Capsule Lesson Summary

Using an arrow picture for support, locate ordered pairs of numbers in a string picture involving the relation **is a multiple of**. Do a similar activity with the relation x10. Let students ask for the location of ordered pairs in a one-string picture to help them decide which of ten relations the string is for.

Materials

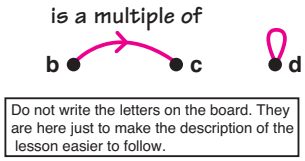
Teacher
- Colored chalk

Student
- Paper
- Colored pencils, pens, or crayons

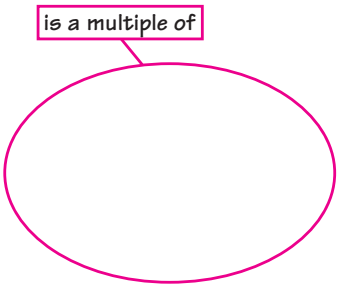
Description of Lesson

Exercise 1

Draw these pictures on the board.

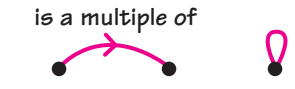


Do not write the letters on the board. They are here just to make the description of the lesson easier to follow.



T: ***What numbers could these dots*** (point to **b** and **c**) ***be for?***

S: ***4 and 2. 4 is a multiple of 2.***



4	2
100	5
30	3
18	6
36	12

Record correct responses in lists below the dots. Continue the activity until four or five possibilities for **b** and **c** have been suggested, as shown here.

Ask for possibilities satisfying certain conditions. For example:

- **b** is more than 30. (32, 8)
- **b** is between 40 and 48. (45, 5)
- **b** is negative. ($^{-}$9, 3)
- **b** is 0. (If **b** is 0, **c** can be any integer)
- **c** is more than 50. (204, 102)
- **c** is 17. (51, 17)
- **c** is negative. ($^{-}$63, $^{-}$7)
- **c** is 1. (If **c** is 1, **b** can be any integer)

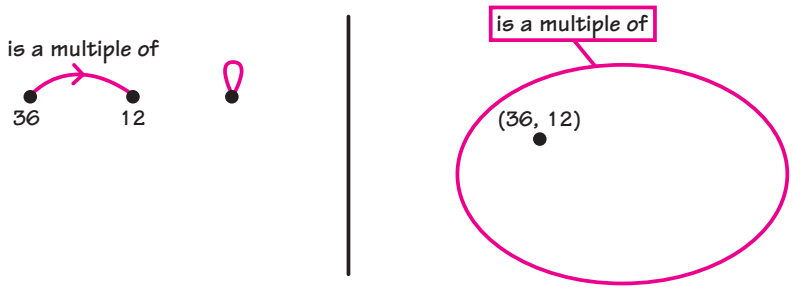
Note: In the series of lessons on *The Relations Game*, we will use raised minus signs for negative numbers since the game involves many calculator relations.

T: ***This string is related to this arrow.***

Point to one possibility for **b** and **c**, for example (36, 12).

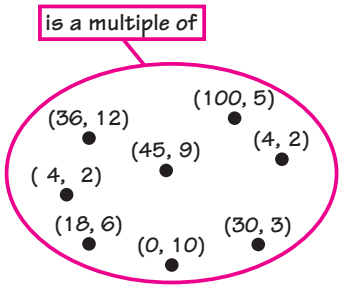
T: ***36 is a multiple of 12. We can show this in the string picture.***

Draw a dot for (36, 12) inside the string.



T: ***We have a first number*** (indicate 36 in each picture) ***and a second number*** (indicate 12 in each picture)***. (36, 12) is called an*** **ordered pair** ***or a*** **couple.** ***What are some other ordered pairs that belong inside the string?***

Let students offer several ordered pairs belonging to the relation **is a multiple of**, as illustrated here.



T (tracing the loop)**:** ***What number could this dot (*****d****) be for?***

Any integer is a multiple of itself, so the dot could be for any integer. A sample dialogue follows. Put the appropriate ordered pairs inside the string as they are mentioned.

S: ***0.***

T (putting (0,0) inside the string)**:** ***Yes, 0 is a multiple of 0.***

S: ***1.***

T: ***That's also right; 1 is a multiple of 1. So what ordered pair belongs inside the string?***

S: ***(1, 1).***

S: ***The dot with a loop could be for 9 because 9 is a multiple of 9, so (9, 9) belongs inside the string.***

T: ***What about (⁻7. ⁻7)?***

S: ***It belongs inside the string because ⁻7 is a multiple of ⁻7.***

T: ***How do you know?***

S: ***1 x ⁻7 = ⁻7, and 1 is an integer.***

T (pointing to **d**)**:** ***Could any integer be here?*** (Yes)

Review what numbers are integers by checking numbers such as these.

2.0	(Yes)	⁻3.0	(Yes)	0.4	(No)
$\frac{1}{2}$	(No)	$\frac{2}{3}$	(No)	$\frac{4}{2}$	(Yes)
100	(Yes)	⁻3143	(Yes)	$\frac{-5}{3}$	(No)

T: ***Both numbers in an ordered pair for the relation*** **is a multiple of** ***are integers.***

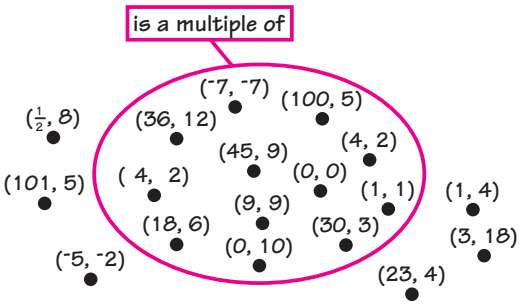
In *The Relations Game*, it will be important for students to remember this restriction.

Note: Although $4 \times \frac{1}{3} = \frac{4}{3}$ and 4 is an integer, the ordered pair $(\frac{4}{3}, \frac{1}{3})$ does not belong to this relation because the numbers are not integers. Likewise, $(2, \frac{1}{2})$ does not belong to this relation.

T: ***Where does (1, 4) belong in this string picture?***

S: ***Outside the string.***

Let students suggest other ordered pairs that belong outsi the string and put them in the picture, as illustrated here.



T: ***Let's find some ordered pairs in which neither number is an integer.***

S: ***(0.4, 96.6).***

S: ***$(\frac{1}{2}, \frac{1}{3})$.***

T: ***Where do such ordered pairs belong in the string picture?***

S: ***Outside the string.***

T: ***What about $(\frac{1}{4}, \frac{1}{8})$?*** (Outside) ***$(1, \frac{1}{2})$?*** (Outside)
What about $(\frac{18}{3}, \frac{6}{3})$?

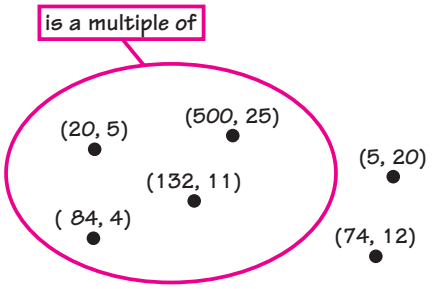
S: ***Inside the string, because $\frac{18}{3} = 6$, $\frac{6}{3} = 2$, and 6 is a multiple of 2.***

Erase all of the ordered pairs from the string picture; then write a list of ordered pairs nearby. For example:

(5, 20)	(500, 25)	(74, 12)
(20, 5)	(132, 11)	(84, 4)

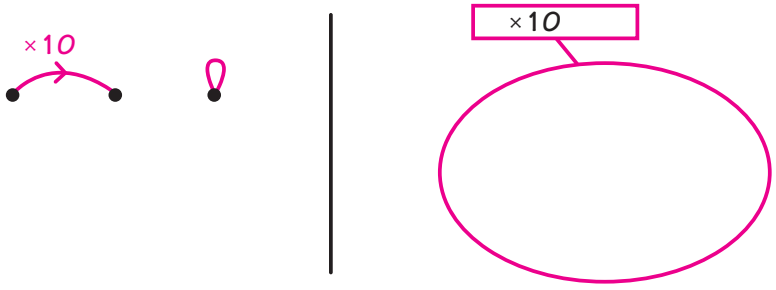
T: ***On a piece of paper, draw a string for the relation* is a multiple of *and locate these ordered pairs in your string picture.***

After a few minutes of individual work, discuss the correct location of each ordered pair.



Exercise 2

Change the pictures in Exercise 1 to consider the relation x10.



T: ***Let's consider the relation x10. This relation is not restricted to integers.***

Proceed in the following manner.

- Ask for ordered pairs that belong inside the string. Use the arrow picture for support. Be sure that some ordered pairs with negative components are included as well as some with positive components.
- Give one component of an ordered pair belonging to the relation x10 and ask for the other. Some examples are given below with answers in boxes. Take advantage of this opportunity to review multiplication of fractions and to find equivalent names for fractions.

$(\frac{1}{3}, \boxed{3\frac{1}{3}})$ $(\frac{3}{2}, \boxed{15})$ $(\boxed{\frac{4}{30}}, \frac{4}{3})$

$(\frac{2}{5}, \boxed{4})$ $(\boxed{\frac{2}{3}}, \frac{20}{3})$ $(\boxed{^{-}9}, {}^{-}90)$

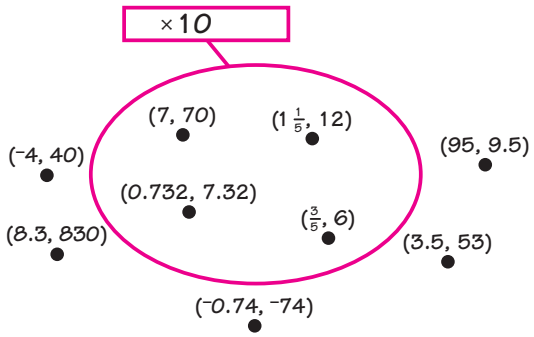
Refer to the opposite relation (return arrow) ÷10, or x$^{1}/_{10}$, whenever it would be helpful.

- Ask what number is at the dot with the loop. (0)
- Ask for ordered pairs that belong outside the string.
- List ordered pairs such as these on the board, and ask students to locate them in a string picture for x10 on their papers.

(7, 70)	$(^{-}0.74, {}^{-}74)$	$(\frac{3}{5}, 6)$
(95, 9.5)	(0.732, 7.32)	$(1\frac{1}{5}, 12)$
(8.3, 830)	$({}^{-}4, 40)$	(3.5, 53)

Allow a few minutes for individual work before discussing the correct location of each ordered pair.

Erase everything on the board except the string before going on to Exercise 3.



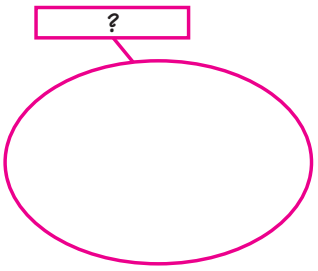
Exercise 3

List these relations on the board. Review them with the class. Mention that the relation **is a positive divisor of** is restricted to integers.

Note: The string is for +5=···, but do not reveal this fact to students yet.

T: ***This string is for one of these ten relations. I'll give you a clue as to which one.***

is less than
is at least 20 less than
is a multiple of
is a positive ivisor of
×10
×2
2=···
+3=···
+4=···
+5=···



Draw a dot for (8, 88) inside the string.

T: ***(8, 88) belongs to the relation. Which of these relations*** (point to the list) ***can we cross off?***

S: **Is a multiple of** ***because 8 is not a multiple of 88.***

S: **x10**, ***because 8 x 10 = 80, not 88.***

S: **x2**, ***because 8 x 2 = 16, not 88.***

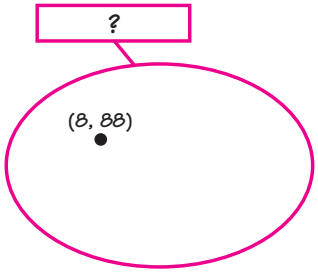
S: ***Cross off*** +3= ···, ***because the difference between 8 and 88 is 80, and 80 is not a multiple of 3.***

S: ***When you add 3s starting at 8, you can get 89 but not 88.***

S: ***8 is two more than a multiple of 3, but 88 is one more than a multiple of 3.***

With the class, check that (8, 88) belongs to the remaining six relations.

is less than
is at least 20 less than
~~is a multiple of~~
is a positive ivisor of
~~x10~~
~~x2~~
+2= ···
~~+3= ···~~
+4= ···
+5= ···



T: ***Suggest an ordered pair whose location, whether inside or outside the string, would eliminate some of these relations as possibilities.***

When a student suggests an ordered pair, place it correctly in the picture. For example:

S: ***(10, 20).***

T: ***(10, 20) goes inside the string. Does this eliminate some relations as possibilities for the string? What can we cross off the list?***

S: **Is at least 20 less than**; ***10 is only 10 less than 20, not at least 20 less than 20.***

S: +4= ··· ***because 10 + 4 = 14, 14 + 4 = 18, and 18 + 4 = 22. 20 is skipped.***

T: ***We need some more information. What is another ordered pair whose location would help us eliminate some other of these relations?***

S: ***(1, 2).***

T: ***(1, 2) goes outside the string. What can we cross off the list?***

S: **Is less than** ***because 1 is less than 2.***

S: **Is a positive divisor of** ***because 1 is a positive divisor of 2.***

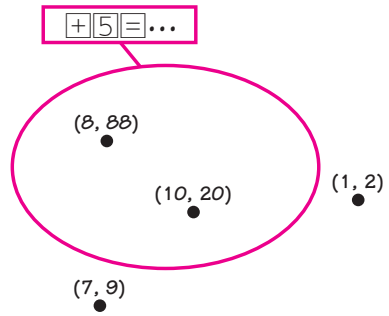
T: ***The relation is either*** +2= ··· ***or*** +5= ··· .

S: ***(7, 9).***

T: ***(7, 9) goes outside the string.***

S: ***The string is for*** +5= ···.

~~is less than~~
~~is at least 20 less than~~
~~is a multiple of~~
~~is a positive ivisor of~~
~~×10~~
~~×2~~
~~+2=...~~
~~+3=...~~
~~+4=...~~
+5=...



Capsule Lesson Summary

Review the idea of locating ordered pairs of numbers in a string for a relation. Let students ask for the location of ordered pairs in a one-string picture to help them decide which of ten relations the string is for. Introduce *The Relations Game*.

Materials

Teacher
- Colored chalk
- Relations Game poster
- Relations Game scoring sheet
- Tape
- Blacklines 7(a) and (b)

Student
- Relations Game analysis sheet

Advance Preparation: Use Blackline L7(a) to make copies of *The Relations Game* analysis sheet for students. Use Blackline L7(b) to make copies of a scoring sheet for use in play of the game (Exercise 3).

Description of Lesson

Exercise 1

Draw these pictures on the board.

T: ***Last week we used both arrows and strings for relations. Can someone explain what the string has to do with the arrow?***

S: ***Suppose the red arrow is for* x10*. If the beginning dot is for 2, then the ending dot is for 20. The ordered pair (2, 20) goes inside the string.***

T: ***(2, 20) goes inside the string for* x10*. Would (20, 2) also go inside the string?***

S: ***No, because 20 x 10 = 200, not 2.***

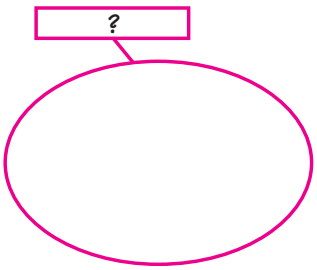
Review this idea with a couple other examples of relations; for example, +3= ··· … and $\times\frac{2}{3}$.

Exercise 2

List these relations on the board near the string picture.

T: ***This string is for one of these ten relations. I'll give you a clue as to which one.***

is less than
is at least 20 less than
is a positive ivisor of
×10
×2
$\times\frac{2}{3}$
+2= ···
+3= ···
+4= ···
+5= ···



Note: The string is for the relation **is at least 20 less than**. Do not reveal the relation to the students at this time. The students will discover it during the activity.

Draw a dot inside the string for (3.5, 63½).

T: ***(3.5, 63½) belongs to the relation. Are there any of these relations*** (point to the list) ***that the string cannot be for?***

S: **x10** ***and*** **x2.** ***3.5*** x ***10 = 35 and 3.5*** x ***2 = 7, not 63½.***

Cross a relation off the list when a student explains why the string cannot be for that relation.

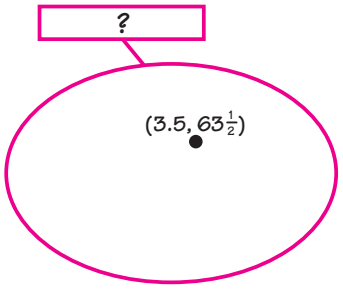
S: $\times\frac{2}{3}$. *3.5* x ⅔ = ⅞.

S: ***The relation*** **is a positive divisor of** ***has only to do with integers, so we can cross it off.***

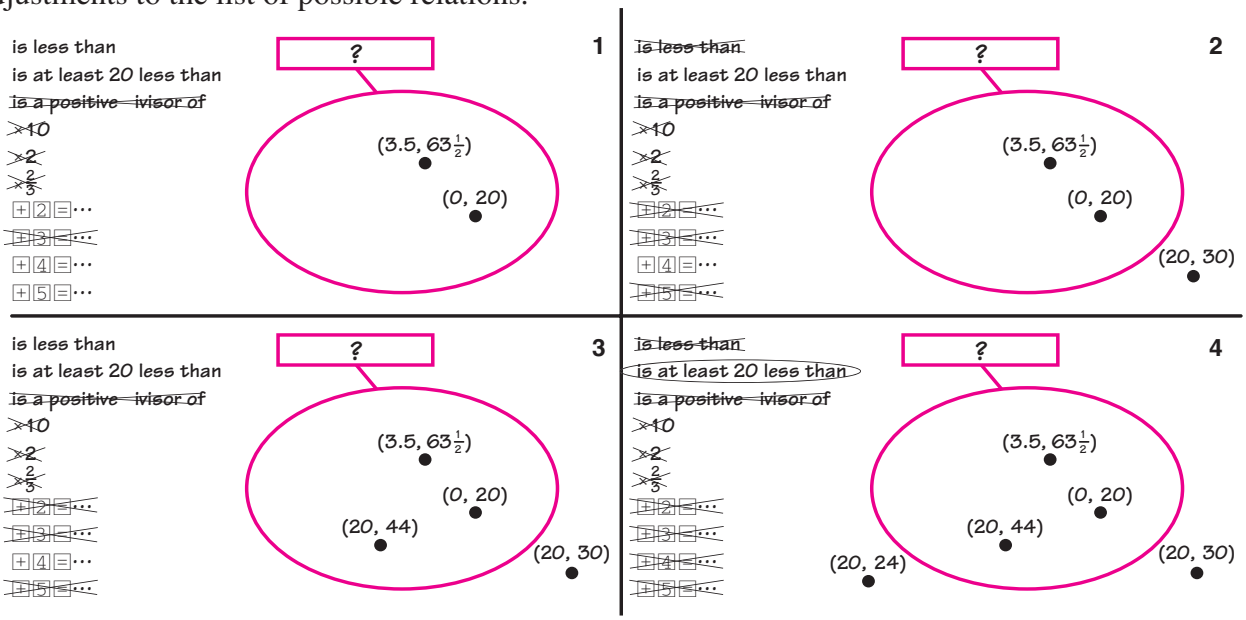
With the class, check that (3.5, 63½) belongs to the remaining relations.

T: ***Can you suggest an ordered pair whose location, whether inside or outside the string, would eliminate some of these other possibilities?***

is less than
is at least 20 less than
~~is a positive divisor of~~
~~×10~~
~~×2~~
~~×⅔~~
+2=…
+3=…
+4=…
+5=…



In the string picture, locate whichever ordered pair is suggested. The class should use the information of where the ordered pair belongs to eliminate, if possible, some of the remaining relations. Continue asking for ordered pairs and eliminating possibilities until the relation is determined. The illustrations that follow record a sequence of suggested ordered pairs and the corresponding adjustments to the list of possible relations.



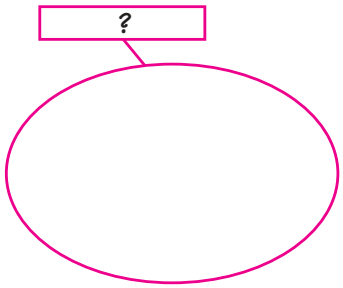
The location of (20, 44) gives no new information.

Exercise 3: The Relations Game

The rules of *The Relations Game* are similar to the rules of *The Table Game*. In *The Relations Game*, players attempt to determine which of 21 given relations (see the poster) a string[†] is for. The game is divided into two parts. In both parts players select ordered pairs, but only in the second part do they predict the location of ordered pairs before they are put correctly in the picture. Scoring rules change from one part to the next. To play the usual version of *The Relations Game*, proceed as follows.

On the board draw a large string and display a copy of *The Relations Game* poster near the string.

is the square of	The Relations Game		
is a multiple of	is a positive divisor of	is less than	is greater than
x2	x10	$\times \frac{2}{3}$	is at least 20 greater than
÷2	÷10	$\div \frac{2}{3}$	is at least 20 less than
+2=···	+3=···	+4=···	+5=···
−2=···	−3=···	−4=···	−5=···



Select a student who will be a reliable scorekeeper[††]. Divide the rest of the class into four (or two) teams. In the following example, the four teams are referred to as Team A, Team B, Team C, and Team D. Distribute Relations Game analysis sheets to students.

The game begins with one to three ordered pairs in the string picture as starting clues. Such ordered pairs can belong either inside or outside the string. Players, either independently or with their team members, use the clues to try to eliminate some of the 21 possibilities for the relation. Before further play, give students the option to turn in their analysis sheets with one of the relations circled. A team gains 15 points each time a member correctly identifies the relation before additional plays are made. If team members are consulting among themselves, you may prefer to ask the team to turn in just one sheet. The scorekeeper tallies the points for each team but does not reveal the score at this time.

After a few minutes, proceed by letting each team make a play. A play consists of selecting an ordered pair to put in the picture as an additional clue. After each play, students or teams may turn in their analysis sheets with a relation circled. Correctly identifying the relation gains a certain number of points (see the scoring sheet) depending upon how many plays (additional clues) have been given. With two teams, each team makes two plays. The last time students (or teams) may turn in their analysis sheets is after the fourth play is made.

Note: You may find reason to vary the number of additional clues—for example, if time is running short, if most players have turned in their analysis sheets after two additional clues, or if very few players have turned in their analysis sheets after four additional clues, and so on.

[†]See Lesson L11 for a description of how to play this game with two strings.

[††] The scorekeeper can be an adult who is available during the class time, such as a teacher's aide or student teacher.

Now the scoring changes. During the second part of the game, each player (alternating teams) selects an ordered pair and says whether it belongs inside or outside the string. Announce whether or not the play is correct, and put the suggested ordered pair correctly in the picture. The scoring for this part of the game is as follows:

- A team gains two points each time a member correctly locates an ordered pair inside the string.
- A team gains one point each time a member correctly locates an ordered pair outside the string.
- A team neither gains nor loses points when a member incorrectly locates an ordered pair.

The second part of the game ends after each player has had a turn. Call on a student to identify the secret relation while the scorekeeper computes the scores. It is a good idea to provide a scoring sheet for the scorekeeper to use. Blackline L7 has a sample.

Secret Relation: _____

Points		**TEAM A**	**TEAM B**	**TEAM C**	**TEAM D**
First part of game					
For **correctly** identifying the relation					
• before additional plays	15 pts.				
• after first play	10 pts.				
• after second play	9 pts.				
• after third play	8 pts.				
• after fourth play	7 pts.				
Second part of game					
For **correctly** locating an ordered pair					
• inside the string	2 pts.				
• outside the string	1 pt.				

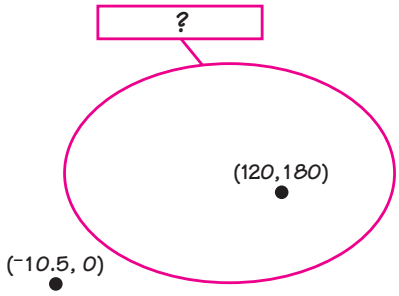
Optional Rule: In the second part of the game, you can encourage students to suggest many kinds of numbers by awarding an extra point for each non-whole number in an ordered pair correctly located inside the string. For a game played with the optional rule, modify the scoring sheet as shown below.

Secret Relation: _____

Points		**TEAM A**	**TEAM B**	**TEAM C**	**TEAM D**
First part of game					
For **correctly** identifying the relation					
• before additional plays	15 pts.				
• after first play	10 pts.				
• after second play	9 pts.				
• after third play	8 pts.				
• after fourth play	7 pts.				
Second part of game					
For **correctly** locating an ordered pair					
• inside the string (bonus for non-whole numbers 1 pt. each)	2 pts.				
• outside the string	1 pt.				

Play the game with your class. The rest of this lesson description presents a possible game. The secret relation is $\div\frac{2}{3}$; there are two ordered pairs—(120, 180) and (⁻10.5, 0)—located as starting clues.

is the square of	The Relations Game		
is a multiple of	is a positive divisor of	is less than	is greater than
x2	x10	$\times\frac{2}{3}$	is at least 20 greater than
÷2	÷10	$\div\frac{2}{3}$	is at least 20 less than
+2=···	+3=···	+4=···	+5=···
−2=···	−3=···	−4=···	−5=···



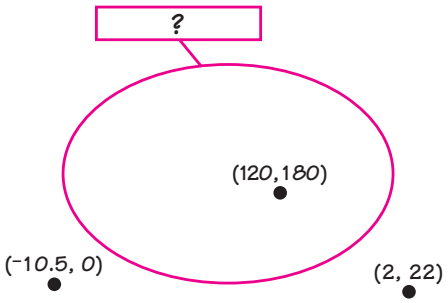
T: ***Knowing that (120, 180) belongs inside the string and that (⁻10.5, 0) belongs outside the string should help you to eliminate some of the possibilities for the relation. On your analysis sheet cross out relations the string cannot be for.***

Allow a few minutes for analysis. Then ask if anyone wants to turn in an analysis sheet for a possible 15 points.

Note: For your information only, there are six remaining possibilities for the secret relation: **is at least 20 less than**, $\div\frac{2}{3}$, +2=···, +3=···, +4=···, and +5=···.

No one turns in an analysis sheet.

Call on a player from Team A to suggest an ordered pair for you to locate.



S: ***(2, 22).***

T: ***(2, 22) belongs outside the string.***

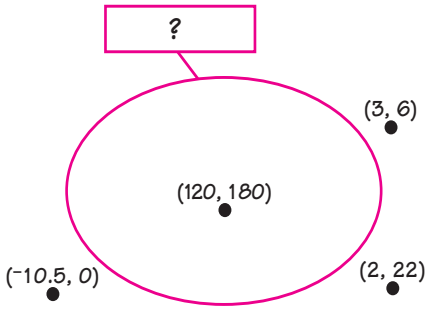
Allow a few minutes for players to analyze the situation.

Note: This clue eliminates four more possibilities and there are two remaining possibilities: $\div\frac{2}{3}$ and +3=···.

T: ***Does anyone want to turn in their sheets for a possible ten points?***

No one turns in an analysis sheet.

Call on a player from Team B to suggest another ordered pair.



S: ***(3, 6).***

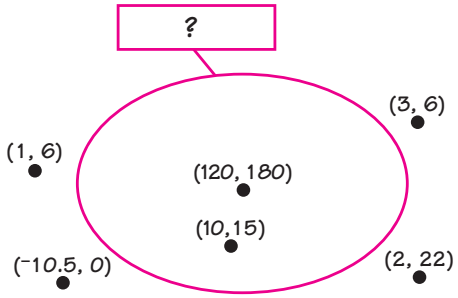
T: ***(3, 6) belongs outside the string.***

Allow a few minutes for the analysis.

T: ***Does anyone want to turn in their sheets for a possible nine points?***

Several students turn in their analysis sheets. (The relation is actually determined at this point.)

Next, players from Team C and Team D suggest (10, 15) and (1, 6) respectively. More students turn in their analysis sheets for eight points or for seven points.



The scorekeeper tallies the points in the game so far but does not reveal the totals to the class.

Team A	Team B	Team C	Team D
34	33	30	33

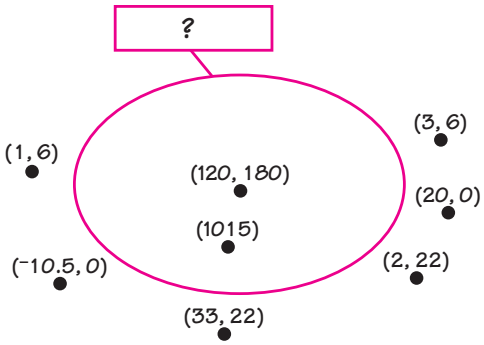
Now the scoring changes. Players select ordered pairs and state whether each belongs inside or outside the string.

S (Team A)**:** ***(20, 0) belongs outside the string.***

T: ***Yes. Team A gains one point.***

S (Team B)**:** ***(33, 22) belongs inside the string.***

T: ***No, (33, 22) belongs outside. Team B neither gains nor loses points.***



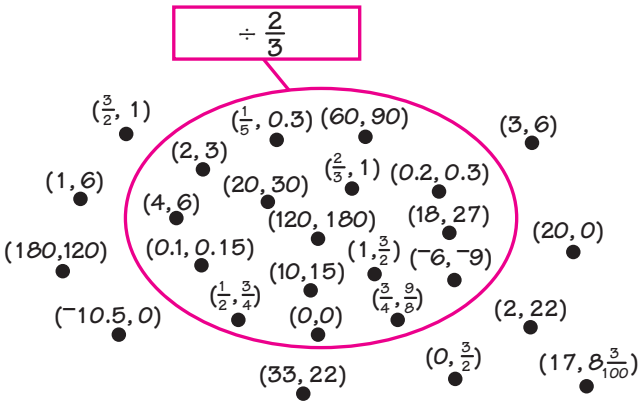
The game continues until every player has had a turn.

T: ***While the scorekeeper is computing the scores, who can tell us which relation the string is for?***

S: $\div \frac{2}{3}$.

The scorekeeper reveals the scores. Team B wins.

The string picture and the scorekeeper's account at the end of the game are shown below. (In this example, the optional rule was in effect.)



Team A	Team B	Team C	Team D
34	33	30	33
1	0	4	1
0	2	2	0
4	4	0	4
2	2	2	3
1	4	4	3
42	45	42	44

Capsule Lesson Summary

Complete ordered pairs of numbers, some which belong and some which do not belong to the relation **is the square of**. In a relations game context, use three clues to eliminate possibilities for a string's label. Then investigate how the location of various ordered pairs would distinguish the four remaining possibilities. Play *The Relations Game*.

Materials

Teacher
- Colored chalk
- Relations Game poster
- Relations Game scoring sheet
- Markers
- Blacklines L7(a) and (b)

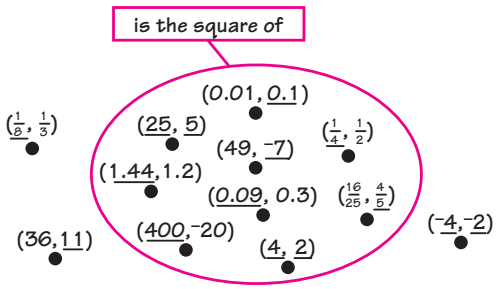
Student
- Relations Game analysis sheet
- Paper
- Colored pencils, pens, or crayons

Advance Preparation: Use Blackline L7(a) to make copies of *The Relations Game* analysis sheet for students. Use Blackline L7(b) to make copies of a scoring sheet for use in play of the game (Exercise 3).

Description of Lesson

Exercise 1

Draw this string picture on the board, but leave blanks where there are underlined numerals. Ask students to copy the picture and to fill in the blanks. Allow about five minutes for students to work individually or with partners; then review possible answers collectively. The illustration here shows one of the many possible ways to complete the picture.

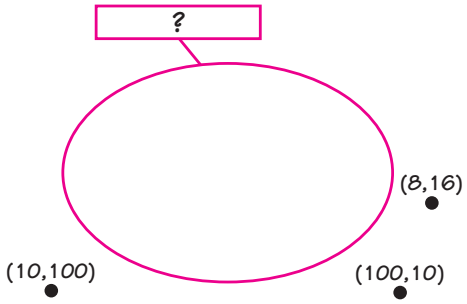


Comments

- Be aware that there are many correct ways to fill in each blank since every number has many names. For example, the square of ½ may be written as ¼, ⅛, 0.25, and so on.
- When multiplying decimals, students may prefer to do the calculation with fractions. For example, 0.3 x 0.3 = $\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$ = 0.09.
- Observe that there is only one number that is the square of another number, but that a positive number is the square of two different numbers. For example, in this exercise there is only one possibility for the first number of the ordered pair (_____, 1.2), namely 1.44. But there are two possibilities for the second number of the ordered pair (49, _____), namely 7 and −7.
- Notice that the square of a number is not always greater than the number; for example, $(\frac{1}{2})^2 = \frac{1}{4}$ and $\frac{1}{4} < \frac{1}{2}$. For your information, the square of a number between 0 and 1 is always less than the number.

Exercise 2

Draw this string picture on the board and display a Relations Game poster nearby.



is the square of	The Relations Game		
is a multiple of	is a positive divisor of	is less than	is greater than
x2	x10	$\times\frac{2}{3}$	is at least 20 greater than
÷2	÷10	$\div\frac{2}{3}$	is at least 20 less than
+2=···	+3=···	+4=···	+5=···
−2=···	−3=···	−4=···	−5=···

T: *The string is for one of these relations* (point to the poster)*. There are three clues in the picture. Which relations can we cross off, and why?*

S: *Cross off* **x10** *and* **÷10**. *10* x *10 = 100 and (10, 100) is outside the string; 100 ÷ 10 = 10 and (100, 10) is outside the string.*

S: *100 is 90 greater than 10 so cross off* **is at least 20 greater than** *and* **is greater than.**

S: *Since 90 (the difference between 100 and 10) is a multiple of 2, 3, and 5, we can cross off* −2=···, −3=···, *and* −5=···. *But we cannot cross off* −4=··· *because 90 is not a multiple of 4.*

S: *Likewise 10 is 90 less than 100. So we can cross off …***is at least 20 less than, is less than,** +2=···, +3=···, *and* +5=···.

Continue the collective analysis until only four labels remain as possibilities.

T: *There are still four possibilities for the string's label. If we knew an ordered pair that belonged inside the string, would we know what the string is for?*

~~is the square of~~	The Relations Game		
~~is a multiple of~~	~~is a positive divisor of~~	~~is less than~~	~~is greater than~~
~~x2~~	~~x10~~	$\times\frac{2}{3}$	~~is at least 20 greater than~~
÷2	~~÷10~~	$\div\frac{2}{3}$	~~is at least 20 less than~~
~~+2=···~~	~~+3=···~~	~~+4=···~~	~~+5=···~~
~~−2=···~~	~~−3=···~~	−4=···	~~−5=···~~

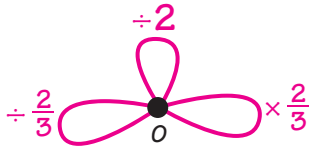
Allow a few minutes for students to consider the situation before discussing it. Perhaps their initial reaction will be that the location of any ordered pair inside the string would determine the string label. To investigate, ask for ordered pairs that could belong inside the string, and ask students what their location tells about the string label. For example:

S: *If (6, 4) belongs inside the string, then the string is for the relation* $\times\frac{2}{3}$*; 6* x ⅔ *= 4. But 6 ÷ 2 = 3, not 4. 6 ÷ ⅔ = 9, not 4. And* 6 − 4 = *… never equals 4.*

S: *If (30, 10) belongs inside the string, then the string is for the relation* −4=··· *because the difference between 30 and 10 is 20, a multiple of 4. But 30 ÷ 2 = 15, not 10. Also, 30* x ⅔ *= 20, not 10, and 30 ÷ ⅔ = 45, not 10.*

T: *Can an ordered pair ever belong to more than one of these four relations?*

If no one suggests (0, 0), do so yourself.



The class should find that (0, 0) belongs to three of the four relations.

T: ***Can an ordered pair ever belong to ÷2 and −4= ···? To both × 2/3 and −4= ···? To both ÷ 2/3 and −4= ···?***

While students consider the situation, draw this table on the board.

÷2	× 2/3	÷ 2/3

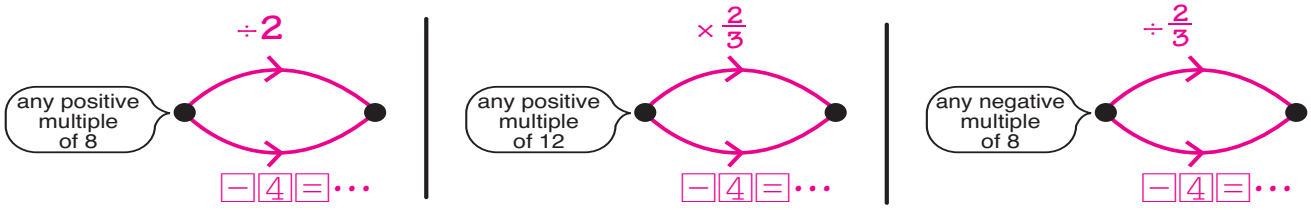
T: ***Let's find some ordered pairs that belong to each of these relations, trying to find some that also belong to the relation −4= ···.***

Accept and record ordered pairs for each relation. Each time an ordered pair is suggested, ask the class to check if it also belongs to the relation −4= ···; circle an ordered pair that does.

Continue until the class finds at least two ordered pairs for each of the three relations that also belong to the relation −4= ···. Your table might look similar to this one.

÷2	× 2/3	÷ 2/3
(10, 5)	(1, 2/3)	(10, 15)
(8, 4)	(6, 4)	(2, 3)
(12, 6)	(9, 6)	(4, 6)
(7, 3 1/2)	(12, 8)	(−4, −6)
(16, 8)	(15, 10)	(−8, −12)
(24, 12)	(24, 16)	(−16, −24)
(−8, −4)	(−12, −8)	(−24, −36)

Note: During the activity, the class may begin to see that there are infinitely many possibilities in each case, as summarized in the pictures below.



T: ***Getting back to our original question, would we necessarily know which of the four relations the string is for if we knew an ordered pair that belonged inside the string?***

S: ***No; knowing that (8, 4) belongs inside the string, for example, would not tell us whether the string is for ÷2 or for −4= ···.***

Any of the circled ordered pairs in the table could be used in an explanation.

Exercise 3

Play *The Relations Game* if there is sufficient time remaining in the class period. (See Lesson L7, Exercise 3 for a description of the game.) For a possible game, let the string be for the relation **is a positive divisor of**, and locate (1.5, 21.5) outside the string as a starting clue.

Capsule Lesson Summary

Play *The Relations Game*. Locate some ordered pairs in each of the four regions of a two-string picture in which one string is for the relation [+][5][=]··· and the other is for the relation **is at least 20 less than**.

Materials

Teacher
- Colored chalk
- Relations Game poster
- Relations Game scoring sheet
- Colored markers or crayons
- Blacklines L7(a) and (b)

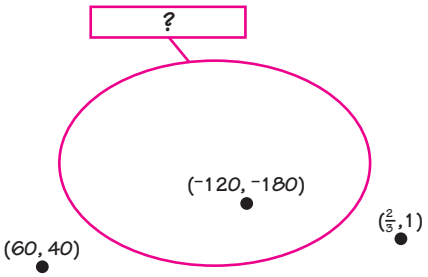
Student
- Relations Game analysis sheet
- Paper
- Colored pencils, pens, or crayons

Advance Preparation: Use Blackline L7(a) to make copies of *The Relations Game* analysis sheet for students. Use Blackline L7(b) to make a scoring sheet for use in play of the game (Exercise 1).

Description of Lesson

Exercise 1

Play *The Relations Game*. For a possible game, let the string be for the relation [−][3][=]···, and locate three ordered pairs in the string picture as starting clues. You may want to tell the class that the relation is determined by these clues.



is the square of	The Relations Game		
is a multiple of	is a positive divisor of	is less than	is greater than
x2	x10	$\times\frac{2}{3}$	is at least 20 greater than
÷2	÷10	$\div\frac{2}{3}$	is at least 20 less than
[+][2][=]···	[+][3][=]···	[+][4][=]···	[+][5][=]···
[−][2][=]···	[−][3][=]···	[−][4][=]···	[−][5][=]···

After the game, collectively analyze the starting situation to find that the relation indeed is determined by the three clues.

Using the information that (−120, −180) is inside the string, cross off these relations.

~~is the square of~~	The Relations Game		
~~is a multiple of~~	~~is a positive divisor of~~	~~is less than~~	is greater than
~~x2~~	~~x10~~	~~$\times\frac{2}{3}$~~	is at least 20 greater than
~~÷2~~	~~÷10~~	$\div\frac{2}{3}$	~~is at least 20 less than~~
~~[+][2][=]···~~	~~[+][3][=]···~~	~~[+][4][=]···~~	[+][5][=]···
[−][2][=]···	[−][3][=]···	[−][4][=]···	[−][5][=]···

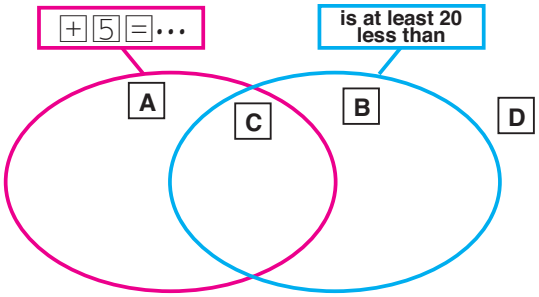
Using the information that (60, 40) and (⅔, 1) are outside the string, cross off additional relations.

~~is the square of~~	The Relations Game		
~~is a multiple of~~	~~is a positive divisor of~~	~~is less than~~	~~is greater than~~
~~x2~~	~~x10~~	~~$\times\frac{2}{3}$~~	~~is at least 20 greater than~~
~~÷2~~	~~÷10~~	~~$\div\frac{2}{3}$~~	~~is at least 20 less than~~
~~[+][2][=]···~~	~~[+][3][=]···~~	~~[+][4][=]···~~	~~[+][5][=]···~~
~~[−][2][=]···~~	[−][3][=]···	~~[−][4][=]···~~	~~[−][5][=]···~~

Your students might not use the clues in any particular order nor exhaust each clue completely. Allow the analysis to flow in a style that your students initiate.

Exercise 2

Draw this string picture on the board.



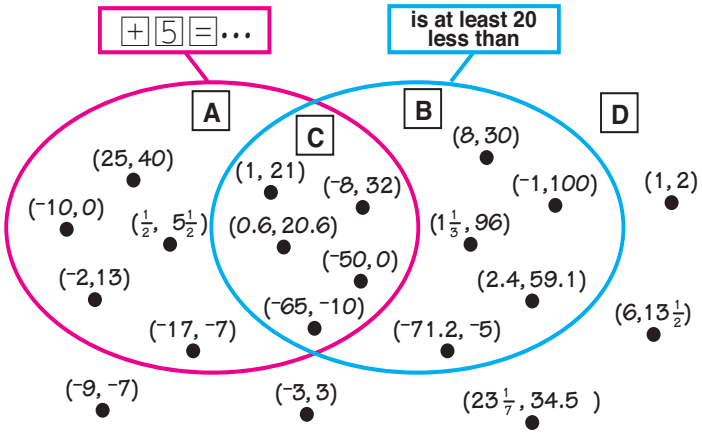
Ask for ordered pairs that belong to each of the four regions. Sometimes place conditions on the numbers, for example:

- Both numbers in the ordered pair are whole numbers.
- Only one number in the ordered pair is a whole number.
- Neither number in the ordered pair is a whole number.
- One number in the ordered pair is negative.
- Both numbers in the ordered pair are negatives.

When a student suggests an ordered pair, ask why it belongs in the region being discussed at that time. A good response might be the following.

S: ***(10, 15) belongs in region* A. *10 + 5 = 15, so (10, 15) belongs in the red string. 10 is not at least 20 less than 15, so (10, 15) does not belong in the blue string.***

After a while, your picture might look similar to this one.



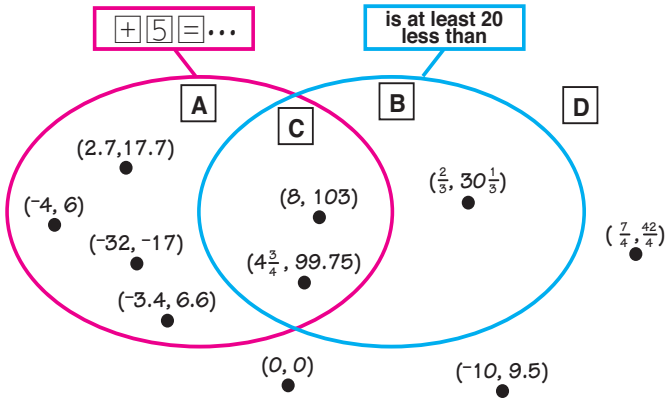
Erase all of the ordered pairs in the string picture; then write this list of ordered pairs near the picture.

(8, 103)	(−4, 6)	(−32, −17)	$(\frac{2}{3}, 30\frac{1}{3})$	(−10, 9.5)
(2.7, 17.7)	(0, 0)	(−3.4, 6.6)	$(\frac{7}{2}, \frac{42}{4})$	$(4\frac{3}{4}, 99.75)$

T: ***Copy the string picture on a piece of paper and locate these ordered pairs*** (point to the list)***.***

After about five minutes of individual or partner work, discuss the location of the pairs.

Accept any correct method, but encourage looking at the difference between the numbers of an ordered pair to decide whether or not the ordered pair belongs inside the red string. For example, the difference between −4 and 6 is 10, a multiple of 5, so (−4, 6) belongs inside the red string. Also, since −4 is not at least 20 less than 6, (−4, 6) belongs outside the blue string.



Capsule Lesson Summary

Determine relations for two strings when given three clues and placement of other ordered pairs suggested by the class. Play *The Relations Game* with two-strings, explaining minor changes in the rules when extending to a two-string game.

Materials

Teacher
- Colored chalk
- Relations Game posters
- Relations Game scoring sheet
- Colored markers
- Blacklines L11(a) and (b)

Student
- Relations Game analysis sheet
- Worksheets L11* and **

Advance Preparation: Use Blackline L11(a) to make copies of *The Relations Game* analysis sheet for students. Use Blackline L11(b) to make a scoring sheet for use in play of a two-string game (Exercise 2).

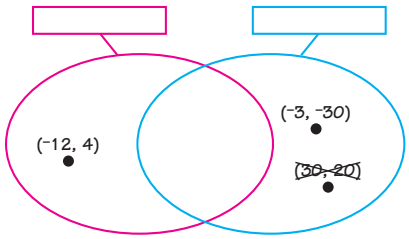
Description of Lesson

Exercise 1

Draw this string picture on the board and display two Relations Game posters, one on each side of the string picture.

Red

is the square of	The Relations Game		
is a multiple of	is a positive divisor of	is less than	is greater than
x2	x10	$\times\frac{2}{3}$	is at least 20 greater than
÷2	÷10	$\div\frac{2}{3}$	is at least 20 less than
+2=···	+3=···	+4=···	+5=···
−2=···	−3=···	−4=···	−5=···



Blue

is the square of	The Relations Game		
is a multiple of	is a positive divisor of	is less than	is greater than
x2	x10	$\times\frac{2}{3}$	is at least 20 greater than
÷2	÷10	$\div\frac{2}{3}$	is at least 20 less than
+2=···	+3=···	+4=···	+5=···
−2=···	−3=···	−4=···	−5=···

Note: The red string is for **is a multiple of** and the blue string is for **is at least 20 greater than.** Do not reveal the string labels at this time. Students will discover them during the activity.

T: ***There are three clues as to which of these relations*** (point to a poster) ***the strings are for. What does it mean to have (30, 20) crossed out?***

S: ***It means that (30, 20) does not belong in that region of the picture.***

T: ***Could (30, 20) belong in the blue string?***

S: ***Yes, but then it would be in the middle region belonging also in the red string.***

T: ***What can we cross off the Red list?***

S: ***All of the "minus" relations.***

T: ***Why?***

S: ***Because 4 is greater than $^-12$. If we subtract a positive number from $^-12$, we get a negative number less than $^-12$.***

Cross −2=···, −3=···, −4=···, and −5=··· off the Red list.

S: *Cross off* $\times \frac{2}{3}$*, because* $-12 \times \frac{2}{3} = -8$.

Continue the analysis until the class finds that only four possibilities remain for the red string. The location of $(-12, 4)$ is sufficient to eliminate 1 8 of the possibilities for the red string.

Red

~~is the square of~~	The Relations Game		
~~is a multiple of~~	~~is a positive divisor of~~	~~is less than~~	~~is greater than~~
~~×2~~	~~×10~~	~~$\times \frac{2}{3}$~~	~~is at least 20 greater than~~
~~÷2~~	~~÷10~~	~~$\div \frac{2}{3}$~~	~~is at least 20 less than~~
~~+2=···~~	~~+3=···~~	~~+4=···~~	~~+5=···~~
~~−2=···~~	~~−3=···~~	~~−4=···~~	~~−5=···~~

Blue

~~is the square of~~	The Relations Game		
~~is a multiple of~~	~~is a positive divisor of~~	~~is less than~~	is greater than
~~×2~~	×10	~~$\times \frac{2}{3}$~~	is at least 20 greater than
~~÷2~~	~~÷10~~	~~$\div \frac{2}{3}$~~	~~is at least 20 less than~~
~~+2=···~~	~~+3=···~~	~~+4=···~~	~~+5=···~~
~~−2=···~~	−3=···	~~−4=···~~	~~−5=···~~

T: *What can we cross off the Blue list?*

Let students suggest relations that can be crossed off. Using the location of $(-3, -30)$, students will be able to eliminate all but four of the possibilities for the blue string.

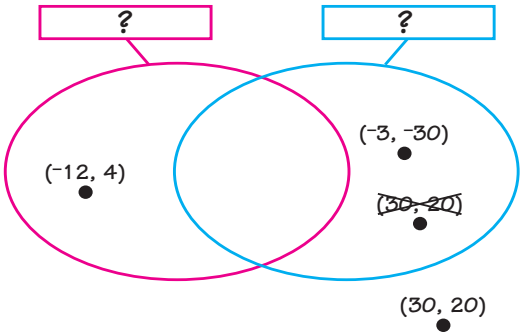
Determining the location of (30, 20) will allow students to eliminate one more possibility. You may need to prompt the use of this clue, as in the following dialogue.

T: *Where does (30, 20) belong? Could it be inside the red string?*

S: *(30, 20) cannot be inside the red string because there are only four possibilities left for the red string and (30, 20) does not belong to any of them: 30 is not a multiple of 20; 30 is not less than 20, and using* +2=··· *or* +4=··· *from 30 we get greater numbers, never 20.*

S: *(30, 20) belongs outside both strings.*

T: *Knowing that (30, 20) belongs outside both strings, what else can we cross off the Blue list?*

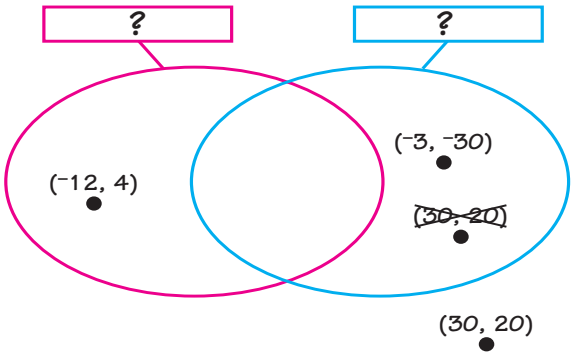


S: **Is greater than** *because 30 is greater than 20.*

T: *There are four possibilities left for the red string and three possibilities left for the blue string.*

Suggest some ordered pairs whose locations would help us determine what the string labels are.

A sample dialogue follows.



S: *(15, 25).*

T: *(15, 25) belongs outside both strings.*

S: *The red string cannot be for* +2=···*, because we can add 2s to 15 and get 25.*

S: *The red string cannot be for* **is less than** *because 15 is less than 25.*

S: *We cannot cross any more relations off the Blue list.*

T: *So now we have two possibilities left for the red string and still three possibilities left for the blue string.*

Red

is the square of	The Relations Game		
is a multiple of	is a positive divisor of	is less than	is greater than
x2	x10	x $\frac{2}{3}$	is at least 20 greater than
÷2	÷10	÷ $\frac{2}{3}$	is at least 20 less than
+2=···	+3=···	+4=···	+5=···
−2=···	−3=···	−4=···	−5=···

Blue

is the square of	The Relations Game		
is a multiple of	is a positive divisor of	is less than	is greater than
x2	x10	x $\frac{2}{3}$	is at least 20 greater than
÷2	÷10	÷ $\frac{2}{3}$	is at least 20 less than
+2=···	+3=···	+4=···	+5=···
−2=···	−3=···	−4=···	−5=···

S: *(1, 10).*

T: *(1, 10) belongs outside both strings.*

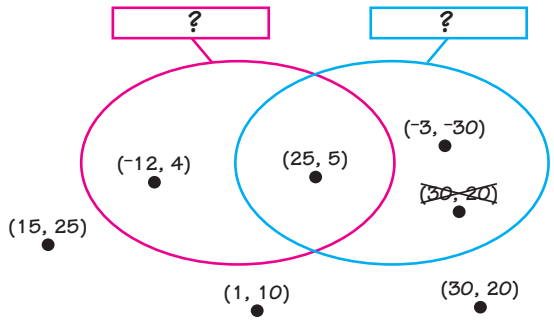
S: *The blue string cannot be for* x10 *because 10 x 1 = 10.*

S: *The two possibilities for the red string are still okay.*

S: *(25, 5).*

T: *(25, 5) belongs inside both strings.*

S: *The red string cannot be for* +4=···, *and the blue string cannot be for* −3=···.



Red

is the square of	The Relations Game		
is a multiple of	is a positive divisor of	is less than	is greater than
x2	x10	x $\frac{2}{3}$	is at least 20 greater than
÷2	÷10	÷ $\frac{2}{3}$	is at least 20 less than
+2=···	+3=···	+4=···	+5=···
−2=···	−3=···	−4=···	−5=···

Blue

is the square of	The Relations Game		
is a multiple of	is a positive divisor of	is less than	is greater than
x2	x10	x $\frac{2}{3}$	is at least 20 greater than
÷2	÷10	÷ $\frac{2}{3}$	is at least 20 less than
+2=···	+3=···	+4=···	+5=···
−2=···	−3=···	−4=···	−5=···

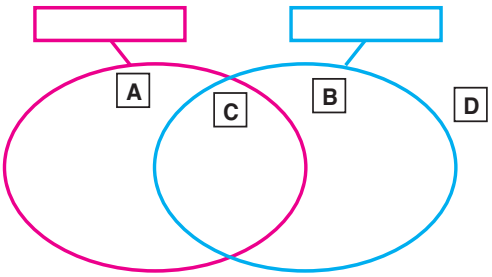
Exercise 2: The Relations Game with Two Strings

Minor modifications need to be made in the rules for *The Relations Game* (see Exercise 3 of Lesson L7) in order to extend it to a two-string game.

First Part of the Game:

In the first part of the game, the point system stays the same except points are awarded only when both relations are correctly identified.

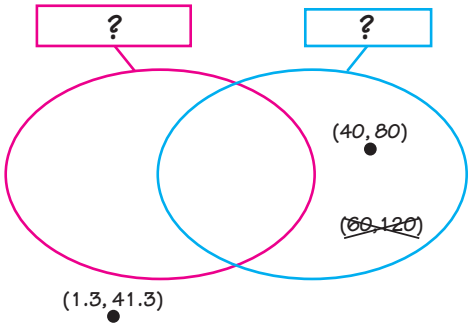
Second Part of the Game:



In the second part of the game, points are awarded as follows:

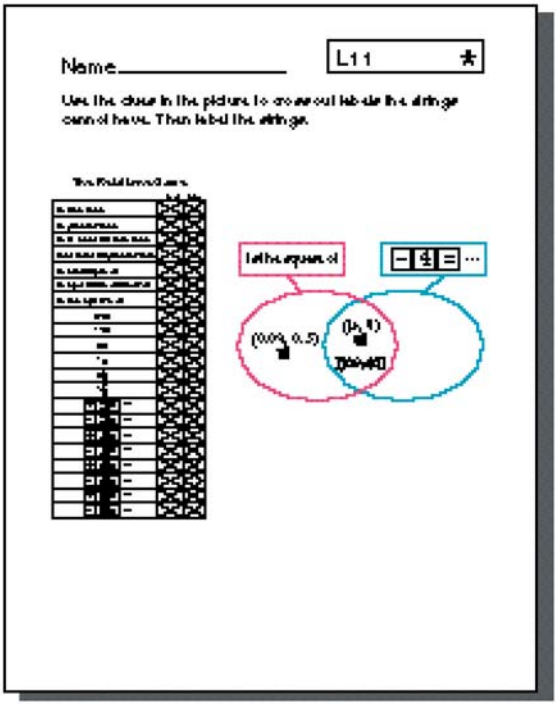
- A team gains four points each time a member correctly locates an ordered pair in region **C**.
- A team gains two points each time a member correctly locates an ordered pair in regions **A** or **B**.
- A team gains one point each time a member correctly locates an ordered pair in region **D**.
- A team neither gains nor loses points when a member incorrectly locates an ordered pair.

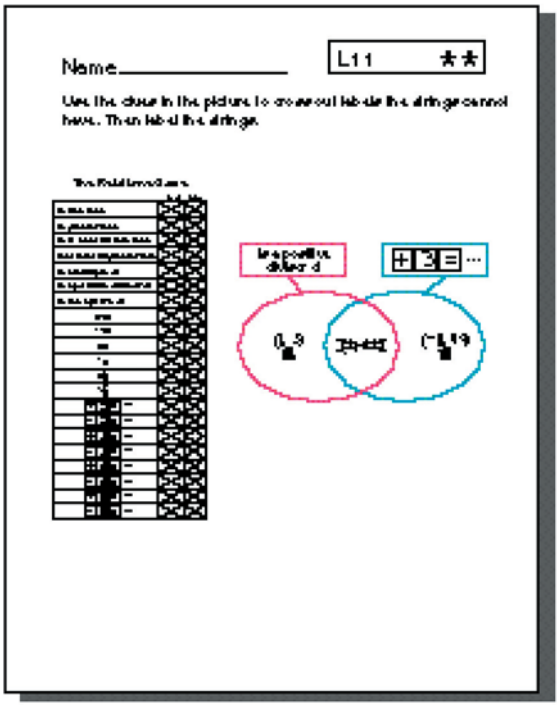
Use a scoring sheet that incorporates these changes (see Blackline L11(b)).



Play a two-string game with your class. As a possible game, let the red string be for +3=··· and the blue string be for **x2**. Give three starting clues as shown here. You may want to tell the class that the red string (relation) is determined by these clues.

Worksheets L11* and ** are available for individual work.





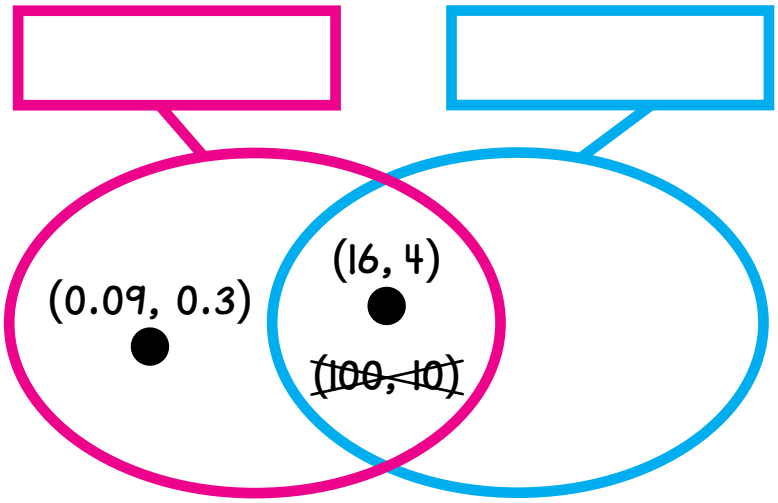
Name _____

L11 *

Use the clues in the picture to cross out labels the strings cannot have. Then label the strings.

The Relations Game

	Red	Blue
is less than		
is greater than		
is at least 20 less than		
is at least 20 greater than		
is a multiple of		
is a positive divisor of		
is the square of		
x10		
÷10		
x2		
÷2		
$\times \frac{2}{3}$		
$\div \frac{2}{3}$		
+ 2 =		
− 2 =		
+ 3 =		
− 3 =		
+ 4 =		
− 4 =		
+ 5 =		
− 5 =		



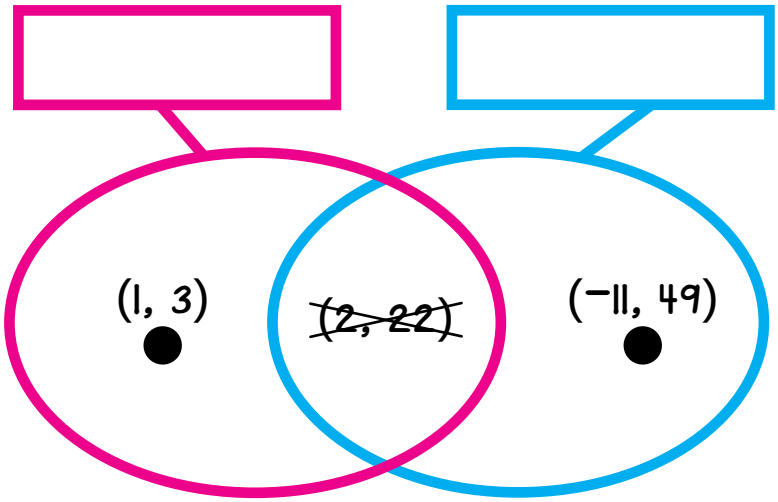
Name _____

L11 **

Use the clues in the picture to cross out labels the strings cannot have. Then label the strings.

The Relations Game

	Red	Blue
is less than		
is greater than		
is at least 20 less than		
is at least 20 greater than		
is a multiple of		
is a positive divisor of		
is the square of		
x10		
÷10		
x2		
÷2		
$\times\frac{2}{3}$		
$\div\frac{2}{3}$		
+2=		
−2=		
+3=		
−3=		
+4=		
−4=		
+5=		
−5=		



L7(a)

is the square of	The Relations Game		
is a multiple of	is a positive divisor of	is less than	is greater than
x2	x10	$\times \frac{2}{3}$	is at least 20 less than
÷2	÷10	$\div \frac{2}{3}$	is at least 20 greater than
+ 2 = ...	+ 3 = ...	+ 4 = ...	+ 5 = ...
− 2 = ...	− 3 = ...	− 4 = ...	− 5 = ...

is the square of	The Relations Game		
is a multiple of	is a positive divisor of	is less than	is greater than
x2	x10	$\times \frac{2}{3}$	is at least 20 less than
÷2	÷10	$\div \frac{2}{3}$	is at least 20 greater than
+ 2 = ...	+ 3 = ...	+ 4 = ...	+ 5 = ...
− 2 = ...	− 3 = ...	− 4 = ...	− 5 = ...

L7(b)

Scoring Sheet for The Relations Game

Secret Relation: _____

Points		TEAM A	TEAM B	TEAM C	TEAM D
First part of game					
For **correctly** identifying the relation					
• before additional plays	15 pts.				
• after first play	10 pts.				
• after second play	9 pts.				
• after third play	8 pts.				
• after fourth play	7 pts.				
Second part of game					
For **correctly** locating an ordered pair					
• inside the string	2 pts.				
• outside the string	1 pt.				

Scoring Sheet for The Relations Game

Secret Relation: _____

Points		TEAM A	TEAM B	TEAM C	TEAM D
First part of game					
For **correctly** identifying the relation					
• before additional plays	15 pts.				
• after first play	10 pts.				
• after second play	9 pts.				
• after third play	8 pts.				
• after fourth play	7 pts.				
Second part of game					
For **correctly** locating an ordered pair					
• inside the string (bonus for non-whole numbers 1 pt. each)	2 pts.				
• outside the string	1 pt.				

L11(a)

The Relations Game

Name _____

Team _____

	Red	Blue
is less than		
is greater than		
is at least 20 less than		
is at least 20 greater than		
is a multiple of		
is a positive divisor of		
is the square of		
x10		
÷10		
x2		
÷2		
$\times\frac{2}{3}$		
$\div\frac{2}{3}$		
+ 2 = ...		
− 2 = ...		
+ 3 = ...		
− 3 = ...		
+ 4 = ...		
− 4 = ...		
+ 5 = ...		
− 5 = ...		

The Relations Game

Name _____

Team _____

	Red	Blue
is less than		
is greater than		
is at least 20 less than		
is at least 20 greater than		
is a multiple of		
is a positive divisor of		
is the square of		
x10		
÷10		
x2		
÷2		
$\times\frac{2}{3}$		
$\div\frac{2}{3}$		
+ 2 = ...		
− 2 = ...		
+ 3 = ...		
− 3 = ...		
+ 4 = ...		
− 4 = ...		
+ 5 = ...		
− 5 = ...		

L11(b)

Scoring Sheet for The Relations Game with Two Strings

Secret Relations: Red String _____ Blue String _____

Points		TEAM A	TEAM B	TEAM C	TEAM D
First part of game					
For **correctly** identifying both relations					
• before additional plays	15 pts.				
• after first play	10 pts.				
• after second play	9 pts.				
• after third play	8 pts.				
• after fourth play	7 pts.				
Second part of game					
For **correctly** locating an ordered pair					
• inside both strings	4 pts.				
• inside only one of the strings	2 pts.				
• outside both strings	1 pt.				

Scoring Sheet for The Relations Game with Two Strings

Secret Relations: Red String _____ Blue String _____

Points		TEAM A	TEAM B	TEAM C	TEAM D
First part of game					
For **correctly** identifying both relations					
• before additional plays	15 pts.				
• after first play	10 pts.				
• after second play	9 pts.				
• after third play	8 pts.				
• after fourth play	7 pts.				
Second part of game					
For **correctly** locating an ordered pair					
• inside both strings	4 pts.				
• inside only one of the strings	2 pts.				
• outside the string	1 pt.				