Capsule Lesson Summary

Introduce *The Divisors Game*, a type of string game in which each of two strings is for the positive divisors of an unknown number. Through selection and placement of numbers in the string picture, discover the unknown numbers. Play a variation of the game involving a three-string picture, or put numbers into a two-string picture with given string labels: **Positive divisors of 36** and **Positive divisors of 42**.

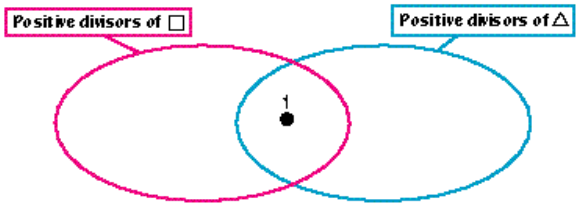
Materials

Teacher	• Colored chalk	**Student**	• None

Description of Lesson

Exercise 1

Draw this string picture on the board.



Introduce a type of string game in which students eventually determine the string labels. Announce that the numbers in the square and in the triangle may be the same or different but are restricted to whole numbers from 1 to 50. Place 1 in the middle region because 1 is a positive divisor of any whole number.

Divide the class into teams. The teams alternate, and the members take turns within each team. During a turn, a student either selects a whole number from 2 to 50 for you to locate in the picture or tries to identify one of the strings. If a student correctly identifies a string label, you record the appropriate number in the square or in the triangle. Otherwise, simple say that the identification is incorrect.

Points are gained or lost during the game in the following ways.

- A team gains one point for each string inside which the selected number belongs when a member selects a number for you to locate. For example, if a member selects a number belonging in both the red string and the blue string, the team gains two points. If a member selects a number belonging in only one of the strings, the team gains one point. If a member selects a number belonging outside both strings, the team gains no point. **Note:** The game starts with 1 placed in the intersection so that the first player does not have the advantage of gaining two points automatically.
- A team gains four points if a member correctly identifies one of the strings.
- A team loses one point if a member incorrectly identifies one of the strings.

The game is over when both strings have been determined. Tally the scores for the teams; the team with the most points wins the game.

L 4

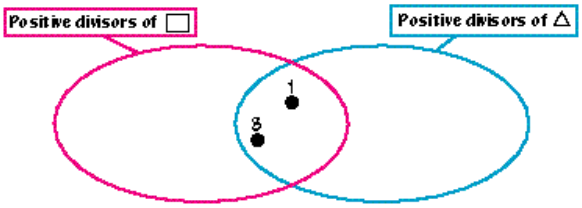
The following dialogue is a condensed description of a game played by two teams. In this game the red string is for **Positive divisors of 30** and the blue string is for **Positive divisors of 42**.

S (Team A)**:** ***3.***

T: ***3 is in the intersection. Two points.***

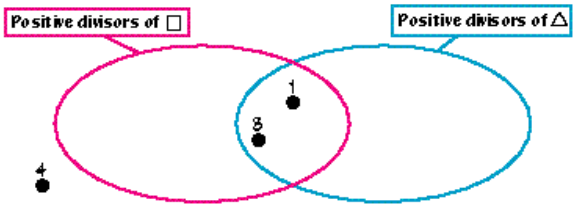
Score

Team A	Team B
2	



S (Team B)**:** ***4.***

T: ***4 is outside both strings.***
No points.

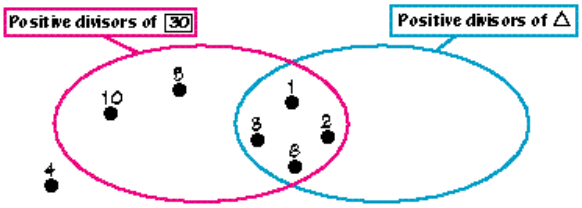


Students next select 5, 2, 10, and 6.

S (Team A)**:** ***The red string is for***
Positive divisors of 30.

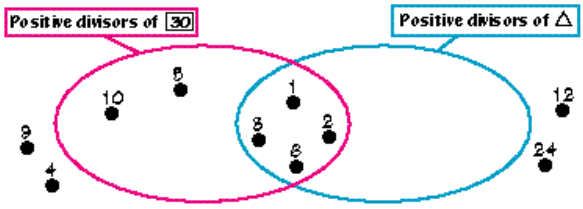
Score

Team A	Team B
2	2
1	2
1	
4	



Students next select 24, 9, and 12.

S (Team A)**:** ***The blue string is for***
Positive divisors of 6.



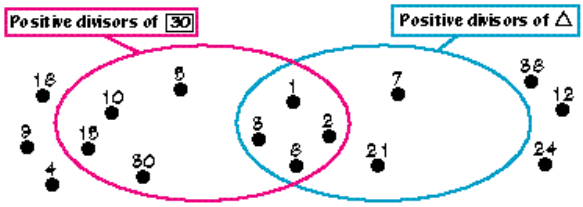
T: ***No. Your team loses a point.***

Score

Team A	Team B
2	2
✳	2
1	1
4	1
1	
1	

Students next select 16, 15, 30, 36, 7, and 21.

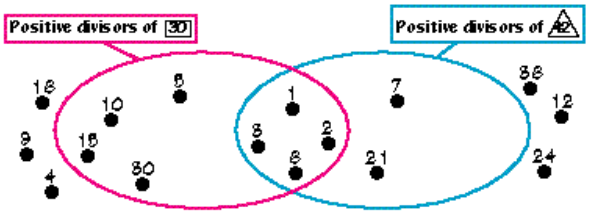
Score	
Team A	Team B
2	2
✳	2
1	1
4	1
1	
1	



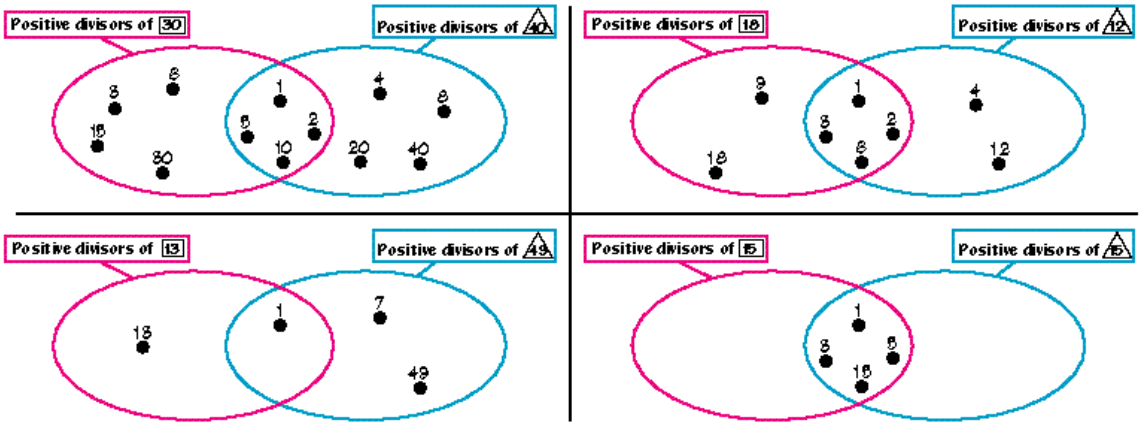
S (Team B)**:** ***The blue string is for*** **Positive divisors of 42.**

T: ***That's correct!***

Score	
Team A	Team B
2	2
✳	2
1	1
4	1
1	4
1	10
9	



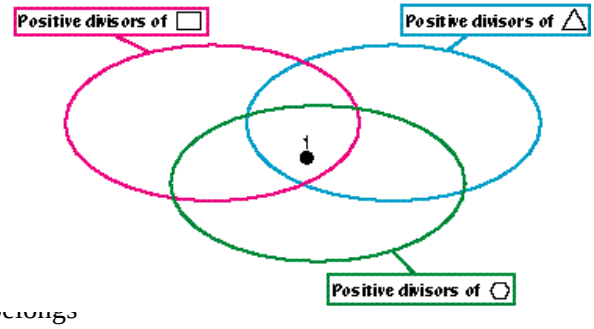
Play this game a couple more times with your class. Partial crib sheets for four possible games are given below. Note that only the numbers inside the strings are shown; assume that any whole number not in the pictures is outside both strings. Remember to place 1 in the intersection before starting a game.



Choose Exercise 2a or 2b according to the ability of your class. Exercise 2 is more challenging than Exercise 2b.

Exercise 2a _____

Play *The Divisors Game* with three strings. Locate 1 in the intersection of the three strings before starting the game.



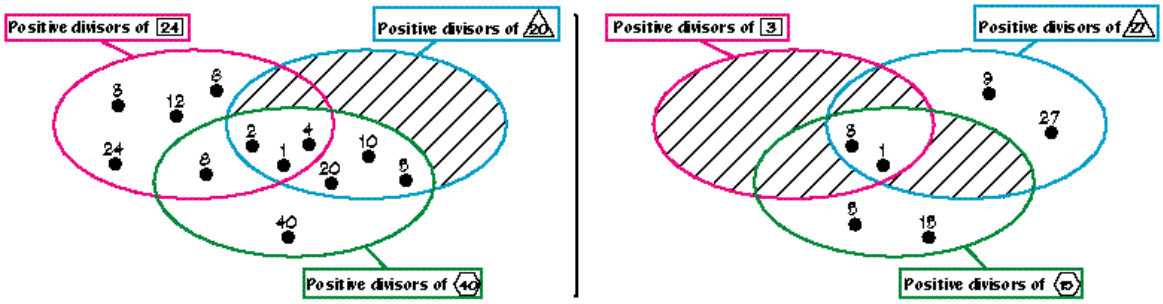
Points are gained or lost during the game in the following ways.

- A team gains one point for each string inside which a selected number belongs (for example, three points if a number belongs inside all three strings; two points if a number belongs inside only two strings, and so on).

- A team gains five points if a member correctly identifies one of the string labels.
- A team loses one point if a member incorrectly identifies one of the string labels.

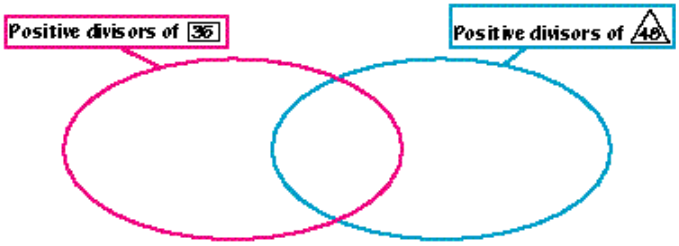
The game is over when all three strings have been determined. The team with the most points wins the game.

Play this game once or twice with your class. Partial crib sheets for two possible games are given below. Note that only the numbers inside the strings are shown; assume that any whole number not in the pictures is outside the strings. The hatching is included for your reference and should not be given as a clue.

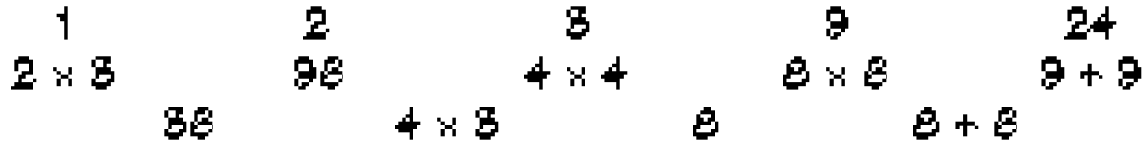


Exercise 2b

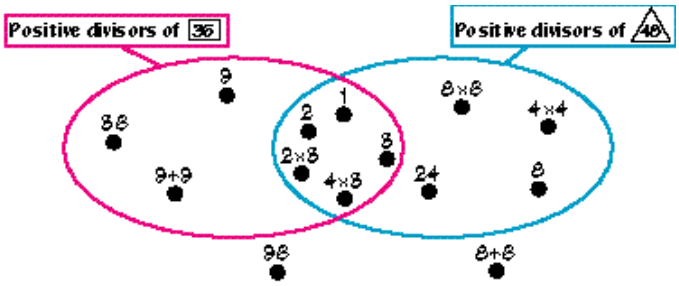
Label the strings in the picture from Exercise 1 in this way.



Ask students to explain where each of the following numbers belongs in the picture.



The following illustration shows the correct placement of these numbers.



Capsule Lesson Summary

Play *The Divisors Game* with either two or three strings. Play a round of *The Divisors Game* in which the class analyzes the placement of each number in the picture. In a game situation, use clues involving the placement of a certain numbers to determine the labels of two strings.

Materials

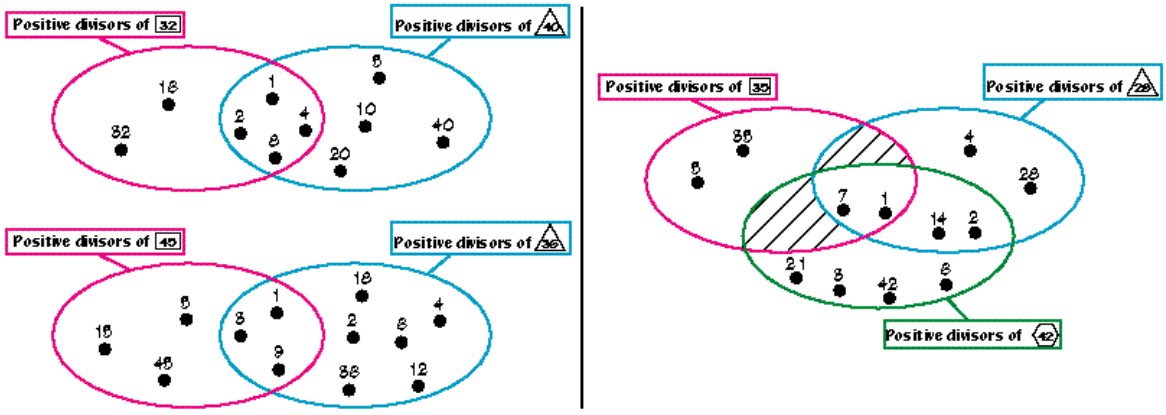
Teacher
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons

Description of Lesson

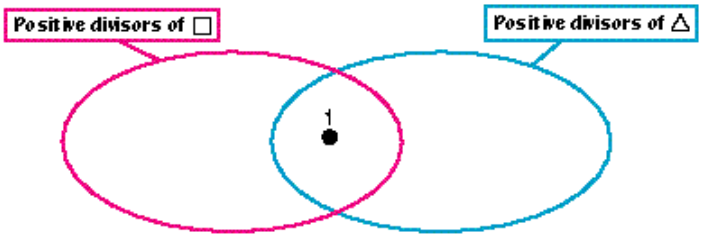
Exercise 1: The Divisors Game

Divide the class into teams and play *The Divisors Game* (described in Lesson L4) once or twice. The next illustration provides partial crib sheets for three possible games; choose from these or make up similar ones. Note that only the numbers inside the strings are shown in the crib sheets; assume that any whole number not shown in the pictures is outside the strings. The hatching is included for your reference and should not be given as a clue.



Exercise 2

Prepare the board for another round of *The Divisors Game*, and list the whole numbers from 1 to 50 next to the string picture.



1	11	21	31	41
2	12	22	32	42
3	13	23	33	43
4	14	24	34	44
5	15	25	35	45
6	16	26	36	46
7	17	27	37	47
8	18	28	38	48
9	19	29	39	49
10	20	30	40	50

L 7

Play a cooperative game where the class tries to identify the strings as quickly as possible. When a number is located in the picture, collectively discuss which numbers cannot be in □, and which numbers cannot be in △. If a number cannot be in □, cross the numeral in the list with a red slash in one direction; if a number cannot be in △, cross the numeral in the opposite direction in blue. Three types of analysis involved are explained in the examples illustrated below.

Note: Here the unknown numbers are 48 for □ and 39 for △.

If 2 is located inside the red string but outside the blue string, then the number in □ must be a multiple of 2. Odd numbers are not multiples of 2 and so can be slashed in red. The number in △ cannot be a multiple of 2, so the even numbers can be slashed in blue.

1	11	21	31	41
2	12	22	32	42
3	13	23	33	43
4	14	24	34	44
5	15	25	35	45
6	16	26	36	46
7	17	27	37	47
8	18	28	38	48
9	19	29	39	49
10	20	30	40	50

If 3 is located inside both the red and blue strings, then the number in □ and the number in △ must both be multiples of 3. Any number which is not a multiple of 3 can be slashed in red and blue.

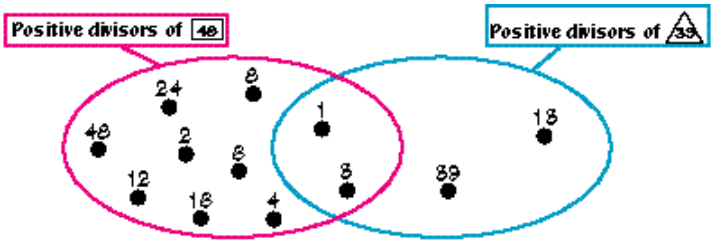
1	11	21	31	41
2	12	22	32	42
3	13	23	33	43
4	14	24	34	44
5	15	25	35	45
6	16	26	36	46
7	17	27	37	47
8	18	28	38	48
9	19	29	39	49
10	20	30	40	50

If 5 is located outside both strings, then the number in □ and the number in △ cannot be multiples of 5. Any number which is a multiple of 5 can be slashed in red and in blue.

1	11	21	31	41
2	12	22	32	42
3	13	23	33	43
4	14	24	34	44
5	15	25	35	45
6	16	26	36	46
7	17	27	37	47
8	18	28	38	48
9	19	29	39	49
10	20	30	40	50

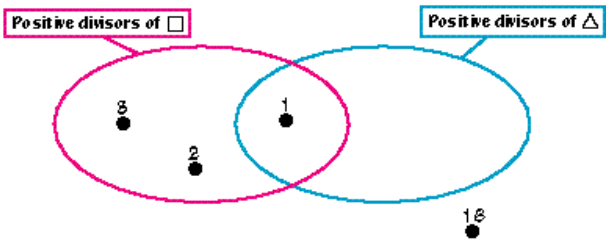
When a student uses a turn to identify the label for one of the strings, ask the student to convince the class, using the numbers in the picture. Continue until both strings are determined and correctly identified.

Here is a partial crib sheet showing only the numbers inside the strings.



Exercise 3 _____

Erase the board except leave the strings for another round of *The Divisors Game*. Put the numbers 1, 2, 3, and 16 in the picture, as shown here.

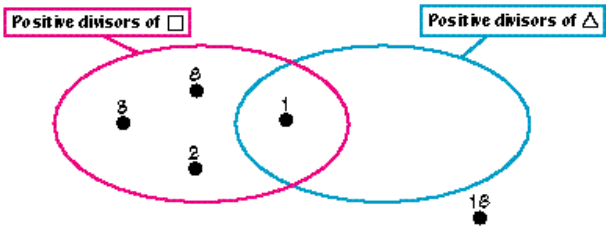


T: ***Suppose we are playing The Divisors Game and so far these numbers have been located in the picture. Also, suppose I stop the game and tell you that the numbers 8 and 11 do not belong outside both strings. Where do they belong? Can you determine what the strings are for? Remember that in The Divisors Game the numbers that can be in the square and in the triangle are restricted to whole numbers from 1 to 50.***

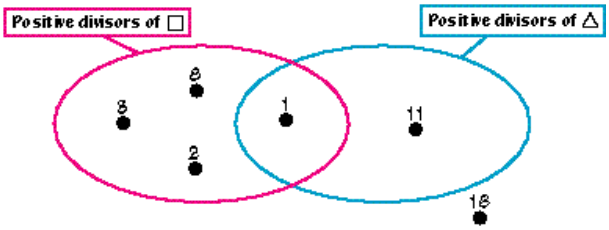
Instruct students to copy the picture on their papers, to locate 8 and 11, and to determine the string labels (there is sufficient information to do this). You may like to organize the class into cooperative groups for this exercise.

After a few minutes, invite students to draw and label dots for 8 and 11 in the picture on the board.

S: ***2 must be in the same string(s) that 8 is in. Since we know where 2 belongs, we also know where 8 belongs.***



S: ***11 cannot be in the red string; otherwise that string would be for the positive divisors of a number greater than 50. If 11, 2, and 3 were in the same string, that string would be for the positive divisors of a multiple of 66 because 66 is the least common multiple of 11, 2, and 3.***



Collectively discuss how to determine the string labels. A sample analysis is given below.

- 8 is in the red string so the number in the square must be a multiple of 8 between 1 and 50, namely:

8 16 24 32 40 48

- 16 is outside the red string so the number in the square cannot be a multiple of 16. Cross out 16, 32, and 48.

8 ~~16~~ 24 ~~32~~ 40 ~~48~~

- 3 is in the red string so the number in the square must be a multiple of 3 between 1 and 50. Cross out 8 and 40.

- 11 is in the blue string so the number in the triangle must be a multiple of 11 between 1 and 50, namely:

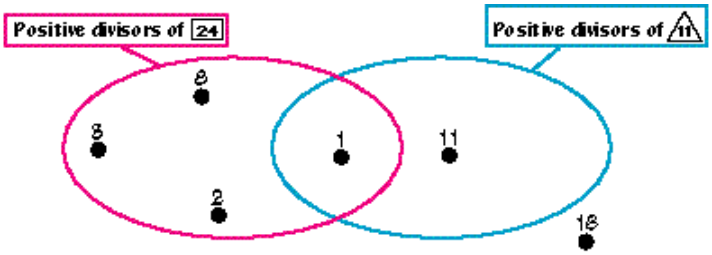
11 22 33 44

- 2 is not in the blue string so the number in the triangle cannot be a multiple of 2. Cross out 22 and 44.

11 ~~22~~ 33 ~~44~~

- 3 is not in the blue string so the number in the triangle cannot be a multiple of 3. Cross out 33.

This picture shows the strings correctly labeled.



Ask students if there are other numbers which belong inside either string. (24, 12, 4, and 6 belong in the red string but outside the blue string.)