masking tape stuck to the backs. A string card should have a loop of masking tape stuck to the front in such a way that what is written on the card is not obscured. With this type of equipment, be prepared to make necessary repairs by having masking tape on hand so that if a loop of tape loses its stickiness it can be replaced on the spot. As an alternative, use a small wad of a plastic caulking compound (Rope Caulk or Mortite, for example) in place of the loop of masking tape.

Preparation for the Game

Draw two (or three, depending on which variation you are using) large, overlapping strings on the playing board using two (or three) different colors. Next to each of these strings attach one string card facedown. Place the team board conveniently nearby. Randomly distribute the game pieces among the sections of the team board. Divide the class into teams using whatever method is acceptable to your class, and assign each team a section of the team board.

Before any student takes a turn, correctly place an equal number (at least one) of each team's game pieces in the string picture. This eliminates the necessity of beginning the game on the basis of pure guesswork. You can influence how long the game will take by the number of pieces you place in the string picture before the game begins.

Object of the Game

Each team tries to place all of its game pieces correctly (according to the facedown string cards) in the string picture. The winning team is the one that places all of its game pieces correctly and identifies the facedown cards correctly first.

Rules of the Game

1) The teams alternate making plays, and the members take turns within each team. A player comes to the board and selects a piece from his or her team's collection to place in one of the regions of the string picture.

2) You are the judge. If the piece is correctly placed, say yes. The piece then remains in the string picture and the player immediately has a second (bonus) turn (no player may have more than two consecutive turns). If the piece is incorrectly placed, say no. The player returns the piece to the team's unplayed collection and play passes to the next team.

   As an aid in judging, prepare a crib sheet showing the correct position of each game piece or at least reminding you of what is on the facedown cards. If at any time you discover that you have made an error, say so immediately and rectify the mistake. Then, either move an incorrectly placed piece to its correct region or replace a correctly placed piece that has been removed.

3) When a team has correctly placed all of its pieces, the player who placed the last piece may then attempt to identify the string cards. If he or she is correct, the team wins. If a mistake is made (even if it is only in the case of one of the string cards), simply indicate that the identification is incorrect and let the game continue.

4) If a team has exhausted its stock of game pieces and the strings have not been identified, that team continues to attempt to identify the strings on its turn, while the other team(s) works to place its game pieces.

The String Game with A-Blocks (Version A)

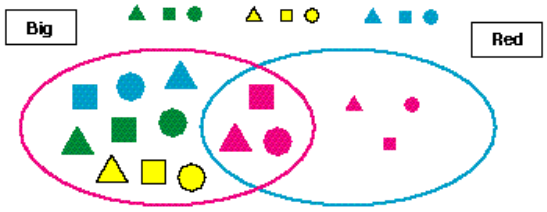
This simplest version of the game uses 24 A-blocks as game pieces and only nine string cards.

This list of string cards should be attached above the team board.

Below are several crib sheets for variations of the game with two and with three strings.

TWO STRINGS

Example 1: No empty regions



D

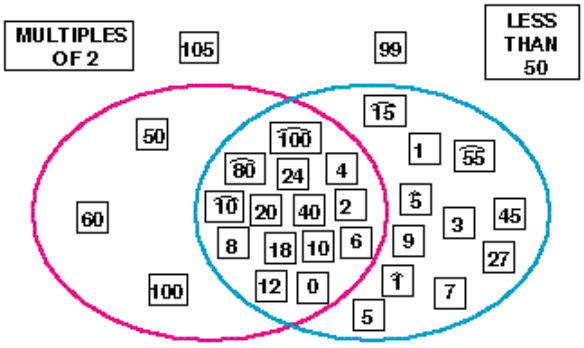
The String Game with Numbers (Version C)

This version of the game uses 30 number cards as game pieces and 16 string cards as described earlier in the section on equipment. Note that The Numerical String Game kit contains three sets of string cards. This allows you to give the same label to more than one string. Example 7 played with two strings and Example 1 played with three strings show this option; however, before using such a game players should have an understanding that this is a possibility.

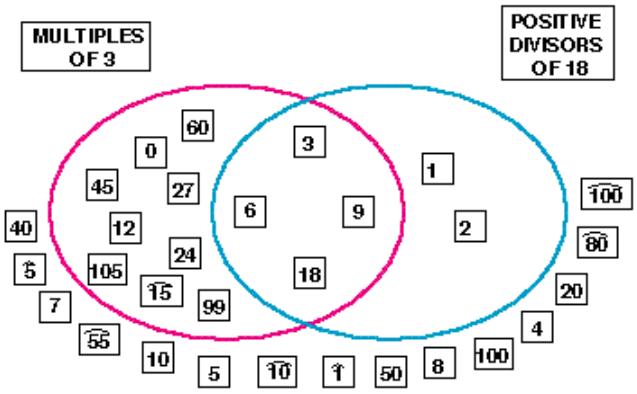
Below are several crib sheets for variations of the game with two and with three strings.

TWO STRINGS

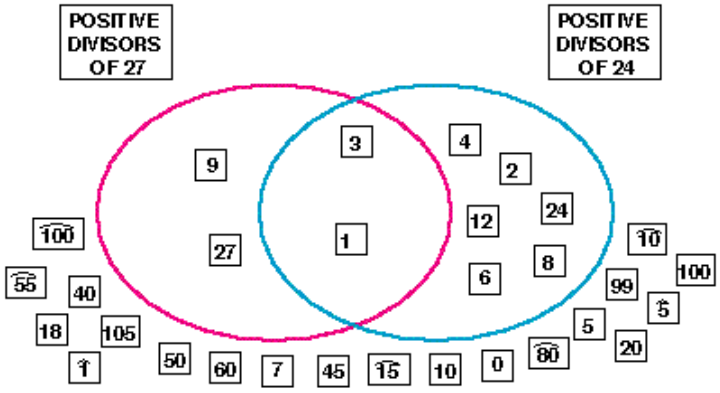
Example 1: No Empty Regions



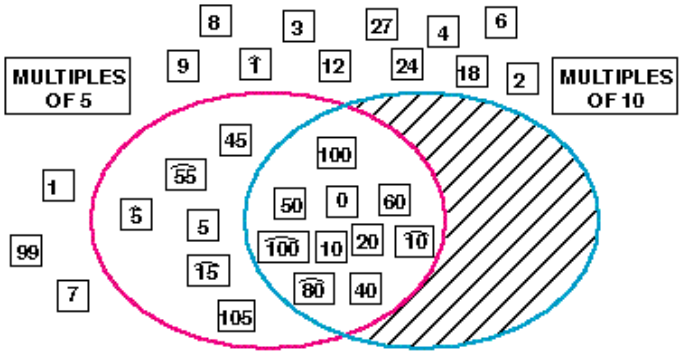
Example 2: No Empty Regions



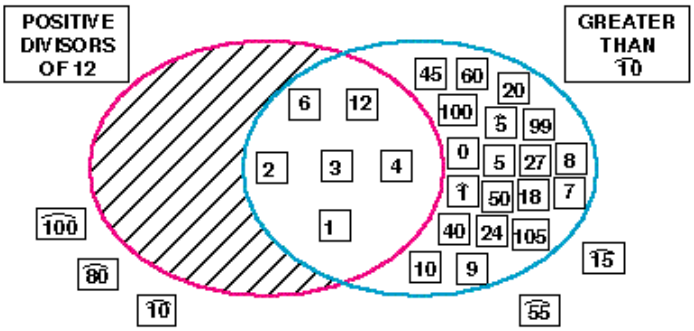
Examples 3: No Empty Regions



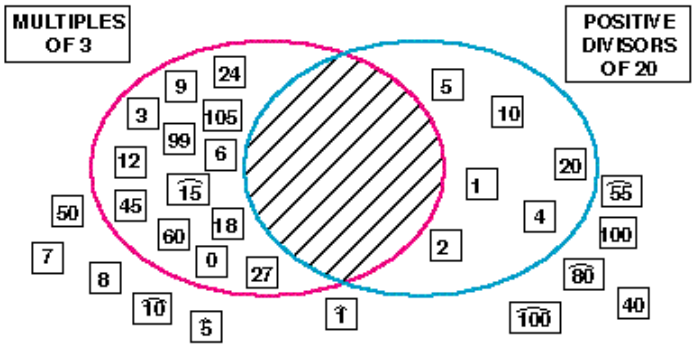
Example 4: One Empty Region



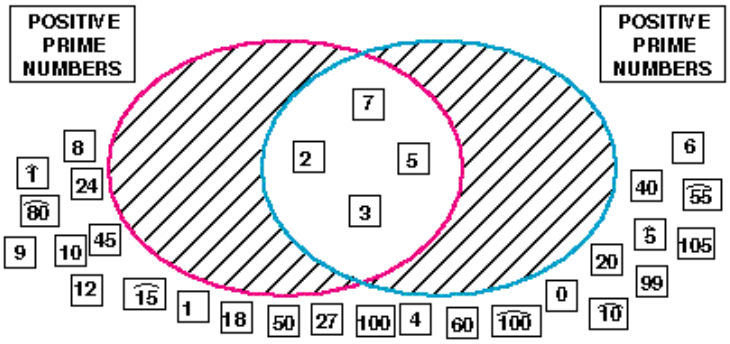
Example 5: One Empty Region



Example 6: One Empty Region



Example 7: Two Empty Regions

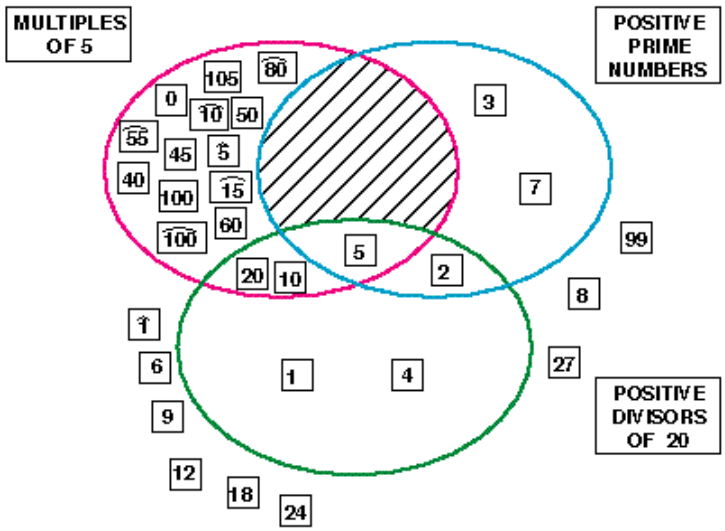


D

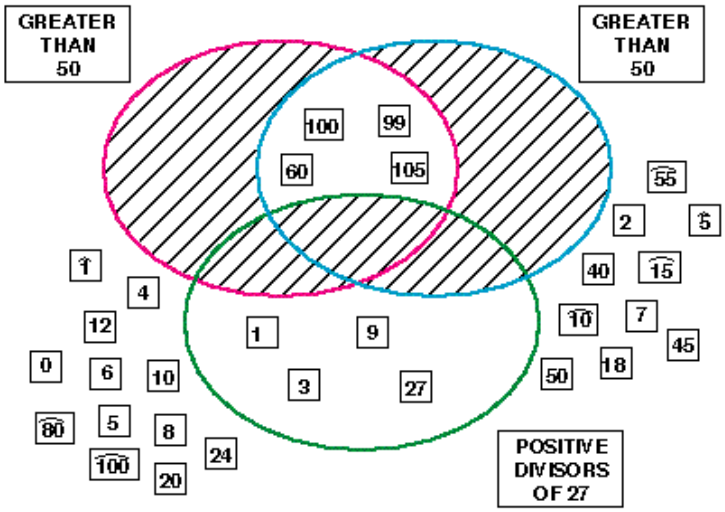
THREE STRINGS

Caution: The jump in difficulty from a two-string game to a three-string game is much greater in *The String Game* with numbers than it was in *The String Game* with A-blocks. Hence you should not expect to proceed to three-string games as rapidly as you might have with A-blocks.

Example 1: One Empty Region



Example 2: Five Empty Regions



D

The String Game with Special Scoring Rules (Version D)

In this version of the game, we introduce special scoring rules to determine the winning team. Both *The String Game* with numbers and with shapes can be played with these scoring rules; however, in the lessons of this book only *The String Game* with numbers is played this way.

Scoring

Each student is given a list of possible string labels from The String Game analysis pad. One round of the game is completed by having one player from each team take a turn. After each round, the students have an opportunity to privately identify the facedown string cards. They do this by circling on their Red and Blue lists what they believe is on the hidden tags and submitting it to the judge (teacher). However, each student has just one chance to make a private identification. If a player correctly identifies the string cards after a particular round, then points are added to the player's team score according to the following schedule.

Round 1 - 200 points	Round 3 - 100 points	Round 5 - 50 points
Round 2 - 150 points	Round 4 - 75 points	Round 6 - 25 points

After Round 6, play out the game without further private identifications. Additional points cannot be scored by a team until the end of the game. The team that correctly identifies the string cards at the end of the game scores an additional 100 points, plus 5 points for each unplayed game piece the opposing team has left on the game board. The winning team is the one with the greater total score.

Comments

1) Allow about two to three minutes before the start of Round 1 for students to analyze the starting clues (correctly placed game pieces).

2) Keep count of the rounds played so that both you and the students know which round has just been completed.

3) After each round, give the students time (perhaps 15 to 30 seconds) to study the picture, to mark their lists, and to submit them, if desired.

4) On each list submitted, write the round after which it was turned in, and sort them by teams. Do not return lists to students. You can total the team scores at the end of the game. Do not tell the students whether their identifications are correct or incorrect, but encourage them to write their findings on a separate sheet of paper for recollection when they play. They should continue to check their identifications during the remainder of the game.

5) It may be even more important to insist on silence during this kind of game because an individual can have more influence on a team's winning or losing.

Purpose

Scoring rules should not be imposed on *The String Game* until students have become very familiar and experienced in the play of the game. Many students already will be playing strategically, analyzing the clues given by correctly played game pieces. Others may still be playing in a random way. The introduction of scoring rules obviously will favor the better players and perhaps challenge them in a way the usual game no longer does. It provides them the opportunity to more dramatically affect their teams' chances of winning. For many students the scoring rules may provide incentive to begin analyzing the game and thinking about what information is given by a correctly played game piece.

D

We suggest that you do not play *The String Game* too often with these scoring rules. They do impose complication and give perhaps too much advantage to the better players. After playing the game with scoring rules a couple of times, you should find that many students have become significantly better players. Returning to the usual way of playing the game, most students will continue to play more thoughtfully.

One advantage of using scoring rules when you play *The String Game* with only a small amount of time available is that you can stop the game before the end and have a reasonable way to determine the winning team. The winning team will be the one with the larger total number of points from identification of string cards.

L 10

Exercise 3

Play *The String Game* in the usual way. The illustration below shows a possible game with starting clues. Encourage students to use their lists (String Game analysis sheet) to eliminate possibilities for string labels during the game.