# Table of Contents

## Looking For Pythagoras

**The Pythagorean Theorem**

- **Unit Introduction** .................................................. 2
  - Goals of the Unit .................................................. 2
  - Developing Students’ Mathematical Habits .................. 2
- **Mathematics of the Unit** ......................................... 3
  - Overview .............................................................. 3
  - Summary of Investigations ................................. 3
  - Mathematics Background ..................................... 3
- **Content Connections to Other Units** .......................... 10
- **Planning for the Unit** ........................................... 11
  - Pacing Suggestions and Materials ......................... 11
  - Pacing for Block Scheduling ................................ 12
  - Vocabulary .......................................................... 12
- **Program Resources** ................................................. 13
  - Components .......................................................... 13
  - Technology ........................................................... 13
- **Assessment Summary** ............................................... 14
  - Ongoing Informal Assessment ................................ 14
  - Formal Assessment ................................................ 14
  - Correlation to Standardized Tests ......................... 14
- **Launching the Unit** .................................................. 15
  - Introducing Your Students to *Looking for Pythagoras* .... 15
  - Using the Unit Opener ............................................ 15
  - Using the Mathematical Highlights ....................... 15

### Investigation 1: Coordinate Grids ................................ 16

- Mathematical and Problem-Solving Goals .................... 16
- **Summary of Problems** ............................................. 16
  - 1.1 Driving Around Euclid: Locating Points and Finding Distances .......................... 17
  - 1.2 Planning Parks: Shapes on a Coordinate Grid ................................................. 23
  - 1.3 Finding Areas ...................................................... 27

- **ACE** Answers to Applications—Connections—Extensions ........ 31
- Possible Answers to Mathematical Reflections ............... 33
### Table of Contents

**Investigation 2** Squaring Off .................................................. 34  
Mathematical and Problem-Solving Goals ........................................... 34  
Summary of Problems ........................................................................ 34  
2.1 Looking for Squares ................................................................. 35  
2.2 Square Roots ............................................................................. 39  
2.3 Using Squares to Find Lengths .................................................... 43  
ACE Answers to Applications—Connections—Extensions ......................... 47  
Possible Answers to Mathematical Reflections ....................................... 50

**Investigation 3** The Pythagorean Theorem .......................................... 51  
Mathematical and Problem-Solving Goals ........................................... 51  
Summary of Problems ........................................................................ 51  
3.1 The Pythagorean Theorem ............................................................ 52  
3.2 A Proof of the Pythagorean Theorem ............................................. 57  
3.3 Finding Distances ........................................................................ 61  
3.4 Measuring the Egyptian Way: Lengths That Form a Right Triangle ........ 65  
ACE Answers to Applications—Connections—Extensions ......................... 69  
Possible Answers to Mathematical Reflections ....................................... 72

**Investigation 4** Using the Pythagorean Theorem .................................... 73  
Mathematical and Problem-Solving Goals ........................................... 73  
Summary of Problems ........................................................................ 73  
4.1 Analyzing the Wheel of Theodorus ............................................... 74  
4.2 Stopping Sneaky Sally: Finding Unknown Side Lengths ..................... 79  
4.3 Analyzing Triangles .................................................................... 83  
4.4 Finding the Perimeter ................................................................... 87  
ACE Answers to Applications—Connections—Extensions ......................... 91  
Possible Answers to Mathematical Reflections ....................................... 96

Blackline Masters
- Labsheets for Students  
  - Dot Paper .................................................................................. 99  
  - Centimeter Grid Paper ................................................................. 100  
  - 1.1, 1.2, 1.3, 1ACE Exercises 15–25 ............................................. 101  
  - 2.1, 2.3 ...................................................................................... 105  
  - 3.2A–C, 3.3 .................................................................................. 107  
  - 4.1, 4.4 ...................................................................................... 111  
  - At a Glance Teacher Form ........................................................... 113

Glossary ......................................................................................... 115
Index ............................................................................................ 116
Acknowledgments ............................................................................ 117

Table of Contents ix
Overview

In Looking for Pythagoras, students explore two important ideas: the Pythagorean Theorem and square roots. They also review and make connections among the concepts of area, distance, and irrational numbers.

Students begin the unit by finding the distance between points on a coordinate grid. They learn that the positive square root of a number is the side length of a square whose area is that number. Then, students discover the Pythagorean relationship through an exploration of squares drawn on the sides of a right triangle. In the last investigation of the unit, students apply the Pythagorean Theorem to a variety of problems.

Summary of Investigations

Investigation 1
Coordinate Grids
Students review coordinate grids as they analyze a map in which streets are laid out on a grid. They make the connection between the coordinates of two points and the driving distance between them. This sets the stage for finding the distance between two points on a grid without measuring. Students investigate geometric figures on coordinate grids. Given two vertices, they find other vertices that define a square, a non-square rectangle, a right triangle, and a non-rectangular parallelogram. And, they calculate areas of several figures drawn on a dot grid.

Investigation 2
Squaring Off
Students explore the relationship between the area of a square drawn on a dot grid and the length of its sides. This provides an introduction to the concept of square root. They find the distance between two points by analyzing the line segment between them: they draw a square using the segment as one side, find the area of the square, and then find the positive square root of that area.

Investigation 3
The Pythagorean Theorem
Students develop and explore the Pythagorean Theorem. They then investigate a geometric puzzle that verifies the theorem, and they use the theorem to find the distance between two points on a grid. In the last problem, they explore and apply the converse of the Pythagorean Theorem.

Investigation 4
Using the Pythagorean Theorem
For students to appreciate the mathematical power of the Pythagorean Theorem, they need to encounter situations that can be illuminated by the theorem. Students explore an interesting pattern among right triangles, apply the Pythagorean Theorem to find distances on a baseball diamond, investigate properties of 30-60-90 triangles, and find missing lengths and angle measures of a triangle composed of smaller triangles.

Mathematics Background

Students’ work in this unit develops an important relationship connecting geometry and algebra: the Pythagorean Theorem. The presentation of ideas reflects the historical development of the concept of irrational numbers. Early Greek mathematicians searched for ratios of integers to represent side lengths of squares with certain given areas such as 2 square units. The square root of 2 is an irrational number, which means that it cannot be written as a ratio of two integers.

Finding Area and Distance
Students find areas of plane figures drawn on dot grids. This reviews some concepts developed in the grade 6 unit Covering and Surrounding. One common method for calculating the area of a figure is to subdivide it and add the areas of the component shapes. A second common method is to enclose the shape in a rectangle and subtract the areas of the shapes that lie outside the figure.
Components

Use the chart below to quickly see which components are available for each Investigation.

<table>
<thead>
<tr>
<th>Invest.</th>
<th>Labsheets</th>
<th>Additional Practice</th>
<th>Transparencies</th>
<th>Formal Assessment</th>
<th>Assessment Options</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Problem</td>
<td>Summary</td>
<td>Check Up</td>
</tr>
<tr>
<td>1</td>
<td>1.1, 1.2, 1.3, 1ACE</td>
<td>Exercises 15–25</td>
<td>✔ 1.1A, 1.1B, 1.2A, 1.2B, 1.3</td>
<td>✔</td>
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<tr>
<td>2</td>
<td>2.1, 2.3</td>
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<td>2.1, 2.2, 2.3A, 2.3B</td>
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</tr>
<tr>
<td>3</td>
<td>3.2A–C, 3.3</td>
<td>✔</td>
<td>3.1, 3.2A, 3.2B, 3.3, 3.4</td>
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<td>✔</td>
</tr>
<tr>
<td>4</td>
<td>4.1, 4.4, Dot Paper</td>
<td>✔</td>
<td>4.1A, 4.1B, 4.2, 4.3A, 4.3B, 4.4</td>
<td>✔</td>
<td>✔</td>
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</tbody>
</table>

For the Unit  

Exأم View  

CD-ROM, Web site  

LBA  

Unit Test, Notebook Check, Self Assessment  

Multiple-Choice Items, Question Bank, ExamView CD-ROM

Also Available for Use With This Unit

* Parent Guide: take-home letter for the unit
* Implementing CMP

Technology

The Use of Calculators

* Connected Mathematics was developed with the belief that calculators should be available and that students should learn when their use is appropriate. For this reason, we do not designate specific problems as "calculator problems." However, students will need access to graphing calculators for much of their work in this unit. Occasionally, students will be asked not to use their calculators to encourage them to think about how they can estimate square roots.

Student Interactivity CD-ROM

* Includes interactive activities to enhance the learning in the Problems within Investigations.

PHSchool.com

For Students  

* Multiple-choice practice with instant feedback, updated data sources, data sets for Tinkerplots data software.

For Teachers  

* Professional development, curriculum support, downloadable forms, and more.

* Spanish Assessment Resources
* Additional online and technology resources

See also www.math.msu.edu/cmp for more resources for both teachers and students.

ExamView® CD-ROM

* Create multiple versions of practice sheets and tests for course objectives and standardized tests.
* Includes dynamic questions, online testing, student reports, and all test and practice items in Spanish.
* Also includes all items in the Assessment Resources and Additional Practice.

TeacherExpress™ CD-ROM

* Includes a lesson planning tool, the Teacher’s Guide pages, and all the teaching resources.

LessonLab Online Courses

* LessonLab offers comprehensive, facilitated, professional development designed to help teachers implement CMP2 and improve student achievement. To learn more, please visit PHSchool.com/cmp2.
### Coordinate Grids

#### Mathematical and Problem-Solving Goals
- Review the coordinate system
- Explore distances on a coordinate grid
- Review properties of quadrilaterals
- Connect properties of figures to coordinate representations
- Draw shapes on a coordinate grid
- Develop strategies for finding areas of irregular figures on a grid

#### Summary of Problems

**Problem 1.1 Driving Around Euclid**
Students analyze a map of a fictitious city in which streets are laid out on a coordinate grid. They find driving distances from one location to another, making the connection between the coordinates of two points and the distance between them. They compare the driving and flying distances between two points.

**Problem 1.2 Planning Parks**
Given two vertices, students find other vertices that define a square, a non-square rectangle, a right triangle, and a non-rectangular parallelogram.

**Problem 1.3 Finding Areas**
Students find areas of irregular figures drawn on a dot grid.

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<table>
<thead>
<tr>
<th>Suggested Pacing</th>
<th>Materials for Students</th>
<th>Materials for Teachers</th>
<th>ACE Assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>All</strong> 4 days</td>
<td>Centimeter rulers, calculators</td>
<td>Transparencies 1.1A and 1.1B (optional)</td>
<td>1–7, 26–28, 30, 35, 36</td>
</tr>
<tr>
<td><strong>1.1</strong> 1/2 days</td>
<td>Labsheet 1.1</td>
<td>Transparencies 1.1A and 1.1B (optional)</td>
<td>1–7, 26–28, 30, 35, 36</td>
</tr>
<tr>
<td><strong>1.2</strong> 1 day</td>
<td>Labsheet 1.2, grid paper (optional; for special-needs students)</td>
<td>Transparency 1.2A and 1.2B (optional)</td>
<td>8–14, 29, 31, 37</td>
</tr>
<tr>
<td><strong>1.3</strong> 1 day</td>
<td>Labsheet 1.3, geoboards (optional), Labsheet 1ACE Exercises 15–25</td>
<td>Transparency 1.3 (optional), overhead geoboard (optional)</td>
<td>15–25, 32–34, 38, 39</td>
</tr>
<tr>
<td>MR 1/2 day</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
1.1 Driving Around Euclid

Goals

- Review the coordinate system
- Explore distances on a coordinate grid

In this problem, students review the concept of the coordinate grid and are introduced to the idea of finding distances between points. Students find two types of distances: distance along grid lines (represented by driving distances along city streets) and straight-line distance (represented by flying distance).

Launch 1.1

To launch this investigation, have students look at the map of Washington, D.C. in their books. Tell students that the system of streets is based on a coordinate grid. Discuss the features of the grid, which are listed in the student book.

Ask students to locate the intersection of 3rd Street and D Street and then share the location they found with the students sitting near them. Students should realize that there is more than one intersection fitting this description. In fact, there are four, one in each quadrant.

Suggested Question  Ask:

- What additional information could I give you so you know which intersection I am referring to? (the quadrant the intersection is in)

Suggested Questions  Discuss the Getting Ready questions. These questions can help you informally assess your students’ understanding of coordinate grids.

- Describe the location of each of these landmarks:
  - George Washington University (Answers may vary slightly. Possible answer: 21st and H St. NW)
  - Dupont Circle (19th and P St. NW)
  - Benjamin Banneker Park (Answers may vary slightly. Possible answer: 10th and G St. SW)
  - The White House (Pennsylvania Ave. between 15th and 17th NW)
  - Union Station (1st and E St. NE)

- How can you find the distance from Union Station to Dupont Circle? (Measure the straight-line distance along Massachusetts Avenue. Note that, if we measure the distance in blocks, these blocks are not the same length as the north-south or the east-west blocks. Students may have a range of suggestions, and many students may struggle with this question.)

- Find the intersection of G Street and 8th Street SE and the intersection of G Street and 8th Street NW. How are these locations related to the Capitol building? (Possible answer: SE indicates that the location is southeast of the Capitol building. NW indicates that the location is northwest of the Capitol building. In addition, by counting the letters up to G and adding this to 8, we can determine that these places are each about 15 blocks from the Capitol.)

Next, talk about the map of the fictitious city of Euclid, which is also shown on Transparency 1.1B. Point out the origin (the location of City Hall), and discuss the meaning of the coordinates. Help students understand that a coordinate system is convenient for locating points, but only if we know where to count from and what scale is being used.

Suggested Questions  Some questions might include:

- What are the coordinates of City Hall? (0, 0)
- What are the coordinates of the art museum? (6, 1)
- What do the 6 and the 1 mean? [They indicate that the art museum is 6 blocks to the right of (east of) and 1 block up from (north of) the origin, or City Hall.]
- Is there more than one way to travel from City Hall to the art museum? (Yes)
- What is the shortest distance, along the streets of Euclid, from City Hall to the art museum? (7 blocks)
• Is there more than one way to follow a shortest path from City Hall to the art museum? (There are several, such as right 2 blocks, up 1 block, and right 4 blocks.)

• A helicopter can fly directly from one location to another; it doesn’t have to travel along the city streets. How can you determine the distance a helicopter travels to get from one point to another in Euclid?

If no one suggests using a ruler, explain that because each centimeter on the map represents one block, you can use a centimeter ruler to find the straight-line distance, in blocks, between two points.

When students seem confident about reading map coordinates and finding distances, have them work individually or in pairs on the problem. Distribute Labsheet 1.1.

### Explore 1.1

As students work, encourage them to look for connections between the coordinates of two points and the driving distance between them.

#### Suggested Questions

Ask:

• What do the first coordinates of the two points tell you about the distance between the points? (The positive difference in the first coordinates is the horizontal distance between the points.)

• What do the second coordinates tell you about the distance between the points? (The positive difference in the second coordinates is the vertical distance between the points.)

• How can you find the total driving distance? (Add the horizontal and vertical distances.)

Check how students are measuring the distance a helicopter travels.

### Summarize 1.1

Establish that students understand that the grid system makes it possible to refer to each landmark in Euclid by a unique pair of coordinates.

#### Suggested Questions

Ask:

• Why might it be important to be able to locate places in a city by using a simple system like grid coordinates?

• What information do you need to be able to locate a point on a grid?

• When we give the coordinates of a point in Euclid, where are we counting from? What scale are we using? (We count from City Hall. The scale is in number of blocks.)

Be sure students can interpret the $x$- and $y$-coordinates of a point. Given a point on the grid, they should be able to name the coordinates. Given the coordinates of a point, they should be able to locate the point on the grid.

#### Suggested Questions

Extend the coordinate idea to include non-integers:

• Where in Euclid is the point $(2, \frac{1}{3})$?

The driving distance between two points is the number of blocks a car would travel from one place to another. Talk with the class about finding the driving distance between two points given their coordinates. You might discuss these three examples:

• The hospital and the cemetery are on the same horizontal line. To find the distance between these points, find the positive difference in the $x$-coordinates.

• City Hall and the police station are on the same vertical line. To find the distance between these points, find the positive difference in the $y$-coordinates.

• The art museum and the gas station do not lie on the same horizontal or vertical line. To find the distance, find the positive difference in $x$-coordinates and the positive difference in $y$-coordinates, and add the two results.

If no one uses the term absolute value to describe the positive difference, you might bring it up yourself. The concept of using absolute value to express distance is explored in the grade 7 unit Accentuate the Negative, but you may want to review this idea with students.
Suggested Questions These questions might help clear up confusion:

• To go from the art museum to the gas station, how many blocks do you travel in a horizontal direction? (2 blocks)

• How is this distance related to the coordinates of the points? (It is the positive difference, or the absolute value of the difference, between the x-coordinates.)

• To go from the art museum to the gas station, how many blocks do you travel in a vertical direction? (3 blocks)

• How is this distance related to the coordinates of the points? (It is the positive difference, or the absolute value of the difference, between the y-coordinates.)

To help students think about direction, ask:

• Suppose you are in Euclid and you are trying to find the library. Someone tells you it is 3 blocks from the stadium. Is this enough information for you to know how to get there? (No.)

• What information do you need to precisely locate the library? (You need directions. For example, you might need to walk 3 blocks south of the stadium, or 2 blocks east and 1 block south from the stadium.)

Verify that everyone understands that to precisely locate a position on the grid, a vertical distance, a horizontal distance, and the direction of each must be given. A coordinate pair gives all of this information in a concise way.

In Question D, students should recognize that the flying distance is the length of the line segment connecting the points.

For Question E, review with the class why the helicopter distance is always shorter than or equal to the driving distance. This is an application of the triangle inequality, which students encountered in the grade 6 unit Shapes and Designs. The triangle inequality states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. The car distance is the sum of the lengths of two sides of a triangle; the helicopter distance is the length of the third side.

This is an opportunity to verify that students connect directions on a coordinate grid with map directions. Going left is traveling west; going up is traveling north, and so on.
Driving Around Euclid

Mathematical Goals

- Review the coordinate system
- Explore distances on a coordinate grid

Launch

Have students look at the map of Washington, D.C. in their books. Discuss the features of the street system, which are listed in the student book.

Discuss the Getting Ready questions.

Discuss the map of Euclid. Point out the origin (the location of City Hall), and discuss the meaning of the coordinates. Help students understand that a coordinate system is convenient for locating points.

- What are the coordinates of City Hall?
- What are the coordinates of the art museum? What do the 6 and the 1 mean?
- What is the shortest distance, along the streets of Euclid, from City Hall to the art museum?
- Is there more than one shortest path from City Hall to the art museum?
- A helicopter can fly directly from one location to another; it doesn’t have to travel along the city streets. How can you determine the distance a helicopter travels to get from one point to another in Euclid?

Explain that because each centimeter on the map represents one block, a centimeter ruler could be used to find the straight-line distance, in blocks, between two points.

Have students work individually or in pairs on the problem.

Explore

As students work, encourage them to look for connections between the coordinates of two points and the driving distance between them.

- What do the first coordinates of the two points tell you about the distance between the points? What do the second coordinates tell you about the distance between the points?
- How can you find the total driving distance?

Check how students are measuring the distance a helicopter travels.

Summarize

Talk with the class about finding the distance between two points given their coordinates.

- To go from the art museum to the gas station, how many blocks do you travel in a horizontal direction? How is this distance related to the coordinates of the points?

Materials

- Centimeter rulers
- Transparencies 1.1A and 1.1B
- Labsheet 1.1

Materials

- Student notebooks

continued on next page
**Summarize**

To go from the art museum to the gas station, how many blocks do you travel in a vertical direction? How is this distance related to the coordinates of the points?

Help students think about direction. Verify that everyone understands that to precisely locate a position on the grid, a vertical distance, a horizontal distance, and the direction of each must be given. A coordinate pair gives all of this information in a concise way.

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**ACE Assignment Guide for Problem 1.1**

<table>
<thead>
<tr>
<th>Core</th>
<th>1–7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Other Connections</td>
<td>26–28, 30; Extensions 35, 36</td>
</tr>
</tbody>
</table>

**Adapted** For suggestions about adapting Exercises 1–6 and other ACE exercises, see the CMP Special Needs Handbook.

**Answers to Problem 1.1**

**A.**
1. (4, 4)
2. (6, −2)
3. (−2, 3)

**B.**
1. Pair 1: Go north (up) 4 blocks.
   - Pair 2: Possible answer: Go east 6 blocks and then north 4 blocks.
   - Pair 3: Possible answer: Go east 12 blocks and then north 5 blocks.

**2.**
- Pair 1: 4 blocks;
- Pair 2: 10 blocks;
- Pair 3: 17 blocks

**C.** Add the positive difference in the x-coordinates to the positive difference in the y-coordinates.

**D.**
- Pair 1: 4 blocks;
- Pair 2: about 7.2 blocks;
- Pair 3: 13 blocks

**E.** The helicopter distance will never be longer than the car distance. Generally, it will be shorter, unless the points are on the same vertical or horizontal line. In this case, the distances will be equal.
Goals

• Review properties of quadrilaterals
• Connect properties of figures to coordinate representations
• Draw shapes in on a coordinate grid

In this problem, students review the properties of quadrilaterals and right triangles. Given the coordinates of two vertices of a polygon, they find the coordinates of other vertices so that the resulting shape will be a square, a non-square rectangle, a right triangle, or a non-rectangular parallelogram.

Launch 1.2

Introduce the context of planning parks in Euclid. Discuss the idea of describing the shapes of the parks by giving the vertices of their borders. Make sure students know what properties define a square, a right triangle, a rectangle, and a parallelogram.

Suggested Questions You may want to display a transparent grid of the Euclid map. Plot two points on the grid and ask questions like these:

• *Suppose we want to draw a right triangle with these points as two of the vertices. Locate such a right triangle and tell us the coordinates of the third vertex. How do you know that this is a right triangle?*

• *Now locate a rectangle that has one of its vertices at the origin. Tell us the coordinates of its vertices. How do you know that this is a rectangle?*

Ask similar questions about a square and a non-rectangular parallelogram. Take this opportunity to assess what students know about the properties of these polygons. Do they know that squares have sides of equal length and four right angles? Do they know that parallelograms have two pairs of parallel sides? Do they know that a figure’s orientation does not matter? (For example, a square is still a square even if it is rotated to look like a “diamond.”)

Explore 1.2

Suggested Questions As students work, ask questions about the reasoning they are applying.

• *How did you figure out where to put the vertices so this park’s sides would all be the same length?*

• *How did you determine where to put the vertices so opposite sides would be parallel?*

• *How did you decide where these vertices had to be to create right angles?*

Encourage students to discuss with the others in their group how they are finding the vertices of each shape so each student should be able to explain the group’s strategies.

If students are struggling to find a square, suggest that they turn their papers slightly to make the given segment horizontal. It is sometimes easier for students to imagine an upright square on a tilted grid than a tilted square on an upright grid.

Summarize 1.2

Ask students to share their strategies for finding the vertices for each park shape. Here are some strategies students might have used:

• Use the concept of slope to check that opposite sides are parallel. Recall (from the grade 7 unit *Moving Straight Ahead*) that parallel lines have the same slope, and then use this fact to establish parallel sides.

• To find the slope of a line, students can count units up and units over to match the slope of an existing segment.
• Use the corner of a piece of paper to check for right angles.
• Use the fact that vertical and horizontal lines are perpendicular (they may recall that the slopes of perpendicular lines are negative reciprocals).
• Use a ruler or the marked edge of a piece of paper to check lengths.
• Use angle rulers to measure angles.
• Find the right triangle by dividing a rectangle or a square in half along one of its diagonals.

For Questions A, B, and D, if no one suggests a park in which the line segment connecting the given vertices is a diagonal rather than a side, introduce this possibility.

Check for Understanding
As a final summary, put a transparent grid on the overhead, and label x- and y-axes. Draw several parallelograms (including squares and non-square rectangles) on the grid.

Ask students to explain what is special about each figure. For example, a parallelogram is a trapezoid and it may be a square or a rectangle. A rhombus is a parallelogram and it could be a square or a rectangle. You may want to organize the relationships in a Venn diagram.

![Venn diagram showing the relationships between different types of quadrilaterals: Quadrilateral, Trapezoid, Parallelogram, Rectangle, Rhombus, Square.](image)
1.2 Planning Parks

**Mathematical Goals**
- Review properties of quadrilaterals
- Connect properties of figures to coordinate representations
- Draw shapes on a coordinate grid

**Launch**
Introduce the context of planning parks in Euclid. Discuss the idea of describing the shapes of the parks by giving the vertices of their borders. Make sure students know what properties define a square, a right triangle, a rectangle, and a parallelogram.

Display a coordinate grid on the overhead and discuss a few examples:
- Suppose we want to draw a right triangle with these points as two of the vertices. Locate such a right triangle and tell us the coordinates of the third vertex. How do you know that this is a right triangle?
- Assess what students know about the properties of squares, rectangles, right triangles, and parallelograms.
- Describe Problem 1.2. Distribute Labsheet 1.2 or centimeter grid paper, and have students work in groups of three or four on the problem.

**Explore**
Ask questions about the reasoning students are applying.
- How did you figure out where to put the vertices so this park’s sides would all be the same length?
- How did you determine where to put the vertices so opposite sides would be parallel?
- How did you decide where these vertices had to be to create right angles?

Encourage students to discuss their reasoning with others in their group.
- If students are struggling to find a square, suggest that they turn their papers slightly to make the given segment horizontal.

**Summarize**
Ask students to share their strategies for finding the vertices for each park shape. For Questions A, B, and D, if no one suggests a park in which the line segment connecting the given vertices is a diagonal rather than a side, introduce this possibility.

**Check for Understanding**
As a final summary, put a transparent grid on the overhead and label x- and y-axes. Draw several parallelograms (including squares and non-square rectangles) on the grid, and ask students what is special about each figure.
**ACE Assignment Guide**

for Problem 1.2

**Core** 8–10, 14

**Other Applications** 11–13; **Connections** 29, 31; **Extensions** 37; unassigned choices from earlier problems

**Adapted** For suggestions about adapting Exercises 8–10 and other ACE exercises, see the CMP Special Needs Handbook.

**Connecting to Prior Units** 29, 31: Moving Straight Ahead, Thinking With Mathematical Models

**Answers to Problem 1.2**

A. There are three possible pairs of vertices: (3, 5) and (0, 4); (5, –1) and (2, –2); and (3, 0) and (2, 3).

B. There are many possible pairs of vertices, including (6, –4) and (3, –5); (1, 2) and (4, 1); and (2, 0) and (3, 3).

C. There are several possible vertices, including (3, –5), (2, 3), and (5, –1).

D. There are many possible pairs of vertices, including (1, –1) and (4, 0); (2, 4) and (–1, 3); (0, 2) and (–3, 1); and (1, 3) and (4, 0).
**Goal**

• Develop strategies for finding areas of irregular figures on a grid

In this problem, students begin by finding areas of figures on a dot grid. Then they move to the coordinate plane to find the area of one of the square parks from Problem 1.2. They will begin to see that, for some figures, it is easy to find areas by subdividing them and adding the areas of the component parts; other figures seem to need another approach.

**Note:** Many activities in this unit are classic geoboard problems. If you have access to geoboards, use them; students will enjoy exploring area with them. If your students have had experience with geoboards, this will go quickly. If not, spend time familiarizing students with them. Demonstrate how to form shapes and how to use extra rubber bands to subdivide a figure or to surround it with a rectangle. You might have students pair up and create figures for each other to find the area of irregular figures. An overhead geoboard would also be helpful in this problem.

**Launch 1.3**

Conduct the following short activity to introduce the idea of finding areas of figures drawn on a dot grid:

Draw a figure on a dot grid on the board, an overhead geoboard, or transparent dot paper. Choose a shape simple enough that students can easily find its area by subdividing it or by enclosing it in a rectangle. For example:

```
  . . . . .
  . . . . .
  . . . . .
  . . . . .
  . . . . .
```

Ask students how they could find the area of the figure. Let students share their ideas. The two strategies students tend to use are outlined here. Students may have variations on these two strategies. It is not necessary to bring both of these strategies out before students work on the problem, but you will want to address both in the summary.

**Strategy 1:**

Subdivide the figure. Find the area of each piece and add these areas to get the total area.

```
  . . . . .
  . . . . .
  . . . . .
  . . . . .
  . . . . .
```

**Strategy 2:**

Enclose the figure in a rectangle. Find the areas of the pieces surrounding the original shape. Then, subtract these areas from the area of the rectangle. This strategy is more efficient for certain figures such as the triangle in ACE Exercise 19.

```
  . . . . .
  . . . . .
  . . . . .
  . . . . .
  . . . . .
```

Have students explore the problem in pairs. Labsheet 1.3 contains the figures for Question A. Students may work on the labsheet, redraw the figures on dot paper, or construct them on geoboards.

**Explore 1.3**

In their work, students will review how to find areas of rectangles and triangles. Look for students who are actively applying this knowledge; they can share their strategies in the summary. Have some students put their work on large poster paper or a transparent grid. Students can count the number of units that cover the...
figure, or they can apply the rules for finding areas of rectangles and triangles. Some students may need help applying the rule for the area of a triangle, \( A = \frac{1}{2}bh \). Help them to see that a triangle is half of a rectangle. This approach was used in the grade 6 unit *Covering and Surrounding*.

For additional practice and challenge, you may also want to have students work on ACE Exercises 15–20 at this time.

**Summarize 1.3**

As students share answers and strategies, help them generalize their methods for finding area.

**Suggested Questions** Ask:

- *We can find areas of some figures by subdividing them and adding the areas of the smaller figures. For which figures in this problem is using this method easy?* (Students will probably mention Figures 1, 2, 3, and 4, although students may also use this strategy on other figures.)

- *We can find areas of some figures by enclosing them in a rectangle and subtracting the areas of the unwanted parts from the rectangle’s area. For which figures in this problem is using this method easy?* (Students’ ideas will vary. Figure 5, for example, can be enclosed in a 2-by-3 rectangle. The areas of four triangles—two with area \( \frac{1}{2} \) square unit and two with area 1 square unit—can then be subtracted from the rectangle’s area, leaving 3 square units.)

- *Did you use different strategies for finding the area of the park on the coordinate grid?* Some students may use the strategy of rearranging parts of a figure to form a rectangle or a triangle with an easy-to-find area. For example, see the answer given for Figure 3.

Students will need to be able to apply these methods for their future work in this unit, so make sure everyone can use at least one of them and explain why it works.
1.3 Finding Areas

Mathematical Goal

- Develop strategies for finding areas of irregular figures on a grid

Launch

Draw a simple figure on a dot grid. Ask students how they could find the area of the figure. Let students share their ideas. There are two main strategies students tend to use: subdividing the figure and finding the areas of the pieces; and enclosing the figure in a rectangle and subtracting the areas of the pieces outside the figure from the area of the rectangle. It is not necessary to discuss both strategies now, but you will want to address both in the summary.

Have students explore the problem in pairs. Students may work on Labsheet 1.3, redraw the figures on dot paper, or construct them on geoboards.

Materials

- Transparency 1.3
- Labsheet 1.3
- Geoboards (optional)
- Centimeter rulers

Explore

In their work, students will review how to find areas of rectangles and triangles. Look for students who are actively applying this knowledge; they can share their strategies in the summary. Some students may need help applying the rule for the area of a triangle \( A = \frac{1}{2}bh \). Help them to see that a triangle is half of a rectangle.

You may want to have students work on ACE Exercises 15–20 at this time.

Summarize

As students share answers and strategies, help them generalize their methods for finding area.

- We can find areas of some figures by subdividing them and adding the areas of the smaller figures. For which figures in this problem is using this method easy?
- We can find areas of some figures by enclosing them in a rectangle and subtracting the areas of the unwanted parts from the rectangle’s area. For which figures in this problem is using this method easy?
- Did you use different strategies for finding the area of the park on the coordinate grid?

Some students may use the strategy of rearranging parts of a figure to form a rectangle or a triangle with an easy-to-find area.

Students will need to be able to apply these methods for their future work in this unit, so make sure everyone can use at least one of them and explain why it works.

Materials

- Student notebooks
ACE Assignment Guide for Problem 1.3

Core 15–25
Other Connections 32–34; Extensions 38, 39; unassigned choices from earlier problems

Adapted For suggestions about adapting ACE exercises, see the CMP Special Needs Handbook.

Connecting to Prior Units 32: Bits and Pieces II;
33, 38, 39: Covering and Surrounding;
34: Accentuate the Negative

Answers to Problem 1.3

A. 1. 2 units²
   2. 1.5 units²
   3. 2 units²
   4. 4 units²
   5. 3 units²
   6. 4 units²
   7. 3.5 units²
   8. 6.5 units²
   9. 8.5 units²
   10. 8.5 units²

B. 10 units² or 5 units², depending on which square the student chooses.

C. Possible strategies include subdividing figures and adding the areas of the smaller figures; enclosing figures in rectangles and then subtracting the areas of the unwanted parts; and rearranging parts to form a rectangle or triangle with an easy-to-find area.
Investigation 1

ACE Assignment Choices

Problem 1.1
Core 1–7
Other Connections 26–28, 30; Extensions 35, 36

Problem 1.2
Core 8–10, 14
Other Applications 11–13; Connections 29, 31; Extensions 37; unassigned choices from earlier problems

Problem 1.3
Core 15–25
Other Connections 32–34; Extensions 38, 39; unassigned choices from earlier problems

Adapted For suggestions about adapting Exercises 1–6, 8–10, and other ACE exercises, see the CMP Special Needs Handbook.

Connecting to Prior Units 29, 31: Moving Straight Ahead, Thinking With Mathematical Models; 32: Bits and Pieces II; 33, 38, 39: Covering and Surounding; 34: Accentuate the Negative

Applications

1. a. (6, 1)  
   b. (−6, −4)  
   c. (−6, 0)  
2. 13 blocks  
3. 18 blocks  
4. There are many 10-block routes, but there are exactly five possible halfway points: (−5, 0), (−4, −1), (−3, −2), (−2, −3), and (−1, −4).  
5. Because there is only one possible route, there is only one possible halfway point: (−3, −2).  
6. a. The art museum and the cemetery  
   b. Possible answer: To get to the art museum, drive 6 blocks east, turn left, and go north 1 block. To get to the cemetery, drive 3 blocks east, turn right, and drive 4 blocks south.

7. a. The hospital is 4 blocks from the greenhouse. There are ten intersections on the map that are 4 blocks by car from the gas station: (1, 5), (0, 4), (1, 3), (2, 2), (3, 1), (4, 0), (5, 1), (6, 2), (7, 3), and (7, 5).

<table>
<thead>
<tr>
<th>School Location</th>
<th>Flying Distance</th>
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<tbody>
<tr>
<td>(1, 5)</td>
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</tr>
<tr>
<td>(0, 4)</td>
<td>4</td>
</tr>
<tr>
<td>(1, 3)</td>
<td>≈ 3.2</td>
</tr>
<tr>
<td>(2, 2)</td>
<td>≈ 2.8</td>
</tr>
<tr>
<td>(3, 1)</td>
<td>≈ 3.2</td>
</tr>
<tr>
<td>(4, 0)</td>
<td>4</td>
</tr>
<tr>
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<td>(7, 3)</td>
<td>≈ 3.2</td>
</tr>
<tr>
<td>(7, 5)</td>
<td>≈ 3.2</td>
</tr>
</tbody>
</table>

8. (−2, 3) and (1, 5); (5, −1) and (2, −3). There is a third possibility with non-integer coordinates, but students do not need to find this one.

9. There are infinitely many possible pairs, including (2, 0) and (5, 2); (0, 2) and (3, 4); (0, −2) and (3, 0); and (2, −1) and (5, 1).

10. There are infinitely many possible vertices, including (0, 2), (3, 0), (4, −6) and (5, −1). Any one of the vertices in Question 8 will work.

11. B

12. There are many possible vertices, including (2, 3), (3, 6), (5, 7), (1, 4), (4, 5), (0, 2), (6, 4). (See the answer to Exercise 13.)
13. An infinite number of right triangles can be drawn. The third vertex can be located at any grid point on the line that goes through (0, 2) and (6, 4) (the line \( y = \frac{1}{3}x + 2 \)) or on the line that goes through (−1, 5) and (5, 7) (the line \( y = \frac{1}{3}x + \frac{16}{3} \)). Each of these lines is perpendicular to the segment connecting (3, 3) and (2, 6), so these lines create the right angle for the triangle. Some students may express this idea as follows: Imagine a line starting from one of the given points and at a right angle to the given side. Any point along that line can be the third vertex of the triangle.

14. Yes. Opposite sides have equal lengths and slopes.

15. 3 units²  
16. 4 units²  
17. 2 units²  
18. 2 units²  
19. 3.5 units²  
20. 5 units²  
21. 5 units²  
22. 2.5 units²  
23. 1 unit²  
24. 5.5 units²  
25. 8.5 units²

Methods used in Exercises 21–25 will vary. Students may subdivide a figure into smaller squares and triangles and add their areas. They might surround a figure with a rectangle and subtract the areas of the shapes outside of the figure from the rectangle’s area. For example, a square of area 4 units² can be drawn around the shape in Exercise 23, and the area of the three 1 unit² triangles can be subtracted, leaving an area of 1 unit².

Connections

26. 8 blocks \( \cdot \) 150 m/block = 1,200 m  
27. 12 blocks \( \cdot \) 150 m/block = 1,800 m  
28. 750 m \( \div \) 150 m/block = 5 blocks. City Hall and the Stadium are 5 blocks, or 750 meters, apart by car. So are the Cemetery and the Animal Shelter, and the Art Museum and the Gas Station.

29. a. She probably found the slopes of all four sides. The slopes of any two adjacent sides are negative reciprocals of each other, so they are perpendicular line segments (in other words, all four angles were 90°).  
b. She probably found the slopes of all four sides. Because the slopes of opposite sides were the same, they were parallel. Because opposite sides of the quadrilateral were parallel, her figure was a parallelogram.

30. a. (−2, −1)  
b. There are three ways to find the shortest route. For example, Cassandra could walk 2 blocks west and 1 block south.  
c. (−1, 4)  
d. There are five ways to find the shortest route. For example, Aida could walk 1 block west and 4 blocks north.  
e. Figure out how many blocks east or west you have to go by comparing the \( x \)-coordinates of the two locations. Figure out how many blocks north or south you have to go by comparing the \( y \)-coordinates. The sum of these is the number of blocks in a shortest route.

31. a. Lines 1, 5, and 8; lines 3 and 6  
b. Lines 2 and 6; lines 3 and 2; lines 8 and 4; lines 1 and 4; lines 5 and 4

32. a. \( \frac{31}{2} \) units²  
b. Answers will vary. Possible figure:

33. a. \( 4\pi \), or about 12.56 units²  
b. \( 16 - 4\pi \), or about 3.43 units²
34. a. \((6, 0)\). It has the greatest \(x\)-coordinate.
   b. \((-5, -5)\). It has the least \(x\)-coordinate.
   c. \((-4, 6)\). It has the greatest \(y\)-coordinate.
   d. \((0, -6)\). It has the least \(y\)-coordinate.

**Extensions**

35. Road maps are typically partitioned into square areas by consecutive letters running along the sides of the map and consecutive numbers running along the top and bottom. This system is similar to a coordinate grid system, but the letters and numbers do not refer to points; they refer to regions. For example, anything in the top-left square might be in region A-1.

36. Answers will vary. Students should include compass directions as well as distances and will need to decide where the distances are to be measured from, such as airports or city centers. For example: Starting at the airport at Grand Rapids, go south 47 mi to the airport at Kalamazoo. From Kalamazoo, go northeast 60 mi to the airport at Lansing. From Lansing, go southeast 80 mi to the airport at Detroit.

**For the Teacher** You may want to point out that pilots need more exact directions than north, south, east, or west because the actual direction may be a few degrees east or west of due north.

37. Possible answer: For each parallelogram, all four sides are the same length. A rhombus is the only parallelogram with perpendicular diagonals. Students may only say that squares—rhombi with right angles—have perpendicular diagonals. You may want to encourage them to look for non-square rhombi.

38. Each triangle has an area of 1 unit\(^2\). They all have base length 1 unit and height 2 units.

39. Each triangle has an area of 3 units\(^2\) because they all have base 3 units and height 2 units.

**Possible Answers to Mathematical Reflections**

1. Driving distances are the same as or greater than flying distances. If the two places do not lie on the same vertical or horizontal line, the flying distance is shorter because the car can’t travel in a straight line between them, but the helicopter can.

2. Note that “distance” is intentionally vague. Students encountered two types of distances in Euclid: driving and flying. The flying distance corresponds to straight-line distance on the plane. Flying distances can be estimated with a ruler. Calculating flying distances exactly requires using the Pythagorean Theorem, which students do not yet know. The driving distance between two landmarks is the sum of the positive differences of the \(x\)- and \(y\)-coordinates. In other words, the driving distance is the sum of the absolute value of the differences between the \(x\)- and \(y\)-coordinates.

3. Sometimes I just counted the units of area. Sometimes I subdivided the figure into smaller shapes like right triangles and rectangles, found the areas of the smaller shapes, and added them to get the large figure’s area. Sometimes I enclosed the figure in a rectangle, found the area of the rectangle, and subtracted the areas of the figures that were not part of the enclosed figure.
Mathematical and Problem-Solving Goals

- Draw squares on 5 dot-by-5 dot grids and find their areas
- Introduce the concept of square root
- Understand square root geometrically, as the side length of a square with known area
- Use geometric understanding of square roots to find lengths of line segments on a dot grid

Summary of Problems

**Problem 2.1 Looking for Squares**
Students search for all the squares that can be drawn on a 5 dot-by-5 dot grid. In the process, they begin to see how the area of a square relates to its side length.

**Problem 2.2 Square Roots**
Students are introduced to the concept of square root. They learn that the positive square root of a number is the side length of a square with that number as area.

**Problem 2.3 Using Squares to Find Lengths**
Students find the lengths of segments on a dot grid by drawing squares with the segment as the side length. The length of the segment is the square root of the square’s area.

<table>
<thead>
<tr>
<th>Suggested Pacing</th>
<th>Materials for Students</th>
<th>Materials for Teachers</th>
<th>ACE Assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>All</strong></td>
<td>4 days</td>
<td>Centimeter rulers, calculators, student notebooks</td>
<td></td>
</tr>
<tr>
<td><strong>2.1</strong></td>
<td>1 day</td>
<td>Labsheet 2.1</td>
<td>Transparency 2.1 (optional)</td>
</tr>
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<td><strong>2.2</strong></td>
<td>1 $\frac{1}{2}$ days</td>
<td>Transparency 2.2 (optional)</td>
<td>4–34</td>
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<tr>
<td><strong>2.3</strong></td>
<td>1 day</td>
<td>Labsheet 2.3, geoboards (optional)</td>
<td>Transparencies 2.3A–C (optional)</td>
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<tr>
<td><strong>MR</strong></td>
<td>$\frac{1}{2}$ day</td>
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</tbody>
</table>
2.1 Looking for Squares

Goal

• Draw squares on 5 dot-by-5 dot grids and find their areas.

In this problem, students draw squares of various sizes on 5 dot-by-5 dot grids. In the process, they begin to see how the area of a square relates to the length of its sides.

Launch 2.1

Display Transparency 2.1 or draw a 5 dot-by-5 dot grid on the board. Draw a unit square on the grid and label it with the numeral 1.

Suggested Question Ask:

• I have drawn a square with an area of 1 square unit on this 5 dot-by-5 dot grid. Can someone come up and draw a square with a different area?

Explain that students are to search for all the different sizes (areas) of squares that will fit on a 5 dot-by-5 dot grid. Distribute Labsheet 2.1 and have students work on the problem in groups of two or three.

Explore 2.1

Some students may find “upright” squares easily (such as a square with an area of 9 square units) but have difficulty finding “tilted” squares (such as a square with an area of 10 square units).

If some students find the same size square more than once, remind them to check the area of each square they draw to verify that the areas are different.

You might want to have some groups put their work on large poster paper to refer to in the discussion and in the rest of the unit.

Summarize 2.1

Ask students to share the various squares they found. Ask them to draw them on Transparency 2.1. Continue until all eight squares are displayed. (If students do not offer all eight, suggest the missing ones yourself.) Discuss the strategies students used to find the squares.

Suggested Questions Ask:

• Which squares were easy to find? Why? (Upright squares, because their sides align with the horizontal and vertical lines of dots in the grid)

• Which squares were not easy to find? Why? (Tilted squares, because their sides must meet at right angles, but they do not align with horizontal and vertical lines of dots in the grid)

• How do you know that the figures you drew were squares? (I checked that the side lengths were equal and all angles were right angles or determined that the sides were perpendicular.)
2.1 Looking for Squares

Mathematical Goal

- Draw squares on 5 dot-by-5 dot grids and find their areas

Launch

Display Transparency 2.1 or draw a 5 dot-by-5 dot grid on the board. Draw a unit square on the grid and label it with the numeral 1.

- I have drawn a square with an area of 1 square unit on this 5 dot-by-5 dot grid. Can someone come up and draw a square with a different area?

Explain that students are to search for all the different sizes (areas) of squares that will fit on a 5 dot-by-5 dot grid. Distribute Labsheet 2.1 and have students work on the problem in groups of two or three.

Materials
- Transparency 2.1
- Labsheet 2.1
- Centimeter rulers or other straightedges

Explore

If students have difficulty identifying tilted squares, display one on the board or overhead. Start with a square of area 2.

Remind students to check the area of each square they draw to verify that the areas are all different.

Summarize

Ask students to share the various squares they found as you draw them on Transparency 2.1. Continue until all eight squares are displayed. (If students do not offer all eight, suggest the missing ones yourself.) Discuss the strategies students used to find the squares.

- Which squares were easy to find? Why?
- Which squares were not easy to find? Why?
- How did you determine that your figure was a square?

Materials
- Student notebooks
ACE Assignment Guide for Problem 2.1

Core 1, 2, 42
Other Applications 3; Extensions 47, 48
Adapted For suggestions about adapting ACE exercises, see the CMP Special Needs Handbook.
Connecting to Prior Units 42: Shapes and Designs, Covering and Surrounding, Moving Straight Ahead

Answers to Problem 2.1

A. Eight different areas are possible:

- 1 unit²
- 4 units²
- 9 units²
- 2 units²
- 5 units²
- 16 units²
- 8 units²
- 10 units²

B. For the examples in this problem, all the upright squares have whole-number side lengths. Some tilted squares also have whole-number side lengths. An example of this (a tilted square with area 25 units²) will be seen in Problem 2.3.
2.2
Square Roots

Goals
• Introduce the concept of square root
• Understand square root geometrically, as the side length of a square with known area

In this problem, the concept of square root is introduced in the context of the relationship between the area of a square and the length of its sides.

Launch 2.2
Discuss the side length of the square with an area of 4 square units. Draw the square on the board or overhead.

Suggested Questions
• This square has an area of 4 square units. What is the length of a side? (2 units)
• How do you know your answer is correct? (You can easily count 2 units along any side, and \(2 \cdot 2 = 4\), or \(2^2 = 4\).)

Introduce the concept of square root.
• What number multiplied by itself is 4? (2) We can say this another way: The square root of 4 is 2.
• A square root of a number is a number that when squared, or multiplied by itself, equals the number. 2 is a square root of 4 because \(2 \cdot 2 = 4\).
• Is there another number you can multiply by itself to get 4? (Yes, \(-2\)).

Write \(\sqrt{4}\) on the board.
• This notation means the positive square root of 4.

Add to the text on the board to get \(\sqrt{4} = 2\).
• If we want to denote the negative square root, we need to add a negative symbol.

Write \(-\sqrt{4} = -2\) on the board.
• Because we are working with lengths, we will be using only the positive square roots of numbers.

On the board or overhead, draw a square with an area of 2 square units on a dot grid.

Suggested Questions
Discuss the Getting Ready questions.
• What is the side length of a square with an area of 2 square units? (\(\sqrt{2}\) units)
• Is this length greater than 1 unit? Is it greater than 2 units? (It is between 1 and 2 units. Students may not be able to answer the first question yet because \(1^2 = 1\) and \(2^2 = 4\).)
• Is 1.5 a good estimate for \(\sqrt{2}\)? Explain. (It depends on how much accuracy we want. \(1.5 \cdot 1.5 = 2.25\), which is not that close to 2, so one could say it is not a good estimate.)
• Can you find a better estimate for \(\sqrt{2}\)? (1.4 is a better estimate because \(1.4 \cdot 1.4 = 1.96\), which is closer to 2.)

When students have some understanding of the concept of a square root, have them work on the problem in groups of two or three. Remind them that they should use a calculator only when the text asks them to do so. There is some important estimation work that would be trivialized by premature use of a calculator.

Explore 2.2
Ask students how they know their answers for Questions A and B are correct. Ask them how they could check their answers.

As the groups finish Questions A and B, ask them to find the negative square roots of 1, 9, 16, and 25 as well. Check their work to see if they are using the square root symbol correctly.
Summarize 2.2

Talk about the side length of the square with an area of 2 square units.

- How can you prove that the area of this square is 2 square units? (Subdivide the square into smaller regions and add their areas, or enclose the square in a larger square and subtract the areas of the four triangles outside the original square from the area of the larger square.)
- What is the exact length of a side of this square? \( \sqrt{2} \) units
- You estimated \( \sqrt{2} \) by measuring a side of the square. What did you get? (Most students will get about 1.4.)
- Is this the exact value of \( \sqrt{2} \)? Does 1.4 squared equal 2? (No, 1.4\(^2\) = 1.96, so 1.4 is too small.)
- You also found \( \sqrt{2} \) by using the square root key on your calculator. What value did your calculator give? \( 1.414213562 \)

Write this number on the board.

- Enter this number into your calculator and square it. Is the result exactly equal to 2? (No; you get 1.999999999.)
- So the value you get by measuring and the value you get with your calculator are both estimates.

For the Teacher  If you use a calculator to find \( \sqrt{2} \), and then square the calculator’s result, you will get 2. However, if you enter the estimate and square it, you will get 1.999999999.

Emphasize that the value students got by measuring and the value they got using a calculator are only approximate values of \( \sqrt{2} \). The exact value is \( \sqrt{2} \). Note that \( \sqrt{2} \cdot \sqrt{2} = \sqrt{4} = 2. 

Continue the discussion with \( \sqrt{5} \):
- What number multiplied by itself is 5? \( \sqrt{5} \)
- Which two whole numbers is \( \sqrt{5} \) between? (2 and 3)
- How do you know this? \( 2^2 \) is 4 and \( 3^2 \) is 9. So, 5 is between \( 2^2 \) and \( 3^2 \). This means \( \sqrt{5} \) is between 2 and 3.)
- Is \( \sqrt{5} \) closer to 2 or 3? (2. Possible explanation: 2.5\(^2\) is 6.25. Because 5 is less than 6.25, \( \sqrt{5} \) must be less than 2.5, so it is closer to 2.)

Ask some students to suggest decimal approximations for \( \sqrt{5} \). As a class, use a calculator to multiply each approximation by itself to check whether the result is 5.

Discuss the answer to Question E.
- What are the side lengths of all the squares you found in Problem 2.1? (1, \( \sqrt{2} \), \( \sqrt{5} \), \( \sqrt{8} \), \( \sqrt{10} \), and 4)
- Which is the least side length? Which is the greatest?

You might display a number line on the board or overhead and invite students to mark and label the location of each side length. To know where to place some of the values, they may need to use a calculator to find approximate square roots or use reasoning similar to that used in Question D to find an estimate for \( \sqrt{5} \).

Question E leads into Problem 2.3, in which students find all the different lengths of line segments that can be drawn on a 5 dot-by-5 dot grid.
### Mathematical Goals
- Introduce the concept of square root
- Understand square root geometrically, as the side length of a square with known area

### Launch
Discuss the side length of the square with an area of 4 square units.
- What is the length of each side? How do you know your answer is correct?

Introduce the concept of square root.
- What number multiplied by itself is 4? We say the square root of 4 is 2.
- A square root of a number is a number that when squared, or multiplied by itself, equals the number. 2 is a square root of 4 because $2 \times 2 = 4$.
- Is there another number you can multiply by itself to get 4?

Introduce square root notation. Write $\sqrt{4} = 2$ and $-\sqrt{4} = -2$ on the board.

Draw a square with an area of 2 square units on a dot grid. Ask:
- What is the side length of this square? Is it greater than 1? Is it greater than 2? Is 1.5 a good estimate for $\sqrt{2}$? Can you find a better estimate?

When students understand the concept of square root, have them work on the problem in groups of two or three. Remind students that they should use a calculator only when the text asks them to do so.

### Explore
Ask students how they know their answers for Questions A and B are correct. Ask them how they could check their answers.

Ask students to find the negative square roots of 1, 9, 16, and 25 as well. Check their work to see if they are using the square root symbol correctly.

### Summarize
Talk about the side length of the square with an area of 2 square units.
- How can you prove that the area of this square is 2 square units?
- What is the exact length of a side of this square?
- You estimated $\sqrt{2}$ by measuring a side of the square. What did you get? Is this the exact value of $\sqrt{2}$?
- You also found $\sqrt{2}$ by using the square root key on your calculator. What value did your calculator give? Enter this number into your calculator and square it. Is the result exactly equal to 2?
Summarize

Emphasize that the results found by measuring and with a calculator are only approximate values for $\sqrt{2}$.

Ask students for decimal approximations for $\sqrt{5}$. As a class, use a calculator to square each approximation to check whether the result is 5.

Discuss Question E.

• What are the side lengths of all the squares you found in Problem 2.1? Which is the least side length? Which is the greatest?

You could have the students write the lengths on a number line.

ACE Assignment Guide for Problem 2.2

Core 4–6, 10, 14–18
Other Applications 7–9, 11–13, 19–34; unassigned choices from earlier problems

Adapted For suggestions about ACE exercises, see the CMP Special Needs Handbook.

Answers to Problem 2.2

A. 1. unit; 3 units; 4 units; 5 units
   2. $\sqrt{1} = 1$, $\sqrt{9} = 3$, $\sqrt{16} = 4$, $\sqrt{25} = 5$.
B. 1. 144 units$^2$; 6.25 units$^2$
   2. $\sqrt{144} = 12$, $\sqrt{6.25} = 2.5$.
C. 1. About 1.4
   2. Using 1.4 as the side length gives an area of $1.4 \cdot 1.4 = 1.96$ units$^2$, which is not equal to 2 units$^2$.
   3. Possible answer: 1.41421356237. The exact number of digits depends on the type of calculator.
   4. The ruler estimate gives only the first few digits of the calculator estimate. In our case, the ruler estimate has only one decimal place. The calculator gives greater accuracy, but its answer is also an approximation, just as the ruler answer is.

D. 1. 2 and 3. Because 5 falls between $2^2$ and $3^2$, $\sqrt{5}$ must be between 2 and 3.
   2. 2 is closer to $\sqrt{5}$. Possible explanation: $2.5^2$ is 6.25. Because 5 is less than 6.25, $\sqrt{5}$ must be less than 2.5.

For the Teacher  We cannot know which whole number a square root is closer to by comparing the squares of the numbers. Consider the preceding example: 6.25 is closer to 4 than to 9, yet $\sqrt{6.25} = 2.5$ is exactly halfway between 2 and 3. This fact is not the point of this problem and need not be made with students just beginning to understand square roots.

3. 2.24. This estimate can be found by trial and error as follows: Find the squares of 2.1, 2.2, 2.3, and so on. You’ll find that 5 is between $2.3^2$ and $2.4^2$. So, $\sqrt{5}$ must be between 2.2 and 2.3. Then, find the squares of 2.21, 2.22, 2.23, and so on. You’ll find that $\sqrt{5}$ is between 2.23 and 2.24. Next, find the squares of 2.231, 2.232, 2.233, and so on. You’ll find that 5 is between $2.236^2$ and $2.237^2$. This means that $\sqrt{5}$ rounded to the hundredths place is 2.24.

E. 1 unit, $\sqrt{2}$ units, 2 units, $\sqrt{5}$ units, $\sqrt{8}$ units, 3 units, $\sqrt{10}$ units, and 4 units
Goal

- Use geometric understanding of square roots to find lengths of line segments on a dot grid

In this problem, students develop a strategy for finding the distance between dots on a grid by examining the line segment between the dots. To find the length of the line segment, students draw a square with the segment as one side, find the area of the square, and then find the square root of the area.

Launch 2.3

As a class, list all the side lengths (in units) students have found so far in their work with 5 dot-by-5 dot grids: 1, √2, 2, √5, √8, 3, √10, and 4.

Suggested Question Ask:

- Can you draw a line segment on a 5 dot-by-5 dot grid with a length that is different from these?

On Transparency 2.3A, draw the segment the class suggests, or draw one of your own. Here is an example:

```
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
```

When students understand the process, distribute Labsheet 2.3 and have students explore the problem in groups of three or four. If geoboards are available, students can put two or more of them together to work on this problem.

Explore 2.3

Groups do not need to find all 14 possible lengths. However, be sure every student is able to draw a square on a line segment and find the length of the segment. You may want to have some groups put their work on poster paper for discussion.

Going Further

Ask students who finish to count the different lengths that can be drawn on a 2 dot-by-2 dot grid, a 3 dot-by-3 dot grid, and a 4 dot-by-4 dot grid. Have them look for a pattern that will help them to predict the number of possible lengths on a 6 dot-by-6 dot and 7 dot-by-7 dot grid. For an \( n \) dot-by-\( n \) dot grid, there are all of the lengths that were in an \((n - 1)\) dot-by-\((n - 1)\) dot grid, plus \( n \) more. Therefore, a 6 dot-by-6 dot grid has the 14 lengths from the 5 dot-by-5 dot grid, plus 6 more, for a total of 20. The 7 dot-by-7 dot grid has 20 + 7, or 27 lengths.
Ask students to share the lengths they found. Draw the lengths on Transparency 2.3A or show them on an overhead geoboard. Continue until all 14 lengths are displayed. Ask students to share strategies they used to make sure they had all the lengths. Arrange the lengths in an orderly way (see below).

Discuss the strategies students used to find the lengths. In some cases, students may have used relationships between line segments rather than drawing a square. For example, the length of segment $AG$ is twice that of segment $AF$, so it is $2\sqrt{2}$. The area of a square with a side length of $2\sqrt{2}$ is 4 times the area of the similar square with an area of 2, or $4 \cdot 2 = 8$. Thus $\sqrt{8} = 2\sqrt{2}$.

Students who find the length of $AG$ by drawing a square will get $\sqrt{8}$ or $2\sqrt{2}$. If your class is ready, talk about this equivalence: $\sqrt{8} = \sqrt{4} \cdot 2 = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$. Or, have students use a calculator to evaluate the various expressions.

**Suggested Questions** To test their understanding of $A(3)$, ask the following:

- *Between what two whole numbers does $\sqrt{17}$ lie?* (4 and 5)
- *Which whole number is it closer to?* (It is closer to 4 because $4.52 = 20.25$, so $\sqrt{17}$ is less than 4.5. A calculator tells us that it is about 4.123105626.)
- *Between what two whole numbers does $\sqrt{32}$ lie?* (5 and 6)
- *How many of the lengths we have listed would you have found on a 4-dot-by-4-dot grid?* (1, 2, and 3 as side lengths of upright squares; $\sqrt{2}, \sqrt{5}, \sqrt{8}, \sqrt{10}, \sqrt{13},$ and $\sqrt{18}$ as side lengths of tilted squares)
- *What is $\sqrt{2} \times \sqrt{2}$?* (2) Why? (Because $\sqrt{2}$ is the side length of a square with area 2)
- *What is $\sqrt{5} \times \sqrt{5}$?* (5) Why?

If Question C has not been discussed, be sure students share their strategies. Ask if there are other line segments whose lengths can be expressed in more than one way. For example, $3\sqrt{2} = \sqrt{18}$ and $2\sqrt{5} = \sqrt{20}$.

- *Are there lengths that cannot be expressed in more than one way?* (Yes, $\sqrt{2}, \sqrt{5} \ldots$)

**Check for Understanding**

Draw another segment on a dot grid. Ask the class to express its exact length using a $\sqrt{\text{symbol}}$ and then to tell which two whole numbers the length is between.

- *Which whole number is it closer to? How do you know?*
- *Is there another way to express this length?* (For example, $\sqrt{8} = 2\sqrt{2}$)
2.3 Using Squares to Find Lengths

Mathematical Goal

• Use geometric understanding of square roots to find lengths of line segments on a dot grid

Launch

List all the side lengths that students have found so far in their work with 5 dot-by-5 dot grids: 1, $\sqrt{2}$, 2, $\sqrt{5}$, $\sqrt{8}$, 3, $\sqrt{10}$, and 4.

• Can you draw a line segment on a 5 dot-by-5 dot grid with a length that is different from these?

On Transparency 2.3A, draw the segment the class suggests, or draw one of your own.

• How do you know the length of this segment is different from others you have found? How might we find the actual length of this line segment?

Explain to students that the squares they draw in the problem will extend beyond the 5 dot-by-5 dot grid. Have students explore the problem in groups of three or four.

Explore

Groups do not need to find all 14 possible lengths. However, be sure every student is able to draw a square on a line segment and find the length of the segment.

Summarize

Ask students to share the lengths they found. Draw the lengths on Transparency 2.3 or show them on an overhead geoboard. Continue until all 14 line segment lengths are displayed. Ask the class for strategies they used to make sure they had all the lengths.

Discuss the strategies that students used to find the lengths. If your class is ready, talk about equivalence: $\sqrt{8} = \sqrt{4} \cdot 2 = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$.

Part (3) of Question A asks students for approximations of some of the square roots they have found. To test their understanding, ask the following:

• Between what two whole numbers does $\sqrt{17}$ lie? Which whole number is it closer to?

• Between what two whole numbers does $\sqrt{32}$ lie?

• How many of the lengths we have listed would you have found on a 4 dot-by-4 dot grid? What is $\sqrt{2} \cdot \sqrt{2}$? What is $\sqrt{5} \cdot \sqrt{5}$? Why?

If Question C has not been discussed, be sure students share their strategies. Share Transparencies 2.3B and 2.3C with your students.
Check for Understanding

Draw another segment on a dot grid. Ask the class to express its exact length using a $\sqrt{}$ symbol and then to tell which two whole numbers the length is between.

ACE Assignment Guide for Problem 2.3

Core 35–37, 41
Other Applications 38–40; Connections 43–46; Extensions 49–53; unassigned choices from earlier problems

Adapted For suggestions about adapting Exercise 41 and other ACE exercises, see the CMP Special Needs Handbook.

Connecting to Prior Units 43: Covering and Surrounding; 45: Bits and Pieces III; 46: Stretching and Shrinking

Answers to Problem 2.3

A. 1 and 2.

The possible lengths in increasing order are $1, \sqrt{2}, 2, \sqrt{5}, \sqrt{8}, 3, \sqrt{10}, \sqrt{13}, 4, \sqrt{17}, \sqrt{18}, \sqrt{20}, 5, \text{ and } \sqrt{32}$. See the Summarize section for pictures and more information.

B. Both are correct. The length of $AC$ is twice the length of $AB$. Because the length of $AB$ is $\sqrt{2}$ (being a side of the small square), the length of $AC$ is $2\sqrt{2}$, or $2\sqrt{2}$. We can also find the length of $AC$ directly by making it a side of a square (the large square in the picture below) whose area is 8. So, the length of $AC$ equals $\sqrt{8}$. So, $2\sqrt{2} = \sqrt{8}$.

C. 1. $\sqrt{40}$, or $2\sqrt{10}$

2. Some examples are $\sqrt{17}$, $\sqrt{13}$, $\sqrt{5}$.

<table>
<thead>
<tr>
<th>Exact Length</th>
<th>Decimal Approximation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{2}$</td>
<td>1.4</td>
</tr>
<tr>
<td>$\sqrt{5}$</td>
<td>2.2</td>
</tr>
<tr>
<td>$\sqrt{8}$</td>
<td>2.8</td>
</tr>
<tr>
<td>$\sqrt{10}$</td>
<td>3.2</td>
</tr>
<tr>
<td>$\sqrt{13}$</td>
<td>3.6</td>
</tr>
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<td>$\sqrt{17}$</td>
<td>4.1</td>
</tr>
<tr>
<td>$\sqrt{18}$</td>
<td>4.2</td>
</tr>
<tr>
<td>$\sqrt{20}$</td>
<td>4.5</td>
</tr>
<tr>
<td>$\sqrt{32}$</td>
<td>5.7</td>
</tr>
</tbody>
</table>
Investigation 2

ACE Assignment Choices

Problem 2.1
Core 1, 2, 42
Other Applications 3; Extensions 47, 48

Problem 2.2
Core 4–6, 10, 14–18
Other Applications 7–9, 11–13, 19–34; unassigned choices from earlier problems

Problem 2.3
Core 35–37, 41
Other Applications 38–40; Connections 43–46; Extensions 49–53; unassigned choices from earlier problems

Adapted For suggestions about adapting Exercise 41 and other ACE exercises, see the CMP Special Needs Handbook.

Connecting to Prior Units 42: Shapes and Designs; 43: Covering and Surrounding; 45: Bits and Pieces III; 46: Stretching and Shrinking

Applications

1. 1, 2, and 4 units²

2. Possible answer:

3. Possible answer: By subdividing the square along its diagonals, you get four triangles, each with an area of $\frac{1}{2}$ unit². Therefore, the square has an area of 2 units².

Note: Ask students to draw the square above inside an upright square with an area of 4 units². Then, ask how the larger square can be used to find the area of the smaller square. Because each triangle formed has an area of $\frac{1}{2}$ unit², the area of the smaller square is $4 - (4 \cdot \frac{1}{2}) = 2$ units².

4. a. 2 units² b. About 1.414 units

5. a. 5 units² b. About 2.236 units

6. Area: 45 units²; side length: $\sqrt{45}$ units, or about 6.708 units

7. $\sqrt{11} \approx 3.3$ 8. $\sqrt{30} \approx 5.5$

9. $\sqrt{172} \approx 13.1$
10. B  
11. 12  
12. 0.6  
13. 31  
14. 5 and 6. Because 27 is between $5^2$ and $6^2$, $\sqrt{27}$ is between 5 and 6.  
15. 31 and 32. Because 1,000 is between $31^2$ and $32^2$, $\sqrt{1,000}$ is between 31 and 32.  
16. True  
17. True  
18. False. $11^2 = 121$  
19. 6,561  
20. 196  
21. 5.3  
22. 10.24  
23. $2^2 = 4$  
24. $3^2 = 9$  
25. 2  
26. 3  
27. 4  
28. 5  
29. 1 and –1  
30. 2 and –2  
31. $\sqrt{2}$ and $-\sqrt{2}$  
32. 4 and –4  
33. 5 and –5  
34. $\sqrt{5}$ and $-\sqrt{5}$  
35. 1 unit, $\sqrt{2}$ units, 2 units, $\sqrt{5}$ units, $\sqrt{8}$ units  
36. a. $\sqrt{29}$ units  
   b. 5 and 6. $5^2$ is 25 and $6^2$ is 36, and 29 is between 25 and 36.  
37. First way: The area of a square with side $AB$ is 5 units$^2$. So, the length of $AB$ is $\sqrt{5}$ units. The length of $AC$ is twice the length of $AB$. So, the length of $AC$ is $2\sqrt{5}$ units.  
   Second way: The area of a square with side $AC$ is 20 units$^2$. So, the length of $AC$ is $\sqrt{20}$ units.  
38. G  
39. $AB = \sqrt{5}$ units; $BC = \sqrt{5}$ units; $CD = \sqrt{2}$ units, $DA = \sqrt{2}$ units  
40. $EF = \sqrt{13}$ units; $FG = 1$ unit; $GH = 1$ unit; $HJ = \sqrt{2}$ units; $JK = \sqrt{2}$ units; $KL = \sqrt{5}$ units; $LE = \sqrt{2}$ units  
41. (Figure 1)  

Connections  

42. a. $U$, $W$, and $X$ are right triangles. Possible reasoning: I used a corner of a piece of paper (or an angle ruler) to check for $90^\circ$ angles.  
   b. Triangle $U$: 2.5 units$^2$; Triangle $W$: 2 units$^2$, Triangle $X$: 9 units$^2$.  
43. a.  

<table>
<thead>
<tr>
<th>Area (units$^2$)</th>
<th>Perimeter (units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>$4\sqrt{2} \approx 5.66$</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>$4\sqrt{5} \approx 8.94$</td>
</tr>
<tr>
<td>8</td>
<td>$4\sqrt{8} \approx 11.31$</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>$4\sqrt{10} \approx 12.65$</td>
</tr>
<tr>
<td>16</td>
<td>16</td>
</tr>
</tbody>
</table>

b. The perimeter is the length of a side multiplied by 4. Symbolically, $P = 4\ell$.  

Figure 1
44. a. Possible answer:

b. Q or S

c. Eight possibilities are shown.

45. (Figure 2)

46. a. Yes. All squares are similar to each other.

b. The coordinates of each vertex of the larger square are twice the coordinates of the corresponding vertex of the smaller square.

c. The area of the larger square is 4 times the area of the smaller square.

d. Squares will vary. However, the coordinates of each vertex of the larger square will be some constant, a, times the coordinates of the corresponding vertex of the smaller square. The area of the larger square will be \( a^2 \) times the area of the smaller square.

**Extensions**

47. Possible answers:

48. Possible answers:

49. No. Possible explanation: \( \sqrt{8} \) is greater than 2, so \( \sqrt{8} + \sqrt{8} \) is greater than 4. However, \( \sqrt{16} \) is 4.
50. a. $\sqrt{10}$
   
b. No. Possible explanations: The rectangles made from putting together two copies of each triangle have different areas, so the triangles must have different areas.

![Diagram of rectangles with areas 4 units$^2$, 5 units$^2$, and 3 units$^2$]

Or, the three triangles have the same base but different heights so they must have different areas.

51. Whole number. $\sqrt{2} \cdot \sqrt{50} = \sqrt{100} = 10$

52. Whole number. $\sqrt{4} \cdot \sqrt{16} = 2 \cdot 4 = 8$ or $\sqrt{4} \cdot \sqrt{16} = \sqrt{64} = 8$

53. Not a whole number. $\sqrt{4} \cdot \sqrt{6} = \sqrt{4} \cdot \sqrt{6} = \sqrt{24} = 4.9$

Possible Answers to Mathematical Reflections

1. To find the length of a horizontal or vertical line segment, you can just count the units. To find the length of a diagonal segment, you can draw a square with the segment as a side and then take the square root of the square's area, which gives the length of a side of the square. Or, you might be able to compare the segment to others for which you know the lengths. For example, the longer segment below is twice the length of the shorter segment. The length of the shorter segment is $\sqrt{2}$, so the length of the longer segment is $2 \cdot \sqrt{2}$ or $2\sqrt{2}$.

![Diagram of segments with lengths 2 and $2\sqrt{2}$]

2. Taking the square root of a number is the opposite of finding the square. For example, if $a \cdot a = 9$, then $a$ is the square root of 9. Every positive number has two square roots. In this case, the square root of 9 is 3 and $-3$, because $3 \cdot 3 = 9$ and $-3 \cdot -3 = 9$. We can show this by writing $3 = \sqrt{9}$ and $-3 = -\sqrt{9}$.
The Pythagorean Theorem

Mathematical and Problem-Solving Goals

- Deduce the Pythagorean Theorem through exploration
- Use the Pythagorean Theorem to find unknown side lengths of right triangles
- Reason through a geometric proof of the Pythagorean Theorem
- Use the Pythagorean Theorem to find the distance between two points on a grid
- Determine whether a triangle is a right triangle based on its side lengths
- Relate areas of squares to the lengths of the sides

Summary of Problems

<table>
<thead>
<tr>
<th>Problem</th>
<th>Title</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>3.1</td>
<td>The Pythagorean Theorem</td>
<td>Students collect information about the areas of the squares on the sides of right triangles and conjecture that the sum of the areas of the two smaller squares equals the area of the largest square.</td>
</tr>
<tr>
<td>3.2</td>
<td>A Proof of the Pythagorean Theorem</td>
<td>Students investigate a puzzle that verifies that the sum of the areas of the squares on the legs of a right triangle is equal to the area of the square on the hypotenuse.</td>
</tr>
<tr>
<td>3.3</td>
<td>Finding Distances</td>
<td>Students use the Pythagorean Theorem to find distances between dots on a grid.</td>
</tr>
<tr>
<td>3.4</td>
<td>Measuring the Egyptian Way</td>
<td>Students explore the converse of the Pythagorean Theorem: If $a$, $b$, and $c$ are the lengths of the sides of a triangle and $a^2 + b^2 = c^2$, then the triangle is a right triangle.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Suggested Pacing</strong></th>
<th><strong>Materials for Students</strong></th>
<th><strong>Materials for Teachers</strong></th>
<th><strong>ACE Assignments</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>Calculators, centimeter rulers</td>
<td>Transparency 3.1</td>
<td>1–14</td>
</tr>
<tr>
<td>3.1</td>
<td>Dot paper</td>
<td>Transparencies 3.2A, 3.2B</td>
<td>18–23, 26</td>
</tr>
<tr>
<td>3.2</td>
<td>Labsheets 3.2A–C, scissors</td>
<td>Transparency 3.3</td>
<td>24, 27–35</td>
</tr>
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<td>3.3</td>
<td>Labsheet 3.3</td>
<td>Transparency 3.4</td>
<td>15–17, 25</td>
</tr>
<tr>
<td>3.4</td>
<td>String; straws or polystrips (optional)</td>
<td>Transparency 3.4</td>
<td></td>
</tr>
<tr>
<td>MR</td>
<td>½ day</td>
<td></td>
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</tbody>
</table>
The Pythagorean Theorem

Goals

- Deduce the Pythagorean Theorem through exploration
- Use the Pythagorean Theorem to find unknown side lengths of right triangles

In this problem, students collect data about the areas of squares on the sides of a right triangle. They use patterns in their data to conjecture that the sum of the areas of the two smaller squares equals the area of the largest square.

Launch 3.1

To introduce the topic, draw a right triangle below on a dot grid at the board or overhead.

- What kind of triangle have I drawn? (A right triangle)

Explain that in a right triangle, the two sides that form the right angle are called the legs of the right triangle. The side opposite the right angle is called the hypotenuse.

- What are the lengths of the two legs of this triangle? (1 unit)

Suggested Questions Ask:

- How can we find the length of the hypotenuse of the triangle? (Draw a square using this segment as a side. Then, find the area of the square and take its square root.)

Explore 3.1

Ask that each student complete a table. Encourage the students in each group to share the work, with each student finding the areas for two or three of the right triangles.

Check to see that students are correctly drawing the squares on the right triangles.

Summarize 3.1

Ask the class to discuss the patterns they see in the table. They should notice that the sum of the areas of the squares on the legs is equal to the area of the square on the hypotenuse.

Suggested Questions Ask:

- What conjecture can you make about your results? (When you add the areas of the squares on the legs, you get the area of the square on the hypotenuse.)
- This pattern is called the Pythagorean Theorem.
Draw and label a right triangle as shown below.

\[ \begin{array}{c}
 a \\
 b \\
 c
\end{array} \]

- **Suppose a right triangle has legs of lengths** \( a \) \( \text{and} \) \( b \) \( \text{and} \) a hypotenuse of length \( c \). Using these letters, can you state the Pythagorean Theorem in a general way? (If \( a \) and \( b \) are the lengths of the legs of a right triangle and \( c \) is the length of the hypotenuse, then \( a^2 + b^2 = c^2 \).)

- **Do you think the Pythagorean Theorem will work for triangles that are not right triangles?**

To help the class explore this question, draw the triangle shown below on the board or overhead (or have the class try this example on their own). Use a corner of a sheet of paper to verify that the triangle does not contain a right angle.

Then, draw squares on each side of the triangle.

**Suggested Questions** Ask:

- **We have shown that this triangle is not a right triangle. What are the areas of the squares on its sides?** (5, 10, and 25 square units)

- **Is the sum of the areas of the squares on the shorter sides equal to the area of the square on the longest side?** (No; \( 5 + 10 \neq 25 \))

Next, ask the class this question:

- **Do you think the Pythagorean Theorem is true for all right triangles, even if the sides are not whole numbers?**

The theorem is true for all right triangles. To help the class explore this, you may want to do ACE Exercises 13 and 14 as a class. The triangle in Exercise 14 has leg lengths \( \sqrt{5} \) units and \( \sqrt{5} \) units, and hypotenuse length \( \sqrt{10} \). The squares of these side lengths are 5, 5, and 10 and \( 5 + 5 = 10 \). This shows that the Pythagorean Theorem applies to a right triangle with side lengths that are not whole numbers. A proof that shows the theorem is true for all right triangles is developed in the next problem.

The Pythagorean Theorem is useful for finding unknown side lengths in a right triangle. In this spirit, you could wrap up by having students add a column to their tables, labeled “Length of Hypotenuse.” Fill in this column together, or give students a short period of time to complete it themselves and then check the results as a class.

**Suggested Question** Choose one of the right triangles in the table, list the lengths of the three sides, and ask students what the Pythagorean Theorem says about these lengths.

- *The lengths of the sides of a right triangle are 2, 3, and \( \sqrt{13} \). What does the Pythagorean Theorem say about these lengths?*
  
  \( (2^2 + 3^2 = (\sqrt{13})^2, \text{or } 4 + 9 = 13) \)

Repeat the question for lengths 5, 12, and 13.
### 3.1 The Pythagorean Theorem

#### Mathematical Goals

- Deduce the Pythagorean Theorem through exploration
- Use the Pythagorean Theorem to find unknown side lengths of right triangles

#### Launch

Draw a tilted line segment on a dot grid at the board or overhead. Ask:

- *How can we find the length of this line segment?*
  
  Using the original line segment as a hypotenuse, draw two line segments to make a right triangle.

- *What kind of triangle have I drawn?*
  
  Explain that in a right triangle, the two sides that form the right angle are called the *legs* of the right triangle. The side opposite the right angle is called the *hypotenuse*.

- *What are the lengths of the two legs of this triangle?*

- *What are the areas of the squares on the legs? What is the area of the square on the hypotenuse?*

  Have students work in groups of three or four on the problem.

#### Explore

Ask that each student complete a table. Encourage the students in each group to share the work, with each student finding the areas for two or three of the right triangles.

As you circulate, check to see that students are correctly drawing the squares on the right triangles.

Discuss the patterns in the table.

- *What conjecture can you make about your results? This pattern is called the Pythagorean Theorem.*

- *Suppose a right triangle has legs of lengths a and b and a hypotenuse of length c. Using these letters, can you state the Pythagorean Theorem in a general way?*

- *Do you think the Pythagorean Theorem will work for triangles that are not right triangles?*

  Help the class explore this question by drawing a non-right triangle and then drawing squares on the sides. Then ask:
Summarize

- Do you think the Pythagorean Theorem is true for all right triangles, even if the sides are not whole numbers?

The theorem is true for all right triangles. To help the class explore this, you may want to do ACE Exercises 13 and 14 as a class.

You could wrap up by having students add a column to their tables, labeled “Length of Hypotenuse.” Fill in this column together, or give students time to complete it themselves and then check the results as a class.

Choose one of the right triangles in the table, list the lengths of the three sides, and ask students what the Pythagorean Theorem says about these lengths.

Materials
- Student notebooks

Vocabulary
- conjecture
- Pythagorean Theorem

ACE Assignment Guide
for Problem 3.1

Core 1, 2, 5, 6, 8–11, 12
Other Applications 3, 4, 7, 13, 14

Adapted For suggestions about adapting Exercises 8–11 and other ACE exercises, see the CMP Special Needs Handbook.
Connecting to Prior Units 18–22: Filling and Wrapping

Answers to Problem 3.1

A. (Figure 1)

B. The area of the square on the hypotenuse is equal to the sum of the areas of the squares on the legs.

C. Possible answer: If the legs of a right triangle are 4 units and 1 unit, then the area of the square on the hypotenuse is 17 units$^2$ because $16 + 1 = 17$.

Figure 1

<table>
<thead>
<tr>
<th>Length of Leg 1 (units)</th>
<th>Length of Leg 2 (units)</th>
<th>Area of Square on Leg 1 (units$^2$)</th>
<th>Area of Square on Leg 2 (units$^2$)</th>
<th>Area of Square on Hypotenuse (units$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
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<tr>
<td>3</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
</tr>
</tbody>
</table>
Goal

• Reason through a geometric proof of the Pythagorean Theorem

In this problem, students investigate a puzzle that verifies that the sum of the areas of the squares on the legs of a right triangle is equal to the area of the square on the hypotenuse. Students are again introduced to this idea in symbolic form: If $a$ and $b$ are the lengths of the legs of a right triangle, and $c$ is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

Launch 3.2

We have seen many examples of right triangles that satisfy the Pythagorean Theorem. While these examples are convincing, we can never be sure that this theorem works for all right triangles. To be sure, we need a mathematical proof which uses reasoning to show that a conjecture is always true.

Explain that there are many proofs of the Pythagorean Theorem. One of the proofs is based on the puzzle they will explore in this problem.

Display a set of puzzle pieces on the overhead. Ask students if they see any relationships among the puzzle pieces. Some may notice that the square pieces fit on the sides of the right triangle.

Summarize 3.2

Have students work in groups of four on the problem. Give each student scissors and a copy of Labsheets 3.2A.

Explore 3.2

Encourage each group to find more than one way to fit the puzzle pieces into the two frames.

Make sure each group compares their results with those of another group.

Pass out a new set of puzzle pieces (Labsheets 3.2B and 3.2C) for some groups to explore.

When groups have finished the problem, ask about any general patterns they noticed. Some may mention the relationship between the squares and the sides of the right triangle. Others may notice that a side length of a puzzle frame is equal to the sum of the lengths of the two legs of each triangle. Demonstrate these relationships at the overhead.

Have a couple of groups show how they arranged their puzzle pieces. The arrangements may differ slightly, but they all lead to the same conclusion. One arrangement is shown below.

I'm handing out sheets containing two puzzle frames and 11 puzzle pieces. Your task is to arrange the puzzle pieces in the two frames using 4 triangles in each frame and to look for a relationship among the areas of the three square pieces.
Suggested Questions  Ask:
• What relationship do these completed puzzles suggest?
• How do the dimensions of the frame relate to the sides of the triangles?
• Can you be sure that the puzzle pieces fit the frame precisely?
• Would you be able to use this “arrangement” proof for any right triangle?

Help students to understand the following argument:
• The areas of the frames are equal.
• Each frame contains four identical right triangles.
• The shapes exactly fit the frame, making straight edges where needed, and matching the “a + b” dimensions.
• If the four right triangles are removed from each frame, the area of the shapes remaining in the frames must be equal. That is, the sum of the areas of the squares in one frame must equal the area of the square in the other frame.

Label a diagram of one of the arrangements suggested by the class as shown below.

Suggest that students apply this method to another right triangle (or use Labsheets 3.2B and 3.2C) as part of their homework. They will need to make eleven puzzle pieces and 2 identical frames. Because this procedure will work for any right triangle, this means that our conjecture about the side lengths of a right triangle is true for all right triangles. This is a geometric proof of the Pythagorean Theorem.

Offer an example to help students apply the theorem.

Suggested Question  Ask:
• How can you use the Pythagorean Theorem to find the length of the hypotenuse of a right triangle? (If we know the lengths of the legs, we can find the areas of the squares on those two sides and add them. This total area is equal to the area of the square on the hypotenuse. Taking the square root of that amount will give us the length of the hypotenuse.)

Check for Understanding

Draw these triangles on the board or overhead:

The diagram shows that if the lengths of the legs of a right triangle are a and b and the length of the hypotenuse is c, then \(a^2 + b^2 = c^2\).
### 3.2 A Proof of the Pythagorean Theorem

**Mathematical Goal**

- Reason through a geometric proof of the Pythagorean Theorem

---

**Launch**

Explain to the class that there are many proofs of the Pythagorean Theorem. One is based on the puzzle they will explore in this problem.

Display a set of puzzle pieces on the overhead. Ask students if they see any relationships among the puzzle pieces.

- Your task is to arrange the puzzle pieces in the two frames and to look for a relationship among the areas of the three square pieces.

Have students work in groups of four on the problem. Give each student scissors and a copy of Labsheet 3.2A.

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**Explore**

Encourage each group to find more than one way to fit the puzzle pieces into the two frames.

Make sure each group compares its results with those of another group.

Pass out a new set of puzzle pieces (Labsheets 3.2B and 3.2C) for some groups to explore.

---

**Summarize**

When groups have finished the problem, ask about any general patterns they noticed. Demonstrate these relationships at the overhead.

Have groups show how they arranged their puzzle pieces.

- What relationship do these completed puzzles suggest?

Help students understand the following argument: The areas of the frames are equal. Each frame contains four identical right triangles. If the four right triangles are removed from each frame, the area remaining in the frames must be equal. That is, the sum of the areas of the squares in one frame must equal the area of the square in the other frame.

Show a diagram of the completed puzzles with sides labeled $a$, $b$, and $c$. Use the diagram to help students see the symbolic form of the Pythagorean Theorem: $a^2 + b^2 = c^2$. Offer an example to help them apply the theorem.

- How can you use the Pythagorean Theorem to find the length of the hypotenuse of a right triangle?

---

**Check for Understanding**

Draw two right triangles on the board. One should have legs labeled 6 and 2, and hypotenuse labeled “?”. The other should have legs labeled 4 and “?”, the hypotenuse labeled 7. Ask students to find the unknown lengths.
ACE Assignment Guide for Problem 3.2

Core 23, 26
Other Connections 18–22, unassigned choices from earlier problems
Adapted For suggestions about adapting ACE exercises, see the CMP Special Needs Handbook.
Connecting to Prior Units 23: Accentuate the Negative; 26: Filling and Wrapping

Answers to Problem 3.2

A. Each side length of the triangle is equal to the lengths of the sides of one of the three squares.

B. 1. Possible arrangement:

2. The sum of the areas of the two smaller squares is equal to the area of the largest square.

3. The sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse.

4. Because the procedure for arranging the triangles and squares for this problem can be applied to any right triangle, the conclusion is true for all right triangles.

C. 1. \(3^2 + 5^2 = 34 \text{ cm}^2\)

2. \(\sqrt{34} \text{ cm}, \text{ or approximately 5.83 cm}\)

D. For a right triangle with legs of lengths \(a\) and \(b\) and hypotenuse of length \(c\), \(a^2 + b^2 = c^2\).
3.3 Finding Distances

Goals

• Use the Pythagorean Theorem to find the distance between two points on a grid
• Relate areas of squares to the lengths of the sides

In this problem, students discover how the Pythagorean Theorem can be used to find the distance between two dots on a grid.

Launch 3.3

Display Transparency 3.3, or a transparent grid, and indicate or label points K and L as shown in the Student Edition.

Suggested Questions Ask:
• How can you find the distance between these two points?

The class may suggest measuring the distance with a ruler. Explain that the Pythagorean Theorem can be used to find an exact length.

Draw line segment KL and ask:
• How can we use the Pythagorean Theorem to find the length of this line segment?

Some students will probably suggest using the segment as the side of a square; others may suggest using it as the hypotenuse of a right triangle.

• What right triangle has this hypotenuse?

Sketch students’ suggestions, which may be either of the triangles shown here:

Explore 3.3

Students should find the problem a review of what they have learned so far. However, Question D is a bit difficult, so you may need to help guide their thinking.

Suggested Questions Ask:
• Can the $\sqrt{13}$-unit line segment be a vertical or a horizontal segment? (No)
• Why not? (Vertical and horizontal segments have whole-number lengths on dot grids.)
• If it is a tilted line segment, can it be the hypotenuse of a right triangle? (Yes)
• Assume this segment is the hypotenuse of a right triangle. What will the area of the square on the hypotenuse be? $[\sqrt{13}]^2$, or 13 square units
• What is the sum of the areas of the squares on the legs of this right triangle? (13)
• What are two square numbers whose sum is 13? (4 and 9) So, what are the lengths of the legs? $\sqrt{4}$ units and $\sqrt{9}$ units, or 2 units and 3 units

Students should draw a right triangle with legs of length 2 units and 3 units. The hypotenuse has length $\sqrt{13}$ units.
Summarize 8.3

Ask students to demonstrate and explain how they found the answers to Questions A–C. Then, go over Question D carefully. After someone has explained how he or she found two points that were $\sqrt{13}$ units apart, offer a similar problem.

- How would you find a line segment with a length of $\sqrt{40}$ units?

Ask one or two students to describe their method. They will likely use a guess-and-check procedure to find the two square numbers with a sum of 40, which are 36 and 4. From this they can determine that leg lengths 6 units and 2 units will give a right triangle with a hypotenuse of length $\sqrt{40}$ units. Students should verify their results: $2^2 + 6^2 = 40$, so $\sqrt{40}$ is the length. You can challenge students to find a few more lengths in this way, such as $\sqrt{50}$ units, $\sqrt{61}$ units, and $\sqrt{72}$ units.

If you want your students to have more practice with this idea, you could have them work on ACE Exercises 27–33, either as a final summary activity or as homework after this problem.

Students should be able to focus on the areas of the three squares on the sides of a right triangle and their relationship to the lengths of the sides.

Typically, two lengths or two areas are known, and we must find the third length or area. Once we know the missing area, we can take its square root to find the length. Conversely, once we know the missing length, we can square it to find the area.

The following visual explanation will help some students understand the essence of the Pythagorean Theorem:

The essential strategy for finding a tilted line with a certain length depends on finding two squares whose sum is equal to the square of that length. In Exercise 27, students create a table of sums of square numbers. This table will help them find the two upright squares whose areas add to the square of the given length. They can use this information to draw a right triangle with the given length as the hypotenuse. As a final check, ask this question:

- Can 7 be the length of a tilted line segment drawn between two dots on a dot grid? (No, because 49 does not equal the sum of two square numbers.)
3.3 Finding Distances

Mathematical Goals

- Use the Pythagorean Theorem to find the distance between two points on a grid
- Relate areas of squares to the lengths of the sides

Launch

Display Transparency 3.3, or a transparent grid, and indicate or label points K and L as shown in the Student Edition.

- How can you find the distance between these two points?
- Draw line segment KL and ask:
  - How can we use the Pythagorean Theorem to find the length of this line segment? What right triangle has this hypotenuse?
- Sketch students’ suggestions.
  - What are the lengths of the legs? How can you use this information to find the length of the hypotenuse? So, what is the distance between points K and L?

Distribute Labsheet 3.3 to each student and have the class work in pairs on the rest of the problem.

Explore

Students should find the problem a review of what they have learned so far. However, Question D is a bit difficult, so you may need to help guide their thinking.

- Can the $\sqrt{13}$-unit line segment be a vertical or a horizontal segment?
- Assume this segment is the hypotenuse of a right triangle. What will the area of the square on the hypotenuse be?
- What is the sum of the areas of the squares on the legs of this right triangle?
- What are two square numbers whose sum is 13? So, what are the lengths of the legs?

Students should draw a right triangle with legs of lengths 2 units and 3 units. The hypotenuse has a length of $\sqrt{13}$ units.

Summarize

Ask students to demonstrate and explain how they found the answers to Questions A–C. Then, go over Question D carefully. Offer a similar problem.

- How would you find a line segment with a length of $\sqrt{40}$?
Summarize continued

Ask one or two students to describe their method. If you want your students to have more practice with this idea, you could have them work on ACE Exercises 27–33, either as a final summary activity or as homework after this problem.

Students should be able to focus on the areas of the three squares on the sides of a right triangle and their relationship to the lengths of the sides.

ACE Assignment Guide for Problem 3.3

Core 24
Other Extensions 27–35; unassigned choices from earlier problems
Adapted For suggestions about adapting ACE exercises, see the CMP Special Needs Handbook.

Answers to Problem 3.3

A. 1. Two right triangles are possible. In the diagram below, they are $KLW$ and $KLV$.

2. 2 and 5

3. $2^2 + 5^2 = 29$, so the length of $KL$ is $\sqrt{29}$ units, or about 5.39 units.

B. 5 units. Draw a right triangle with hypotenuse $MN$. Because $3^2 + 4^2 = 25$, the hypotenuse has a length of 5 units.

C. $\sqrt{45}$ units, $3\sqrt{5}$ units, or about 6.71 units. Draw a right triangle with hypotenuse $PQ$. Since $3^2 + 6^2 = 45$, the hypotenuse has a length of $\sqrt{45}$ units. This can also be written as $3\sqrt{5}$.

D. Because $(\sqrt{13})^2 = 2^2 + 3^2$, the hypotenuse of a right triangle with legs of lengths 2 units and 3 units will have a length of $\sqrt{13}$ units.
**Goals**

- Determine whether a triangle is a right triangle based on the lengths of its sides
- Relate areas of squares to the lengths of the sides

In this problem, students investigate the converse of the Pythagorean Theorem: If \( a, b, \) and \( c \) are the lengths of the sides of a triangle and \( a^2 + b^2 = c^2 \), then the triangle is a right triangle.

**Launch 3.4**

Discuss the two questions in the introduction to Problem 3.4. Remind students that, so far, they have learned that *if* a triangle is a right triangle, *then* its side lengths satisfy the relationship \( a^2 + b^2 = c^2 \). However, they do not yet know whether a triangle whose side lengths satisfy this relationship must be a right triangle.

Have students work on the activity in the Getting Ready in pairs. Each pair will need a string, a marker, a ruler, and some tape. The activity is easiest if the strings are cut to lengths that can easily be divided into 12 equal intervals (for example, 48 cm). (If students miscalculate and have string left over, they can just cut off the excess.) Emphasize that students should tape the ends of the string together so there is no overlap.

**Suggested Questions** After most students have successfully formed a right triangle with whole-number side lengths, discuss the questions.

- **What are the side lengths of the right triangle you formed?** (3 units, 4 units, and 5 units)
- **Do the side lengths satisfy the relationship \( a^2 + b^2 = c^2 \)?** (Yes)
- **How do you think the Egyptians used the knotted rope?** (Possible answer: They formed a triangle with side lengths 3 units, 4 units, and 5 units. This triangle is a right triangle. They used the right angle of the triangle to mark the corners of the rectangular plots.)

Distribute straws, string, or polystrips, and have the class work in pairs on the problem. (Note: Students used polystrips in the grade 6 unit Shapes and Designs to explore the triangle inequality and to investigate the rigidity of triangles and quadrilaterals with fixed side lengths. You may be able to borrow them from a sixth-grade teacher. They are a very useful tool for this problem.)

**Explore 3.4**

If necessary, help students form one of the triangles in Question A so they know what to do.

If you have students who need more practice checking whether three side lengths form a right triangle, you might make up a few examples for them.

**Challenge**

Ask students to think about the multiples of side lengths of 3-4-5 and 5-12-13, such as 6-8-10 and 10-24-26.

**Suggested Questions** Ask:

- **Do triangles whose sides have these lengths form a right triangle as well?** (Yes)
- **How do you know?** (Because \( 6^2 + 8^2 = 10^2 \) and \( 10^2 + 24^2 = 26^2 \); because these enlarged triangles are similar to the original triangles, so they have the same angle measure.)

You could also challenge some students to find different sets of whole-number side lengths that make a right triangle. Ask them to explain why.

**Summarize 3.4**

Have someone demonstrate at the overhead how to arrange the string, straws, or polystrips to form a triangle with side lengths 3 units, 4 units, and 5 units. Ask the student how he or she knows it is a right triangle. Explain that this triangle is sometimes called a “3-4-5 right triangle.” Other right triangles are referred to in a similar way.

**Suggested Questions** Ask:

- **Are multiples of a 3-4-5 triangle, such as 6-8-10 and 9-12-15 triangles, also right triangles?** (Yes, they are all similar triangles, so the measures of corresponding angles are equal. Students might use the language of scale factors or ratios of corresponding sides from their work with the Stretching and Shrinking unit to answer this question.)
Have students check these triangles.

- What about the multiples of 5-12-13? Do these lengths form a right triangle? (Yes, $10^2 + 24^2 = 26^2$, $15^2 + 36^2 = 39^2$, and so on.)

Tell students that sets of three numbers that satisfy the Pythagorean relationship are called Pythagorean triples. Other whole-number triples are 7-24-25 and 9-40-41.

Spend some time discussing the side lengths that did not form a right triangle.

- Which of these sets of side lengths did not form a right triangle? (5, 6, 10; 4, 4, 4; and 1, 2, 2)
- Does $a^2 + b^2 = c^2$ for these sets? (No)
- If the side lengths of a triangle satisfy the condition $a^2 + b^2 = c^2$, is it a right triangle? (Yes)
- Can we rearrange the sides of a right triangle to form another triangle that is not a right triangle? (No; for three given side lengths, there is only one possible triangle. This idea was explored in the grade 6 unit Shapes and Design.)

You might want to review students’ understanding of the conditions for side lengths of a triangle called the triangle inequality, which was explained in Shapes and Designs.

- What about a triangle that has side lengths of 2 units, 6 units, 10 units? Is it a right triangle? (There is no triangle with these side lengths. These lengths do not satisfy the triangle inequality, which says that the sum of any two side lengths must be greater than the length of the third side length.)

Question B, part (3), asks students if their conjecture will always work. It works for the examples they have tried. Remind students that a few examples are not a proof. A proof for this theorem is given on page 7. You could try to demonstrate this proof or suggest that some students may want to think about a proof (or reasons) for homework.
3.4 Measuring the Egyptian Way

Mathematical Goals

- Determine whether a triangle is a right triangle based on its side lengths
- Relate areas of squares to the lengths of the sides

Launch

Discuss the two questions in the introduction to Problem 3.4. Remind students that, so far, they have learned that if a triangle is a right triangle, then its side lengths satisfy the relationship $a^2 + b^2 = c^2$. However, they do not yet know whether a triangle whose side lengths satisfy this relationship must be a right triangle.

Have students work on the activity in the Getting Ready in pairs, or do the activity as a demonstration.

Distribute rulers and straws, string, or polystrips, and have the class work in pairs on the problem.

Explore

If necessary, help students form one of the triangles in Question A.

If you have students who need more practice checking whether three side lengths form a right triangle, you might make up a few examples for them.

Challenge some students to think about the multiples of side lengths of 3-4-5 and 5-12-13, such as 6-8-10 and 10-24-26.

- Do triangles whose sides have these lengths form a right triangle as well? How do you know?

You could also challenge some students to find different sets of whole-number side lengths that make a right triangle.

Summarize

Have someone demonstrate how to arrange the string, straws, or polystrips to form a triangle with side lengths 3 units, 4 units, and 5 units and to explain how he or she knows it is a right triangle. Explain that this triangle is sometimes called a “3-4-5 right triangle.”

- Are multiples of a 3-4-5 triangle, such as 6-8-10 and 9-12-15 triangles, also right triangles?

Have students demonstrate each set of lengths on a grid at the overhead, checking for right angles with an angle ruler or a corner of a piece of paper.

- What about the multiples of 5-12-13? Do these lengths form a right triangle?

Also, discuss the side lengths that did not form a right triangle.

- Which of these sets of side lengths did not form a right triangle? Does $a^2 + b^2 = c^2$ for these sets?
ACE Assignment Guide for Problem 3.4

Core 15–17
Other Connections 25; unassigned choices from earlier problems
Adapted For suggestions about adapting ACE exercises, see the CMP Special Needs Handbook.
Connecting to Prior Units 25: Filling and Wrapping

Answers to Problem 3.4
A. | Side Lengths (units) | Do the side lengths satisfy \(a^2 + b^2 = c^2\) | Is the triangle a right triangle? |
---|---|---|---|
3, 4, 5 | yes | yes |
5, 12, 13 | yes | yes |
5, 6, 10 | no | no |
6, 8, 10 | yes | yes |
4, 4, 4 | no | no |
1, 2, 2 | no | no |

B. 1. If a triangle’s side lengths satisfy the relationship \(a^2 + b^2 = c^2\), the triangle is a right triangle.
2. If a triangle’s side lengths do not satisfy the relationship \(a^2 + b^2 = c^2\), the triangle is not a right triangle.
3. Possible answers: The side lengths 1, 1, and 2 do not satisfy the relationship \(a^2 + b^2 = c^2\) and are not lengths of sides of a right triangle. Side lengths 15, 8, and 17 do satisfy \(a^2 + b^2 = c^2\) and are side lengths of a right triangle.

C. 1. Yes. \(12^2 + 16^2 = 20^2\)
2. Yes. \(8^2 + 15^2 = 17^2\)
3. No. \(12^2 + 9^2 \neq 16^2\)

D. M, N, Q, and R. The side lengths of these triangles satisfy the relationship \(a^2 + b^2 = c^2\).
Answers

Investigation 3

ACE Assignment Choices

Problem 3.1
Core 1, 2, 5, 6, 8–12
Other Applications 3, 4, 7, 13, 14

Problem 3.2
Core 23, 26
Other Connections 18–22; unassigned choices from earlier problems

Problem 3.3
Core 24
Other Applications 27–35; unassigned choices from earlier problems

Problem 3.4
Core 15–17
Other Connections 25; unassigned choices from earlier problems

Adapted For suggestions about adapting Exercises 8–11 and other ACE exercises, see the CMP Special Needs Handbook.

Connecting to Prior Units 18–22, 25, 26: Filling and Wrapping; 23: Accentuate the Negative

Applications

1. a. \(5^2 + 12^2 = 169\) in.\(^2\)
   b. 13 in.
2. \(c^2 = 3^2 + 6^2 = 45\), \(c = \sqrt{45}\) cm, or about 6.7 cm.
3. WX is the hypotenuse of a right triangle with legs of length 4 units and 1 unit. Because \(4^2 + 1^2 = 17\), the length of segment WX is \(\sqrt{17}\) units. Therefore, W and X are \(\sqrt{17}\) units apart.

4. YZ is the hypotenuse of a right triangle with legs of length 4 units and 2 unit. Because \(4^2 + 2^2 = 20\), the length of segment YZ is \(\sqrt{20}\) units. Therefore, the distance between Y and Z is \(\sqrt{20}\) units.

Note: There are many triangles with a hypotenuse length of \(\sqrt{20}\) units (for example, one with legs 3 and \(\sqrt{11}\)). However, in this case, we want to use integer lengths so we can draw the triangle on dot paper.

5. \(h^2 = 4^2 + 3^2 = 25\), so \(h = \sqrt{25}\) in. = 5 in.
6. \(k^2 = 3^2 + 8^2 = 73\), so \(k = \sqrt{73}\) cm \(\approx 8.5\) cm.
7. \(x^2 = 7^2 - 4^2 = 33\), so \(x = \sqrt{33}\) m \(\approx 5.7\) m.
   \(y^2 = 21^2 - 4^2 = 425\), so \(y = \sqrt{425}\) m \(\approx 20.6\) m.
8. Because \(4^2 + 3^2 = 25\), the distance is 5 blocks.
9. Because \(6^2 + 5^2 = 61\), the distance is \(\sqrt{61}\) blocks \(\approx 7.8\) blocks.
10. The distance is 4 blocks.
11. Because \(4^2 + 4^2 = 32\), the distance is \(\sqrt{32}\) \(\approx 5.7\) blocks.
12. D
13. a. 2 units, 2 units, 4 units
   b. The side lengths are \(\sqrt{2}\) units, \(\sqrt{2}\) units, and 2 units, and \((\sqrt{2})^2 + (\sqrt{2})^2 = 2^2\) (that is, \(2 + 2 = 4\)), so the side lengths satisfy the Pythagorean Theorem.
14. The sides have lengths \(\sqrt{5}\) units, \(\sqrt{5}\) units, and \(\sqrt{10}\) units and, because \((\sqrt{5})^2 + (\sqrt{5})^2 = (\sqrt{10})^2\) (that is, \(5 + 5 = 10\)), the triangle satisfies the Pythagorean Theorem.
Note: This is a nice place to remind students that \( \sqrt{5} + \sqrt{5} \neq \sqrt{10} \), even though \((\sqrt{5})^2 + (\sqrt{5})^2 = (\sqrt{10})^2\). They can use the diagram to show \( \sqrt{5} + \sqrt{5} > \sqrt{10} \) or they can use estimation.

15. F

16. This is a right triangle. \( 10^2 + 10^2 = (\sqrt{200})^2 \)

17. This is not a right triangle. \( 9^2 + 16^2 \neq 25^2 \).

For the Teacher  In fact, these side lengths will not form a triangle of any kind. As in Exercise 16, watch for students who incorrectly write that \( \sqrt{9} + \sqrt{16} = \sqrt{25} \).

Connections

18. a. 6.5 cm

b. You do not need to know the value of \( a \) to find the volume, but it is needed to find the surface area. To find the volume, you multiply 4 by the area of the triangular face, which you can find using only the given base and height. To find the surface area, you need to find the areas of the rectangular faces. For one of these faces, you need to know the value of \( a \).

c. 30 cm\(^3\): \( 0.5(6 \cdot 2.5) \cdot 4 = 30 \)

d. 75 cm\(^2\): \( (2.5 \cdot 4) + 2[0.5(6 \cdot 2.5)] + (6 \cdot 4) + (6.5 \cdot 4) = 10 + 15 + 24 + 26 = 75 \)

e. Possible sketch:

23. a. 4 blocks

b. \( \sqrt{10} \) blocks. Find the length of the segment connecting the points. It is the hypotenuse of a right triangle with leg lengths 1 and 3. The leg lengths are the vertical and horizontal distances between the two points \([ (5 - 2) \text{ units and } -3 - (-4) \text{ units} ]\)

\[ a^2 + b^2 = c^2 \]

\[ 3^2 + 1^2 = 10, \text{ so the distance is } \sqrt{10} \text{ blocks.} \]

24. Points A and B are 5 units apart. Point F is also 5 units from point A.

25. a. Using the Pythagorean Theorem,

\[ 2^2 + h^2 = 29, \text{ so the height } h \text{ of the cone is } 5 \text{ units.} \]

b. The volume of the cylinder is \( \pi(2)^2(5) = 20\pi \text{ units}^3 \). So the volume of the cone is \( \frac{20\pi}{3} \text{ units}^3 \), or about 20.94 units\(^3\).

26. a. 72 cubic units. The volume of the cube is \( 6 \cdot 6 \cdot 6 = 216 \text{ units}^3 \). The volume of the pyramid is \( \frac{1}{3} \) of the cube's volume, or 72 units\(^3\).

b. \( \frac{1}{3}x^3 \). The cube has volume \( x^3 \). The volume of this pyramid is one-third the volume of the cube, so it is \( \frac{1}{3}x^3 \).
**Extensions**

27. a. (Figure 2)  
   b. i. 1 and 9  
   c. 9 and 16  
   d. 25 and 64  
   e. $1 + 25 = 26$, so a triangle with leg lengths of 1 unit and 5 units has a hypotenuse of length $\sqrt{26}$ units.  
   f. $36 + 64 = 100$, so a triangle with leg lengths of 6 units and 8 units has a hypotenuse of length 10 units.

28. Yes. $\sqrt{2}$ units is the length of the hypotenuse of a right triangle with leg lengths of 1 unit.

29. No. 3 is not the sum of two square numbers.

30. Yes. $\sqrt{4} = 2$, so just draw a horizontal or vertical segment with length 2 units.

31. Yes. $\sqrt{5}$ units is the length of the hypotenuse of a right triangle with leg lengths of 2 units and 1 unit.

32. No. 6 is not the sum of two square numbers.

33. No. 7 is not the sum of two square numbers.

34. a. Possible answer: Draw a right triangle as shown below, and use the Pythagorean Theorem to find the hypotenuse, which is the radius.

   **Figure 2**

<table>
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</tbody>
</table>

   b. 5 units
35. a. $J(1, 1); K(4, 7)$

b. About 6.7 units. You can draw a right triangle with hypotenuse $JK$. The length of one leg is the positive difference of the $x$-coordinates, which is $4 - 1$, or 3. The length of the other leg is the positive difference of the $y$-coordinates, which is $7 - 1 = 6$. So the length of $JK$ is $\sqrt{9 + 36} = \sqrt{45} \approx 6.7$ units.

Note: In high school, students will see the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, or $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$. The distance formula follows directly from the Pythagorean Theorem. If you use the segment between two points as the hypotenuse of a right triangle, the length of the horizontal leg will be $|x_2 - x_1|$ and the length of the vertical side will be $|y_2 - y_1|$, so the distance between the points, which is the length of the hypotenuse, is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

c. 2.8 units. $\sqrt{(7 - 5)^2 + (10 - 8)^2} = \sqrt{4 + 4} = \sqrt{8} \approx 2.8$

Note: You can give additional extension problems to interested students. For example, you might ask students to find the length of a diagonal of a square with side length $a$. Or, you could ask them to draw a square of side $a$ inscribed in a circle and then to find the radius and area of the circle in terms of $a$.

Possible Answers to Mathematical Reflections

1. If we know the lengths of the legs, the length of the hypotenuse can be found by taking the square root of the sum of the squares of the leg lengths. If we know the lengths of one leg and the hypotenuse, we can find the length of the other leg by subtracting the square of the given leg length from the square of the hypotenuse length; this is the square of the missing leg length. Take the square root of that difference to get the missing leg length.

2. Think of the segment between the two points as the hypotenuse of a right triangle. Find the lengths of the legs of the right triangle (which lie on a vertical line and a horizontal line). Apply the Pythagorean Theorem by adding the squares of these two lengths and taking the square root of that sum.

3. Check whether the side lengths satisfy the relationship $a^2 + b^2 = c^2$, where $a$ and $b$ are the lengths of the shorter sides, and $c$ is the length of the longest side. If they do, then the triangle is a right triangle.
Investigation 4

Using the Pythagorean Theorem

Mathematical and Problem-Solving Goals

• Learn the meanings of rational number and irrational number
• Estimate the values of square roots that are irrational numbers
• Estimate lengths of hypotenuses of right triangles
• Apply the Pythagorean Theorem to a problem situation
• Investigate the special properties of equilateral and 30-60-90 triangles
• Use the properties of special right triangles to solve problems

Summary of Problems

Problem 4.1 Analyzing the Wheel of Theodorus

Students apply the Pythagorean Theorem to find the exact lengths of hypotenuses of right triangles. Then, they use a number-line ruler to estimate the lengths. Finally, they compare their ruler estimates to those made with a calculator.

Problem 4.2 Stopping Sneaky Sally

Students apply the Pythagorean Theorem to find distances on a baseball diamond.

Problem 4.3 Analyzing Triangles

Students investigate properties of equilateral and 30-60-90 triangles by applying the Pythagorean Theorem.

Problem 4.4 Finding the Perimeter

Students draw from their experiences in the previous three problems to find missing lengths and angles in a triangle made up of other triangles.

<table>
<thead>
<tr>
<th>Suggested Pacing</th>
<th>Materials for Students</th>
<th>Materials for Teachers</th>
<th>ACE Assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>All 5 days</td>
<td>Centimeter rulers, student notebooks</td>
<td>Transparencies 4.1A and 4.1B</td>
<td>1, 2, 13–16</td>
</tr>
<tr>
<td>4.1 1 day</td>
<td>Labsheet 4.1, scissors</td>
<td>Transparency 4.2</td>
<td>3–9, 17–25, 36–46</td>
</tr>
<tr>
<td>4.2 1 day</td>
<td></td>
<td>Transparencies 4.3A and 4.3B</td>
<td>10, 11, 26–34, 47–52</td>
</tr>
<tr>
<td>4.3 1½ days</td>
<td></td>
<td>Transparency 4.4</td>
<td>12, 35, 53–58</td>
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<tr>
<td>4.4 1 day</td>
<td>Labsheet 4.4, scissors</td>
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Goals

- Learn the meanings of **rational number** and **irrational number**
- Estimate the values of square roots that are irrational numbers
- Estimate lengths of hypotenuses of right triangles

In this problem, students explore an intriguing pattern of triangles called the *Wheel of Theodorus*. In this series of right triangles, the hypotenuse of one triangle is the longer leg of the next triangle. Students apply the Pythagorean Theorem to find the length of the hypotenuse of each triangle in the wheel. Then, they estimate the hypotenuse lengths with both a number-line ruler and a calculator.

Launch 4.1

Introduce the problem by discussing how to find a decimal approximation for a square root.

- **Think back to when we found the side lengths of tilted squares.**
- The side length of a square with an area of 2 square units is \( \sqrt{2} \) units, the positive number you can multiply by itself to get 2. We estimated this value first by measuring and then by using the \( \sqrt{ } \) key on a calculator.
- **Just how large is \( \sqrt{2} \)? Can you find a decimal number that is equal to \( \sqrt{2} \)? Where is \( \sqrt{2} \) on the number line?**

Draw a simple number line on the board and ask students where \( \sqrt{2} \) should be placed.

To answer this, students will have to consider where 1.4 and 1.5 are on the number line. Encourage them to briefly discuss the difficulty of placing a number on a number line when the number cannot be written as an exact decimal. Specifically, you know what decimals it is between, but not exactly where it is in that interval. For example, is it closer to 1.4 or 1.5?

Explore 4.1

Ask that each student label his or her own number-line ruler. As students work, check on their understanding of measuring lengths and writing decimals.

On a dot grid, draw a square with an area of 2 square units on a number line as shown below.

Suggested Questions Ask:

- **What is the length of a side of this square? (\( \sqrt{2} \))**
- **If we mark off a segment on the number line with the same length as the side, where will the segment end?** (At about 1.4)

Mark off the length of the segment on the edge of a sheet of paper and transfer it to the number line.

- **So, \( \sqrt{2} \) is approximately equal to 1.4. Is 1.4 exactly equal to \( \sqrt{2} \)?** (No, because \( 1.4^2 = 1.96 \).)
- **Suppose we try 1.41. Does 1.41 = \( \sqrt{2} \)?** (No, it is too small; \( 1.41^2 = 1.9881 \).)
- **Try 1.42. Does it equal \( \sqrt{2} \)?** (No, it is too large: \( 1.42^2 = 2.0164 \).)
- **Can you find a number that is closer to \( \sqrt{2} \) than 1.41 and 1.42 are?**

Students should try numbers between 1.41 and 1.42, such as 1.415, 1.413, and 1.414.

Display the Wheel of Theodorus, which is on Transparency 4.1A. Discuss with the class how the wheel was constructed and ask for the lengths of the second and third hypotenuses. Cut out the number-line ruler and demonstrate how to transfer these lengths to the ruler.

Distribute Labsheet 4.1 and scissors to each student. Have students work in groups of two to four on the problem.
Summarize 4.1

Display the Wheel of Theodorus. Ask for the lengths of the hypotenuses and write them on the wheel. Then, have students come to the front and mark the length of each hypotenuse on the number-line ruler.

Ask for approximations to the nearest tenth for each length. As a class, check each approximation by squaring it on a calculator.

Suggested Question Ask:

* Is this estimate too large? Is it too small? What might be a better estimate? How do you know?

Students should square each decimal estimate and compare the result to the square of the length of the hypotenuse. Take this opportunity to assess students’ understanding of the ordering of decimals. Students sometimes need to review and practice comparing such numbers as 1.41, 1.415, and 1.42.

Ask students to compare their ruler estimates to the estimates they obtained with a calculator. Calculators display varying numbers of decimal places, but students are usually convinced that, no matter what decimal number their calculators display for \( \sqrt{2} \), they have not found an exact decimal equivalent. (Note: On many calculators, if the approximation is not cleared before it is squared, the calculator will display the original square as the answer.)

Discuss the Did You Know? that appears after the problem. Tell students that numbers like \( \sqrt{2} \), \( \sqrt{7} \), and \( \pi \) are called irrational numbers. They cannot be written as the ratio of two whole numbers. The set of irrational and rational numbers is called the real numbers.

Mathematics Background

For background on real numbers, see page 9.
4.1 **Analyzing the Wheel of Theodorus**

**Mathematical Goals**

- Learn the meanings of *rational number* and *irrational number*
- Estimate the values of square roots that are irrational numbers
- Estimate lengths of hypotenuses of right triangles

**Launch**

Introduce the problem by discussing how to find a decimal approximation for a square root.

On a dot grid, draw a square with an area of 2 square units on a number line, with the “bottom vertex” at point 0.

- **What is the length of a side of this square? If we mark off a segment on the number line with the same length as the side, where will the segment end?**
- **So, \( \sqrt{2} \) is approximately equal to 1.4. Is 1.4 exactly equal to \( \sqrt{2} \)?**
  - Suppose we try 1.41. Does 1.41 = \( \sqrt{2} \)? Try 1.42. Does it equal \( \sqrt{2} \)?
  - Can you find a number that is closer to \( \sqrt{2} \) than 1.41 and 1.42 are?

Display the Wheel of Theodorus. Explore with the class how the wheel was constructed and ask for the lengths of the second and third hypotenuses. Cut out the number-line ruler and demonstrate how to transfer these lengths to the ruler.

Distribute Labsheet 4.1 and scissors to each student and have students work in groups of two to four on the problem.

**Explore**

Ask that each student label his or her own number-line ruler. Check on students’ understanding of measuring lengths and writing decimals.

**Summarize**

Display the Wheel of Theodorus. Ask for the lengths of the hypotenuses and write them on the wheel. Then, have students come to the front and mark the length of each hypotenuse on the number-line ruler.

Ask for approximations to the nearest tenth for each length. As a class, check each approximation by squaring it on a calculator.

- **Is this estimate too large? Too small? What might be a better estimate? How do you know?**

Take this opportunity to assess students’ understanding of the ordering of decimals.

Ask students to compare their estimates to the numbers they obtained with a calculator. Tell the class that the numbers \( \sqrt{2}, \sqrt{3}, \sqrt{5}, \ldots \) are called irrational numbers.

**Materials**

- Transparency 4.1A
- Labsheet 4.1
- Scissors

**Vocabulary**

- irrational number
ACE Assignment Guide
for Problem 4.1

Core 1, 2
Other Connections 13–16
Adapted For suggestions about adapting ACE exercises, see the CMP Special Needs Handbook.

Answers to Problem 4.1

A. The lengths of the hypotenuses (in units), from least to greatest, are \(\sqrt{2}, \sqrt{3}, 2\) (or \(\sqrt{4}\), \(\sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, 3\) (or \(\sqrt{9}\)), \(\sqrt{10}, \sqrt{11},\) and \(\sqrt{12}\).

B. (Figure 1) Note: \(\sqrt{4} = 2\) and \(\sqrt{9} = 3\).

C. 1. \(\sqrt{2}\) and \(\sqrt{3}\) are between 1 and 2; \(\sqrt{5}, \sqrt{6}\), \(\sqrt{7}\), and \(\sqrt{8}\) are between 2 and 3; and \(\sqrt{10}, \sqrt{11}\), and \(\sqrt{12}\) are between 3 and 4.

2. \(\sqrt{2}\) is between 1.4 and 1.5; \(\sqrt{3}\) is between 1.7 and 1.8; \(\sqrt{5}\) is between 2.2 and 2.3; \(\sqrt{6}\) is between 2.4 and 2.5; \(\sqrt{7}\) is between 2.6 and 2.7; \(\sqrt{8}\) is between 2.8 and 2.9; \(\sqrt{10}\) is between 3.1 and 3.2; \(\sqrt{11}\) is between 3.3 and 3.4; \(\sqrt{12}\) is between 3.4 and 3.5.

3. \(\sqrt{2} \approx 1.414213562, \sqrt{3} \approx 1.732050808, \sqrt{4} \approx 2, \sqrt{5} \approx 2.236067978, \sqrt{6} \approx 2.449489743, \sqrt{7} \approx 2.645751311, \sqrt{8} \approx 2.828427125, \sqrt{10} \approx 3.16227766, \sqrt{11} \approx 3.31662479, \sqrt{12} \approx 3.464101615.

The numbers obtained using the ruler and the calculator are both approximations, but the calculator gives greater accuracy.

D. Both students have a valid point. Odakota’s number is an estimate accurate to nine decimal places (although most calculators store 13 digits), while Geeta is correct in pointing out that the square of this decimal approximation does not equal exactly 3. (Note: It would be impossible to write all the decimal places in the decimal expansion for \(\sqrt{3}\).)
4.2 Stopping Sneaky Sally

Goals

• Estimate lengths of hypotenuses of right triangles
• Apply the Pythagorean Theorem to a problem situation

In this problem, students apply the Pythagorean Theorem to determine distances on a baseball diamond.

Launch 4.2

Introduce the baseball scenario described in the student edition. Talk about the layout of a baseball diamond, which is pictured on Transparency 4.2. The baseball diamond is a square.

Suggested Questions

Ask:

• Does anyone know the distance between bases on a standard baseball field? (90 ft)

• How far do you think a catcher would need to throw the ball to get a runner out at second base?

Let students offer a few estimates, and then have them work in pairs on the problem.

Explore 4.2

Suggested Questions

Some students may need help in recognizing the right triangles that are the key to solving the problem.

• Suppose you draw a line segment from home plate to second base. What is special about the line segment? (It is the hypotenuse of a right triangle whose legs are the segments from home plate to first base and from first base to second base.)

Summarize 4.2

Have several students share their strategies for solving the problem. Look for specific references to the Pythagorean Theorem.

There are a couple of common misconceptions that may arise during this discussion. First, students may add the lengths of the legs and then square the sum to find the square of the hypotenuse. If this happens, you may need to demonstrate with actual numbers that $(a + b)^2 \neq a^2 + b^2$:

\[
(90 + 90)^2 \neq 90^2 + 90^2 \\
180^2 \neq 90^2 + 90^2 \\
32,400 \neq 8,100 + 8,100 \\
32,400 \neq 16,200
\]
A second misconception involves taking square roots: some students will try to find the length of the hypotenuse by calculating $\sqrt{a^2} + \sqrt{b^2}$ rather than $\sqrt{a^2 + b^2}$. Again, offer numerical examples to help students understand that these expressions are not equivalent. Stress the correct procedure:

Students should square each leg length first, add the squares, and then take the square root of the sum. For some students, the symbolic expression, $\sqrt{a^2 + b^2}$, will be an aid to memory. For some, it may be confusing.
Mathematical Goals

• Estimate lengths of hypotenuses of right triangles
• Apply the Pythagorean Theorem to a problem situation

Launch

Introduce the baseball scenario described in the Student Edition.
Talk about the layout of a baseball diamond, which is pictured on
Transparency 4.2. The baseball diamond is a square.

• Does anyone know the distance between bases on a standard baseball
  field?
• How far do you think a catcher would need to throw the ball to get a
  runner out at second base?

Let students offer a few estimates, and then have them work in pairs on
the problem.

Explore

Some students may need help in recognizing the right triangles that are the
key to solving the problem.

• Suppose you draw a line segment from home plate to second base. What
  is special about the line segment?
• What do you know about the side lengths of this right triangle? How can
  you find the length of the hypotenuse?

Repeat these questions, if necessary, for Question B.

Summarize

Have several students share their strategies for solving the problem. Look
for specific references to the Pythagorean Theorem.

Stress the correct procedure: Square each leg length first, add the
squares, and then take the square root of the sum to get the length of the
hypotenuse.
**ACE Assignment Guide for Problem 4.2**

**Core** 3–5, 24, 25  
**Other Applications** 6–9; **Connections** 17–23, 36, 37; **Extensions** 38–46; unassigned choices from earlier problems  
**Adapted** For suggestions about adapting Exercise 8 and other ACE exercises, see the CMP Special Needs Handbook.  
**Connecting to Prior Units** 17–18: Moving Straight Ahead

**Answers to Problem 4.2**

**A.** Because $90^2 + 90^2 = 16,200$, the distance from home plate to second base is $\sqrt{16,200}$ ft, or about 127.28 ft.

**B.** The shortstop is standing on the baseline at a distance of $90 + 45 = 135$ ft from third base. Because $90^2 + 45^2 = 10,125$, the distance from home plate to the shortstop is $\sqrt{10,125}$ ft, or about 100.62 ft.

**C.** The pitcher's mound is not exactly halfway between home plate and second base. The distance from the pitcher's mound to second base is $127.28 - 60.5 = 66.78$ ft.

To find the distance to first base, you need to find the halfway point between home and second base, which is about $127.28 \div 2$, or about 63.64 ft from home plate. Then, draw a right triangle with vertices at the halfway point, the pitcher’s mound, and first base.

The lengths of the legs are 63.64 ft and 3.14 ft. (3.14 ft is the distance between the pitcher’s mound and the halfway point between home plate and second base. 63.64 ft is half of the distance between first and third bases, which is the same as the distance between home plate and second base.)

Use the Pythagorean Theorem to find the distance between the pitcher’s mound and first base:

$$\sqrt{(3.14)^2 + (63.64)^2} \approx 63.72$$ ft. The distance between the pitcher’s mound and third base is also about 63.72 ft.
**Goal**

- Investigate the special properties of a 30-60-90 triangle

**Launch 4.3**

Show a transparency of the Getting Ready for Problem 4.3.

**Suggested Questions** Ask:

- This is an equilateral triangle. What is true about the lengths of the sides of an equilateral triangle? (They are all equal.)

- What is true about the sum of the angles in any triangle? (It is 180°.)

- What is true about the measures of the angles of an equilateral triangle? (They are all equal.)

- What is the measure of each angle in an equilateral triangle? (The sum of the angles in any triangle is 180°, so each angle must measure 60°.)

Tell the class that AP is a reflection line of symmetry.

- What is a reflection line of symmetry? (It is a line that divides a triangle into two identical shapes.)

Some students may need to be reminded about reflection line symmetries. Cut out a copy of triangle ABC and fold it along the reflection line. Ask the students what they observe about the two shapes (smaller triangles) that are created. Students should discover that line segment AP divides triangle ABC into two congruent triangles. You may want to remind students that triangles are congruent if each pair of corresponding sides has the same length. More informally, in one triangle fits on another triangle exactly, or if two triangles have the same size and shape, they are congruent.

- What can you say about the measures of angles CAP, BAP, CPA, and BPA? (Angles CAP and BAP are equal. So each has a measure of 30°. Angles CPA and BPA are also equal. Since the two angles form a straight angle and they are equal, they must each be 90°.)

- What can you say about line segments CP and PB? (These segments have equal lengths or each of them is half of the length of a side of the equilateral triangle.)

- What can you say about triangles ACP and ABP? (The triangles are congruent or have identical shapes. Each is a right triangle.)

Label the angles of the triangle as 30, 60, and 90 degrees. Students can work in pairs on the problem.

- We have just explored some interesting relationships in an equilateral triangle that occur when a line of reflection is drawn. In Problem 4.3, you will continue to explore these relationships about angles and side lengths.

**Explore 4.3**

If students are having trouble, ask questions to help them see that two right triangles were formed by the line of symmetry. Then, ask what else they know about these right triangles.
Make sure students are determining the side lengths by using the Pythagorean Theorem, not by measuring.

**Summarize 4.3**

Let several pairs share their reasoning about each question, demonstrating their work at the board or overhead.

For Question A, students should be able to reason that both triangles have angles of measure $30^\circ$, $60^\circ$, and $90^\circ$. The reflection line (also called the median or midpoint line) forms two congruent angles along the base of the original equilateral triangle. As the sum of the angles along a straight line is $180^\circ$, the two congruent angles both measure $90^\circ$. In each triangle, the larger acute angle measures $60^\circ$, so the smaller acute angle measures $30^\circ$.

**Suggested Questions** Students should also discover that the length of the side opposite the $30^\circ$ angle is half the length of the hypotenuse. If not, ask:

- *What is the length of segment CP?* (Since it is half the length of segment BC, it has a length of 2.)
- *What is the length of the hypotenuse of right triangle ACP?* (4)
- *What is the relationship between the side opposite the $30^\circ$ angle and the hypotenuse?* (The side opposite the $30^\circ$ angle is half the length of the hypotenuse.)

In Question B, for an equilateral triangle with side lengths $s$, all students should be able to find the length of the third side of the right triangle (or the reflection line in this example) using the Pythagorean Theorem. Many students will struggle to see that the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg (see answers for calculations). Depending on time, interest, and your students’ sophistication with these ideas, you can help them to see this.

There are two common approaches, which refer back to ideas from the seventh-grade unit *Stretching and Shrinking*.

**Approach 1: Ratios** Have students compare the decimal approximation for the longer leg to the length of the shorter leg using ratios. In each 30-60-90 triangle, the result will be about 1.732, which is approximately the square root of 3. The conclusion is that the longer leg is 1.732 times (or the square root of 3 times) the length of the shorter leg.

**Approach 2: Scale factor** Have students find the scale factor from the original 30-60-90 triangle, whose hypotenuse is 1 unit, to each of the others in this problem. For instance, the scale factor from the original triangle to the 30-60-90 triangle whose hypotenuse is 4. Therefore, the length of the longer leg in the 30-60-90 triangle with hypotenuse of 4 units is $2 \cdot \sqrt{3}$. For a general 30-60-90 right triangle with hypotenuse of $s$ units, the legs of the triangle are $\frac{s}{2}$ and $\frac{s}{2} \cdot \sqrt{3}$.

**Suggested Questions** Ask:

- *Suppose you had started with a larger equilateral triangle. Would your rule have been different? What if you had started with a smaller equilateral triangle?* (If students are still having difficulties, give them another equilateral triangle with side lengths of 5 or 6 units to try.)
- *Would your rule be true of any 30-60-90 triangle?*

You may need to cut out several 30-60-90 triangles to demonstrate that two copies can always be placed back to back to make an equilateral triangle. This is an opportunity to review the properties of similar triangles. Students may need to review that all 60-60-60 triangles are equilateral and are similar. In similar triangles, the ratios of the lengths of corresponding sides are equal. So, in a 30-60-90 triangle, the ratio of the length of the side opposite the $30^\circ$ angle to the length of the hypotenuse is also 1 to 2, or $\frac{1}{2}$. If necessary, use other lengths for the sides of the equilateral triangle so students can see that the relationship among the sides remains the same.

Question C of the problems reviews the relationship in a 30-60-90 triangle.

**Check for Understanding**

As a final summary, you might have students look for the same kinds of relationships in the triangles formed by drawing one diagonal in a square.

**Mathematics Background**

For background on 45-45 and 30-60 right triangles, see pages 7–8.
**4.3 Analyzing Triangles**

**Mathematical Goal**
- Investigate the special properties of a 30-60-90 triangle

**Launch**

Show a transparency of the Getting Ready for Problem 4.3. Tell the class that triangle ABC is an equilateral triangle and discuss reflection line of symmetry.

- What is true about the lengths of the sides of an equilateral triangle?
- What is true about the sum and measures of the angles of an equilateral triangle?

Students should discover that line segment AP divides triangle ABC into two congruent triangles. Remind students of the formal and informal meaning of congruent triangles.

- What can you say about the measures of angles and segments of the two congruent triangles?
- What can you say about triangles ACP and ABP?

In Problem 4.3, students will continue to explore these relationships about angles and side lengths. Students can work on this problem in pairs.

**Materials**
- Transparency 4.3A
- Scissors

**Vocabulary**
- 30-60-90 triangle

**Explore**

If students are having trouble, ask questions to help them see that two right triangles were formed by the line of symmetry. Then, ask what else they know about these right triangles.

Make sure students are determining the side lengths by using the Pythagorean Theorem, not by measuring.

**Summarize**

Let several pairs share their reasoning about each question, demonstrating their work at the board or overhead.

Students should also discover that the length of the side opposite the 30° angle is half the length of the hypotenuse. If not, ask:

- What is the length of segment CP? Segment AC?

In Question B, all students should be able to find the length of the third side of the right triangle using the Pythagorean Theorem. Use one of the two possible approaches to help clarify student confusion related to Question B.

Question C reviews the relationships in a 30-60-90 triangle.
**ACE Assignment Guide for Problem 4.3**

Core 10, 11  
Other Connections 26–34; Extensions 47–52; unassigned choices from earlier problems  
Adapted For suggestions about adapting ACE exercises, see the CMP Special Needs Handbook.  
Connecting to Prior Units 26: Filling and Wrapping; 28: Stretching and Shrinking; 29–31: Bits and Pieces I

**Answers to Problem 4.3**

A. Since triangles $ACP$ and $APB$ are congruent, measures of corresponding angles and sides are equal.

1. Angle $CAP$ measures 30°; Angle $CAB$ measures 60° because it is an angle of the original equilateral triangle, angle $CAP$ has a measure equal to half of angle $CAB$ or 30° because $AP$ is a line of reflection.

2. Angle $BAP$ measures 30° since it is congruent to angle $CAP$.

3. Angle $CPA$ measures 90° because each is half of 180°.

4. Angle $BPA$ measures 90° because each is half of 180°.

5. Length of $CP$ is 2 units. The length of side $CP$ is equal to half of a side of the equilateral triangle or half of 4, since $AP$ is a line of reflection.

6. Length of $PB$ is 2 units.

7. Length of $AP$ is $\sqrt{3}$ units; Because triangle $APB$ is a right triangle, and $4^2 - 2^2 = 12$, the length of side $AP$ is $\sqrt{12}$ or $2\sqrt{3}$ units.

B. The same pattern will hold for any triangle $ABC$ with side length $s$: There are two congruent triangles for each case; angle measures of the triangles obtained by a line of reflection are again 30-60-90 degrees.

1. Angle $CAP$ measures 30°.


3. Angle $CPA$ measures 90°.

4. Angle $BPA$ measures 90°.

5. Length of $CP$ is $\frac{1}{2}s$ units.

6. Length of $PB$ is $\frac{1}{2}s$ units.

7. Length of $AP$ is $\frac{1}{2}\sqrt{3}$ units; The ratio of the length of the side opposite the 30° angle to the length of the hypotenuse is always 1 to 2, so the lengths of $PB$ and $CP$ are $\frac{1}{2}s$ units. The length of $AP$ is $\sqrt{s^2 - \left(\frac{s^2}{4}\right)} = \sqrt{\frac{3s^2}{4}} = \frac{s}{2}\sqrt{3}$.

Notice that the ratio of the length of the side opposite the 60° to the length of the side opposite the 30° angle is always $\sqrt{3}$. Therefore, in a 30-60-90 triangle the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg. Not all students will notice this. They can, however, always apply the Pythagorean Theorem.

C. 1. The length of the side opposite the 30° angle is half the length of the hypotenuse, or 3 units. Because $6^2 - 3^2 = 27$, the length of the other leg is $\sqrt{27}$ units or $3\sqrt{3}$ units.

2. As explained in Question B, the ratio of the length of the side opposite the 30° angle to the length of the hypotenuse is always 1 to 2, and the ratio of the length of the side opposite the 30° to the length of the side opposite the 60° angle is always 1 to $\sqrt{3}$. 

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86 Looking for Pythagoras
**Goal**

- Use the properties of special right triangles to solve problems

In this problem, students will apply what they have learned about the Pythagorean Theorem and the special properties of 30-60-90 triangles.

**Launch 4.4**

Display Transparency 4.4 on the overhead.

**Suggested Questions** Ask:

- *Look at triangle ABC. What do you need to know to find its perimeter? (The lengths of the sides)*
- *How can we find those lengths?*

Let students offer their ideas. They may notice that the length of the side opposite the 30° angle in triangle ABC must be half the length of the hypotenuse but that neither of those two lengths is given. Some may notice that the measure of angle CAB must be 60°, because the sum of the measures of the other two angles in triangle ABC is 120°.

- *The challenge for you in this problem is to reason about the relationships in 30-60-90 triangles and the measures that are given to find the side lengths of triangle ABC and calculate the perimeter.*

Have the class work in groups of four on the problem.

**Explore 4.4**

Circulate as groups explore the problem. Some may need help identifying the three 30-60-90 triangles embedded in the figure. Suggest they draw the three triangles separately as shown here:

Students may have different strategies for determining the missing measures. Some may start with triangle BCD, some with triangle ABC.

**Suggested Questions** Ask:

- *How can you find the measure of angle BCD? (This is a right triangle, so the measure is 180° − (90° + 30°) = 60°.)*
- *How can you find the measure of angle CAD? (You can use triangle ABC or triangle ACD. In the latter case, you will need to find the measure of angle ACD first.)*

Encourage groups to keep track of their calculations in an orderly way so they will be able to explain their reasoning to the class.
Ask one of the groups to describe how they found the perimeter of $ABC$. Here is one possible explanation:

Because the two labeled angles in triangle $ABC$ have measures $30^\circ$ and $90^\circ$, the measure of angle $CAB$ must be $180^\circ - 120^\circ$, or $60^\circ$. Therefore, angle $ACD$ measures $180^\circ - 150^\circ = 30^\circ$, and angle $DCB$ measures $90^\circ - 30^\circ = 60^\circ$.

The side opposite the $30^\circ$ angle in right triangle $ACD$ has a length of 8 units. The length of the hypotenuse, side $AC$, must be twice that, or 16 units.

Because side $AB$ is the hypotenuse of the 30-60-90 triangle $ABC$ and the length of the side opposite the $30^\circ$ angle is 16 units, the length of the hypotenuse, or side $AB$, must be twice that, or 32 units.

We can now apply the Pythagorean Theorem to find the missing side length of triangle $ABC$. Because one leg and the hypotenuse measure 16 units and 32 units, respectively, the length of side $BC$ is the square root of $32^2 - 16^2$, or $\sqrt{768}$.

The perimeter of triangle $ABC$ is thus $16 + 32 + \sqrt{768} \approx 16 + 32 + 27.7 \approx 75.7$ units.

Move on to the rest of the questions. Some students may recall the properties of a 30-60-90 triangle and realize that the length of $BC$ is $16\sqrt{3}$.

Suggested Questions Once students have discussed how they found the areas of the triangles, ask:

- What is the relationship between the areas of the two smaller triangles and the area of the largest triangle? (The sum of the areas of the two smaller triangles is equal to the area of the largest triangle.)
- Which triangles are similar? Why?
- For each pair of similar triangles, what is the ratio of the short leg to the long leg? The short leg to the hypotenuse?
Mathematical Goal

• Use the properties of special right triangles to solve problems

Launch

Display Transparency 4.4 on the overhead.

• Look at triangle ABC. What do you need to know to find its perimeter? How can we find those lengths?

Let students offer their ideas.

• The challenge for you in this problem is to reason about the relationships in 30-60-90 triangles and about the measures that are given to find the side lengths of triangle ABC and then to calculate the perimeter.

Have the class work in groups of four on the problem.

Explore

Circulate as groups explore the problem. Some may need help identifying the three 30-60-90 triangles embedded in the figure. Suggest they draw the three triangles separately.

• How can you find the measure of angle BCD? How can you find the measure of angle CAD?

Encourage groups to keep track of their calculations in an orderly way so they will be able to explain their reasoning to the class.

Summarize

Ask one of the groups to describe how they found the perimeter of ABC.

Move on to the rest of the questions. Once students have discussed how they found the areas of the triangles, ask:

• What is the relationship between the areas of the two smaller triangles and the area of the largest triangle?
ACE Assignment Guide
for Problem 4.4

Core 12, 35
Other Extensions 53–58; unassigned choices from earlier problems
Adapted For suggestions about adapting ACE exercises, see the CMP Special Needs Handbook.
Connecting to Prior Units 35: Stretching and Shrinking

Answers to Problem 4.4

A. Side $AC$ has a length of 16 units, side $AB$ has a length of 32 units, and side $BC$ has a length of $\sqrt{768}$, or $16\sqrt{3}$ units. The perimeter of triangle $ABC$ is thus $16 + 32 + \sqrt{768} \approx 75.7$ units. Answers will vary. See the possible explanation in the Summarize section.

B. The area of triangle $ABC$ is $\frac{1}{2}bh = \frac{1}{2} \cdot 16 \cdot \sqrt{768} \approx 221.7$ units$^2$ or equivalently $\frac{1}{2} \cdot 32 \cdot \sqrt{192} \approx 221.7$ units$^2$.

C. Using the Pythagorean Theorem, because $16^2 - 8^2 = 192$, the length of side $CD$ is $\sqrt{192}$, or $8\sqrt{3}$ units. So, the area of triangle $ACD$ is $\frac{1}{2}bh = \frac{1}{2} \cdot 8 \cdot \sqrt{192} \approx 55.4$ units$^2$. The length of side $BD$ is $32 - 8 = 24$ units, so the area of triangle $BCD$ is $\frac{1}{2} \cdot 24 \cdot \sqrt{192} \approx 166.3$ units$^2$.

Alternatively, some students may argue that the areas of the two smaller triangles need to add to the area of the largest triangle. These students will use the area formula to find the area of one of the smaller triangles, and then subtract from the area of the largest triangle to find area of the other smaller triangle.
Answers

Investigation

ACE Assignment Choices

Problem 4.1
Core 1, 2
Other Connections 13–16

Problem 4.2
Core 3–5, 24, 25
Other Applications 6–9; Connections 17–23, 36, 37; Extensions 38–46; unassigned choices from earlier problems

Problem 4.3
Core 10, 11
Other Connections 26–34; Extensions 47–52; unassigned choices from earlier problems

Problem 4.4
Core 12, 35
Other Extensions 53–58; unassigned choices from earlier problems

Adapted For suggestions about adapting Exercise 8 and other ACE exercises, see the CMP Special Needs Handbook.


Applications

1. 12 cm

2. a. The 12th triangle has leg lengths 1 unit and \( \sqrt{12} \) units and hypotenuse length \( \sqrt{13} \) units. The 13th triangle has leg lengths 1 unit and \( \sqrt{13} \) units and hypotenuse length \( \sqrt{14} \) units. The 14th triangle has leg lengths 1 unit and \( \sqrt{14} \) units and hypotenuse length \( \sqrt{15} \) units.

b. \( \frac{1}{2} \) sq. unit, \( \frac{1}{2} \cdot \sqrt{2} \) units\(^2\), \( \frac{1}{2} \cdot \sqrt{3} \) units\(^2\), \( \frac{1}{2} \cdot \sqrt{4} \) units\(^2\), 1 unit\(^2\), \( \frac{1}{2} \cdot \sqrt{5} \) units\(^2\).

The number under the square root sign increases by 1 for every new triangle. Or, the area of the \( n \)th triangle is \( \frac{1}{2} \cdot \sqrt{n} \).

c. 5 is the square root of 25. So, the hypotenuse length of the 24th triangle is 5 units.

3. \( \sqrt{900 - 100} = \sqrt{800} \approx 28.28 \) in.

4. \( \sqrt{144 - 16} = \sqrt{128} \approx 11.31 \) ft

5. a. Because \( 500^2 + 600^2 = 610,000 \), the distance is \( \sqrt{610,000} \approx 781 \) m.

b. \( 1,100 - 781 \approx 319 \) m

6. a. They are congruent.

b. 45\(^\circ\), 45\(^\circ\), 90\(^\circ\). The diagonal divides the corner angles into two equal angles, so the smaller angles must each be half of 90\(^\circ\), or 45\(^\circ\). Some students may use a protractor or angle ruler.

c. The legs of the right triangle each have a length of 1 unit, and \( 1^2 + 1^2 = 2 \). So the diagonal—which is the hypotenuse of a right triangle—has a length of \( \sqrt{2} \) units.

d. The measures of the angles would still be 45\(^\circ\), 45\(^\circ\), and 90\(^\circ\). Because \( 5^2 + 5^2 = 50 \), the length of the diagonal would be \( \sqrt{50} \) units. (Note: Some students may notice that \( \sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2} \), or that this square is larger than the original by a scale factor of 5; thus, the diagonal must be 5 times as long, or \( 5\sqrt{2} \) units.)

7. a. All 45-45-90 triangles are similar to each other. If corresponding angles of a triangle are congruent, then the triangles are similar.

b. The other leg must also be 5 units long because 45-45-90 triangles are isosceles.
Applying the Pythagorean Theorem we have \( \text{hypotenuse}^2 = 5^2 + 5^2 = 50 \), so \( \text{hypotenuse} = \sqrt{50} = 5\sqrt{2} \approx 7.07 \) units.
So, the perimeter is \( 5 + 5 + 5\sqrt{2} \approx 17.07 \) units.

Investigation 4 Using the Pythagorean Theorem
8. 1012.4 m. The first segment along the ground is the leg of an isosceles right triangle. Because the other leg is 15 m long, this leg also has a length of 15 m. The same argument holds for the last segment along the ground. Therefore, the horizontal portion of cable is $1000 - (2 \times 15) = 970$ m long. Each angled part of the cable is the hypotenuse of an isosceles right triangle with legs of length 15 units. Because $15^2 + 15^2 = 450$, each angled piece has length $\sqrt{450} \approx 21.2$ m. The overall length of the cable is thus $970 + 21.2 + 21.2 < 1012.4$ m.

9. 22 ft. Because $25^2 - 15^2 = 400$, the tallest tree that can be braced is $\sqrt{400} = 20$ ft tall at the point of attachment. Adding 2 ft gives a total height of 22 ft. (Note: You can point out to students that this is a 3-4-5 Pythagorean Triple with a scale factor of 5.)

10. About 105.5 ft. The leg along the bottom of the 30-60-90 triangle measures 58 ft. The hypotenuse (from Denzel’s eyes to the top of the tower) is twice as long, or 116 ft. Because $116^2 - 58^2 = 10,092$, the vertical leg measures $\sqrt{10,092} \approx 100.5$ ft. Adding the distance from the ground to Denzel’s eyes, the tower is about 105.5 ft tall.

11. a. $ABC$, $ADE$, and $AFG$ are 30-60-90 triangles. The measure of angle $A$, which is in all three triangles, is $60^\circ$. Angles $ACB$, $AED$, and $AGF$ all have measure $90^\circ$ because the segments that form their sides are perpendicular (one side is horizontal and the other is vertical). So, the third angles of the three triangles—$ABC$, $ADE$, and $AFG$—must all have measure $30^\circ$. These triangles are all similar because if corresponding angles of a triangle are congruent, then the triangles are similar.

b. $\frac{BA}{AC} = \frac{4}{2} = 2$. The length of $AC$ is 2 units and, because triangle $ABC$ is a 30-60-90 triangle, $BA$ is twice the length of the side opposite the $30^\circ$ angle, which is $AC$. Therefore, the length of $BA$ is 4 units. The corresponding ratio for the other two triangles must be the same because the triangles are similar.

c. $\frac{BC}{AC} = \frac{2\sqrt{3}}{2} = \sqrt{3}$. Possible explanation: In a 30-60-90 triangle, the length of the side opposite the $60^\circ$ angle is $\sqrt{3}$ times the length of the side opposite the $30^\circ$ angle, which is $AC$. $AC$ has length 2 units, so $BC$ has length $2\sqrt{3}$. So, $\frac{BC}{AC} = \frac{2\sqrt{3}}{2} = \sqrt{3}$. The corresponding ratio for the other two triangles must be the same because the triangles are similar.

d. $\frac{BC}{AB} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$. The corresponding ratio for the other two triangles must be the same because the triangles are similar.

e. 24 units and $12\sqrt{3}$ units. Possible explanation: Triangle $XYZ$ fits the description given in the problem.

[Diagram]

The ratio $\frac{YZ}{XZ}$ must be equal to $\sqrt{3}$ because $XYZ$ is similar to triangle $ABC$ in part (a). Therefore, $YZ = XZ \cdot \sqrt{3} = 12\sqrt{3}$.

In all 30-60-90 triangles, the ratio of the hypotenuse to the shortest side is 2:1. So $XY = 2 \times 12 = 24$.

12. About 28.39 m. All triangles in the diagram are 30-60-90 triangles. The hypotenuse of the large triangle is 12 m (twice the shorter leg that is given). The longer leg is $6\sqrt{3}$, or $\sqrt{108} \approx 10.39$. This last leg can be found with the Pythagorean Theorem or by applying a scale factor of 6 to the 30-60-90 triangle in Question A of Problem 4.3.

Connections

13. $\sqrt{121} = 11$; rational
14. $\sqrt{0.49} = 0.7$; rational
15. $\sqrt{15} \approx 3.9$; irrational
16. $\sqrt{1000} \approx 31.6$; irrational
17. See Figure 2. The distance between the cars increases by 78.1 mi each hour. (Note: Students will probably calculate the distance apart by adding the sum of the squares and taking the square root of that sum.)

18. After 2 hr, the northbound car has traveled 80 mi. Use this distance as one leg of a right triangle and the distance apart (100 mi) as the hypotenuse. Using the Pythagorean Theorem, \(100^2 - 80^2 = 3,600\), so the distance the eastbound car has traveled must be \(\sqrt{3,600} = 60\) mi. This distance was traveled in 2 hr, so the eastbound car is traveling at 30 mph. (Note: This is a 3-4-5 right triangle with a scale factor of 20.)

19. \(\frac{2}{5} = 0.4\); terminating

20. \(\frac{3}{8} = 0.375\); terminating

21. \(\frac{5}{6} = 0.833\ldots\); 3 repeats

22. \(\frac{35}{10} = 3.5\); terminating

23. \(\frac{8}{99} = 0.0808080\ldots\); 08 repeats

24. Right triangle. \(5^2 + 7^2 = (\sqrt{74})^2\)

25. Right triangle.
\[(\sqrt{2})^2 + (\sqrt{7})^2 = 2 + 7 = 9 = 3^2\]

26. a. \(\sqrt{32} \approx 5.66\) cm

27. B

28. a. Two pairs of corresponding angles are equal, so the triangles are similar.

b. Because the triangles are similar, the corresponding sides are proportional. The given side length of the smaller triangle is a third of the corresponding side length of the larger triangle, so the other two side lengths of the smaller triangle must also be a third the length of the corresponding sides of the larger triangle. The sides of the larger triangle are 6 units, 3 units, and 3\(\sqrt{3}\) or \(\sqrt{27}\) units (or about 5.2 units), so the sides of the smaller triangle are 2 units, 1 unit, and \(\sqrt{3}\) or \(\frac{1}{3}\sqrt{27}\) units (or about 1.7 units).

c. The larger triangle’s area is 9 times the smaller triangle’s area.

29. Possible answers: \(\frac{35}{100}\) or \(\frac{7}{20}\)

### Figure 2

<table>
<thead>
<tr>
<th>Hours</th>
<th>Distance Traveled by Northbound Car (mi)</th>
<th>Distance Traveled by Eastbound Car (mi)</th>
<th>Distance Between Cars (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
<td>50</td>
<td>(\sqrt{60^2 + 50^2} \approx 78.1)</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
<td>100</td>
<td>(\sqrt{120^2 + 100^2} \approx 156.2)</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
<td>150</td>
<td>(\sqrt{180^2 + 150^2} \approx 234.3)</td>
</tr>
<tr>
<td>4</td>
<td>240</td>
<td>200</td>
<td>(\sqrt{240^2 + 200^2} \approx 312.4)</td>
</tr>
<tr>
<td>(n)</td>
<td>60(n)</td>
<td>50(n)</td>
<td>78.1(n)</td>
</tr>
</tbody>
</table>
30. Possible answer: $\frac{21456}{10000}$

31. Possible answer: $\frac{89050}{10000}$

32. False. $0.06 \times 0.06 = 0.0036$

33. True. $1.1 \times 1.1 = 1.21$

34. False. $20 \times 20 = 400$

35. a. About 37.9 units. $AC = 16$ units,

   $CD = \sqrt{192}$ units, or $8\sqrt{3}$ units, or about 13.9 units. So the perimeter is about $16 + 8 + 13.9$, or 37.9 units.

b. Because triangle $BDC$ is a 30-60-90 triangle, we can use the length of $AC$ to get the length of $AB$, which is 32 units, and of $BC$, which is $16\sqrt{3}$ units. So the perimeter of triangle $ABC$ is $32 + 16 + 16\sqrt{3}$, or about 75.7 units. We could have arrived at this answer without any calculation by noticing that the triangles are similar and the scale factor is 2. Therefore, the perimeter of triangle $ACD$ is twice the perimeter of triangle $ACD$.

c. The area of triangle $ABC$ is 4 times the area of triangle $ACD$.

36. 6 and 7. $6^2 = 36$ and $7^2 = 49$. Because 39 is between 36 and 49, $\sqrt{39}$ is between 6 and 7.

37. 24 and 25. $24^2 = 576$ and $25^2 = 625$. Because 600 is between 576 and 625, $\sqrt{600}$ is between 24 and 25.

Extensions

38. a. | Fraction | Decimal |
<table>
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<tr>
<td>$\frac{1}{9}$</td>
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</tr>
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<td>0.7777...</td>
</tr>
<tr>
<td>$\frac{8}{9}$</td>
<td>0.8888...</td>
</tr>
</tbody>
</table>

39. $\frac{1}{9} = 0.010101...$, $\frac{2}{9} = 0.020202...$.

40. $\frac{1}{99} = 0.001001001...$, $\frac{2}{99} = 0.002002002...$.

41. $\frac{3}{99} = 0.030303...$. A fraction with a denominator of 99 is equal to a repeating decimal. For numerators less than 99, the repeating part has two digits: either a 0 followed by the number in the numerator if that number is less than 10 or the number in the numerator if that number is greater than 10.

42. $\frac{4}{99} = 0.040404...$, $\frac{5}{99} = 0.050505...$.

43. $\frac{6}{99} = 0.060606...$, $\frac{7}{99} = 0.070707...$.

44. $\frac{8}{99} = 0.080808...$, $\frac{9}{99} = 0.090909...$.

45. $\frac{10}{99} = 0.1010101...$, $\frac{11}{99} = 0.1111111...$.

46. $\frac{12}{99} = 0.12121212...$, $\frac{13}{99} = 0.13131313...$.

47. The bottom of the box has sides of length 3 cm and 4 cm. Because $3^2 + 4^2 = 25$, the diagonal of the bottom has length $\sqrt{25}$ cm, or 5 cm. Using this as a leg of a right triangle with hypotenuse $d$, $d^2 = 5^2 + 12^2 = 169$, so $d = \sqrt{169} = 13$ cm.

48. The bottom has sides of length 6 cm and 7 cm. Because $6^2 + 7^2 = 85$, the diagonal of the bottom has length $\sqrt{85}$ cm. Using this as a leg of the right triangle with hypotenuse $d$, $d^2 = (\sqrt{85})^2 + (\sqrt{111})^2 = 85 + 111 = 196$, so $d = \sqrt{196} = 14$ cm.
49. a. (3.54, 3.54). Draw a vertical segment from \( B \) down to the \( x \)-axis to create a 45-45-90 triangle \( ABC \).

As observed in Exercise 7, in 45-45-90 triangles, the length of the hypotenuse is \( \sqrt{2} \) times the length of the leg. So \( BC = AC = \frac{5}{\sqrt{2}} \) units, which is approximately 3.54 units. So, the coordinates of \( B \) are (3.54, 3.54).

b. The half-circle on the leg of length 3 units has area \( \frac{\pi}{4} \times 3^2 = 7.06858 \approx 3.54 \text{ sq. units} \).

50. a. The half-circle on the leg of length 3 units has area \( \frac{\pi}{4} \times 3^2 = 7.06858 \approx 3.54 \text{ sq. units} \). The half-circle on the leg of length 4 units has area \( \frac{\pi}{4} \times 4^2 = 12.56637 \approx 6.28 \text{ sq. units} \). The half-circle on the hypotenuse has area \( \frac{\pi}{4} \times 5^2 = 19.63495 \approx 9.81748 \text{ sq. units} \).

b. The sum of the areas of the half-circles on the legs is equal to the area of the half-circle on the hypotenuse: \( 3.54 + 6.28 = 9.81748 \approx 9.81748 \text{ sq. units} \).

51. a. Each equilateral triangle can be divided into two 30-60-90 triangles. The equilateral triangle on the leg of length 3 units is composed of two right triangles, each with a leg of length 1.5 units and a hypotenuse of length 3 units. Because \( 3^2 - 1.5^2 = 6.75 \), the longer leg (which is the height of the equilateral triangle) has length \( \sqrt{6.75} \approx 2.6 \) units. This equilateral triangle has an area of about \( \frac{\sqrt{3}}{4} \times 3^2 = 3.9 \text{ sq. units} \). The equilateral triangle on the leg of length 4 units is composed of two right triangles, each with a leg of length 2 units and a hypotenuse of length 4 units. Because \( 4^2 - 2^2 = 12 \), the longer leg has length \( \sqrt{12} \approx 3.46 \) units. This equilateral triangle has an area of about \( \frac{\sqrt{3}}{4} \times 4^2 = 6.9 \text{ units}^2 \). The equilateral triangle on the hypotenuse is composed of two right triangles, each with a leg of length 2.5 units and a hypotenuse of length 5 units. Because \( 5^2 - 2.5^2 = 18.75 \), the longer leg has length \( \sqrt{18.75} \approx 4.3 \) units. This equilateral triangle has an area of about \( \frac{\sqrt{3}}{4} \times 5^2 = 10.8 \text{ units}^2 \).

b. The sum of the areas of the equilateral triangles on the legs is equal to the area of the equilateral triangle on the hypotenuse: \( 3.9 + 6.9 = 10.8 \text{ units}^2 \).

52. a. Each hexagon can be divided into six equilateral triangles, the areas of which were found in ACE Exercise 51. The hexagon on the leg of length 3 units has an area of about \( 6 	imes 3.9 = 23.4 \text{ units}^2 \). The hexagon on the leg of length 4 has an area of about \( 6 	imes 6.9 = 41.4 \text{ units}^2 \). The hexagon on the hypotenuse has an area of about \( 6 	imes 10.8 = 64.8 \text{ units}^2 \).

b. The sum of the areas of the hexagons on the legs is equal to the area of the hexagon on the hypotenuse: \( 23.4 + 41.4 = 64.8 \text{ units}^2 \).

53. Possible answers: \( \sqrt{39}, \sqrt{40} \), and \( 2\pi \).

54. a. \( 100x = 15.15151515 \ldots \)
\[ -x = 0.15151515 \ldots \]
\[ 99x = 15 \]
\[ x = \frac{15}{99} = \frac{5}{33} \]

b. \( 10x = 7.77777 \ldots \)
\[ -x = 0.77777 \ldots \]
\[ 9x = 7 \]
\[ x = \frac{7}{9} \]

c. \( 1,000x = 123.123123123123 \ldots \)
\[ -x = 0.123123123123 \ldots \]
\[ 999x = 123 \]
\[ x = \frac{123}{999} = \frac{41}{333} \]

55. a. \( \sqrt{100 - 36} = \sqrt{64} = 8 \text{ ft} \)

b. The farmer is saying that the barn is not perpendicular to the ground.

c. \( \sqrt{225 - 144} = \sqrt{81} = 9 \text{ ft} \)
d. Possible answer: She could use a 5-foot pole that would touch the barn 4 ft high and rest on the ground 3 ft from the base of the barn.

56. 78 units. Triangle $CDB$ is similar to triangle $ABC$, because both have angle $B$ and a right angle. Because $12^2 + 5^2 = 169$, the length of side $BC$ is $\sqrt{169} = 13$ units. The leg of length 5 units on the small triangle corresponds with the leg of length 13 units on triangle $ABC$, so the scale factor from triangle $CDB$ to triangle $ABC$ is $\frac{13}{5}$, or 2.6. Multiplying the side lengths of triangle $CDB$ by 2.6, side $AC$ has length $12 \cdot 2.6 = 31.2$ units and side $BA$ has length $13 \cdot 2.6 = 33.8$ units. The perimeter of triangle $ABC$ is thus $13 + 31.2 + 33.8 = 78$ units. (Note: Students may also calculate that triangle $CDB$ has a perimeter of $5 + 12 + 13 = 30$ and then apply the scale factor to find that the perimeter of triangle $ABC$ is $30 \cdot 2.6 = 78$.)

57. a. Using the Pythagorean Theorem, the length of half of the edge of the base is 3 units, so the edge length of the base is 6 units. Therefore, the base area is 36 units$^2$.

b. The surface is made up of 4 congruent triangles plus a base. Each triangle has area $\frac{1}{2}(6)(4) = 12$ units$^2$. So the surface area is $36 + 4(12) = 84$ units$^2$.

c. The height of the pyramid is found from the right triangle with sides 3 units (half of the base edge) and 4 units (the slant height). We need to solve $3^2 + h^2 = 4^2$. $h$ is $\sqrt{7}$ units, or about 2.65 units.

d. $\frac{1}{2}(36)(2.65) \approx 31.8$ units$^3$.

58. a. 31.81 in.$^3$. Because the diameter is 4.5 in., the radius is 2.25 in. The height is 6 in., so the volume is $\frac{1}{3} \pi (2.25)^2 (6) = 31.81$ in.$^3$

b. $26\pi$ in.$^3$. $7^2 = r^2 + 6^2$, so $r = \sqrt{13}$ in., or about 3.6 in. So the volume is $\frac{1}{3} \pi (\sqrt{13})^2 (6) = 26\pi$ in.$^3$, or about 81.7 in.$^3$.

Possible Answers to Mathematical Reflections

1. The Pythagorean Theorem is useful for finding the length of one side of a right triangle if you know the lengths of the other two sides.

An example of this is finding the distance between two points when the coordinates of the points are known. We connect the points with a line segment and then use the segment as the hypotenuse of a right triangle. We draw two legs, find their lengths, and then find the sum of the squares of the lengths. The distance between the two points is the square root of this sum.

Another example is finding the length of the diagonal $d$ of a rectangle. If the side lengths of the rectangle are $a$ and $b$, then the Pythagorean Theorem tells us $d^2 = a^2 + b^2$, or $d = \sqrt{a^2 + b^2}$.

2. In a 30-60-90 triangle, the length of the side opposite the $30^\circ$ angle is half the length of the hypotenuse. The length of the longer leg is $\sqrt{3}$ times the length of the leg opposite the $30^\circ$ angle. (Note: In a 30-60-90 triangle, if the leg opposite the $30^\circ$ angle has length $a$, then the hypotenuse has length $2a$. So, the longer leg has length $\sqrt{4a^2 - a^2} = \sqrt{3a^2} = a\sqrt{3}$.)

Answers to Looking Back and Looking Ahead

1. a. 12.5 units$^2$

b. $2.5\sqrt{2}$ units, or $\sqrt{12.5}$, or approximately 3.536 units

c. 10 units, $5\sqrt{2}$ units, and $5\sqrt{2}$ units

d. Triangle $B$: scale factor is 1 (in other words, triangles $A$ and $B$ are congruent);
   Triangle $F$: scale factor from $F$ to $A$ is 2 and from $A$ to $F$ is $\frac{1}{2}$;
   Triangle $D$: scale factor from $D$ to $A$ is 2 and from $A$ to $D$ is $\frac{1}{2}$;
   Triangle $G$: scale factor from $G$ to $A$ is $\sqrt{2}$ and from $A$ to $G$ is $\frac{1}{\sqrt{2}}$.
Possible Answers to Mathematical Reflections

1. The Pythagorean Theorem is useful for finding the length of one side of a right triangle if you know the lengths of the other two sides.

An example of this is finding the distance between two points when the coordinates of the points are known. We connect the points with a line segment and then use the segment as the hypotenuse of a right triangle. We draw the two legs, find their lengths, and then find the sum of the squares of the lengths. The distance between the two points is the square root of this sum.

Another example is finding the length of the diagonal \( d \) of a rectangle. If the side lengths of the rectangle are \( a \) and \( b \), then the Pythagorean Theorem tells us \( d^2 = a^2 + b^2 \), or \( d = \sqrt{a^2 + b^2} \).

2. In a 30-60-90 triangle, the length of the side opposite the 30° angle is half the length of the hypotenuse. The length of the longer leg is \( \sqrt{3} \) times the length of the leg opposite the 30° angle. \((\text{Note: In a 30-60-90 triangle, if the leg opposite the 30° angle has length } a, \text{ then the hypotenuse has length } 2a. \text{ So, the longer leg has length } \sqrt{4a^2 - a^2} = \sqrt{3a^2} = a\sqrt{3}.)\)

Answers to Looking Back and Looking Ahead

1. a. 12.5 units²
   b. 2.5\(\sqrt{2} \) units, or \(\sqrt{12.5} \), or approximately 3.536 units
   c. 10 units, 5\(\sqrt{2} \) units, and 5\(\sqrt{2} \) units
   d. Triangle B: scale factor is 1 (in other words, triangles A and B are congruent);
      Triangle F: scale factor from F to A is 2 and from A to F is \( \frac{1}{2} \);
      Triangle D: scale factor from D to A is 2 and from A to D is \( \frac{1}{2} \);
      Triangle G: scale factor from G to A is \( \sqrt{2} \) and from A to G is \( \frac{1}{\sqrt{2}} \)

   d. Possible answer: She could use a 5-foot pole that would touch the barn 4 ft high and rest on the ground 3 ft from the base of the barn.

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   b. The surface is made up of 4 congruent triangles plus a base. Each triangle has area \((\frac{1}{2})(6)(4) = 12 \) units². So the surface area is \( 36 + 4(12) = 84 \) units².
   c. The height of the pyramid is found from the right triangle with sides 3 units (half of the base edge) and 4 units (the slant height). We need to solve \( 3^2 + h^2 = 4^2 \).
   \( h \) is \( \sqrt{7} \) units, or about 2.65 units.
   d. \( \frac{1}{3}(36)(2.65) = 31.8 \) units³.

58. a. 31.81 in³. Because the diameter is 4.5 in., the radius is 2.25 in. The height is 6 in., so the volume is \( \frac{1}{3}\pi(2.25)^2(6) = 31.81 \) in³.
   b. 26\(\pi \) in³. \( 7^2 = r^2 + 6^2 \), so \( r = \sqrt{13} \) in., or about 3.6 in. So the volume is \( \frac{1}{3}\pi(\sqrt{13})^2(6) = 26\pi \) in³, or about 81.7 in³.

Looking for Pythagoras
2. a. The length of the side opposite a $30^\circ$ angle in a 30-60-90 triangle is half the length of the hypotenuse. Thus the wire is attached to the ground 30 ft from the base of the tower.

b. Use the Pythagorean Theorem to find the length of the other leg. The height of the tower is $30\sqrt{3}$ or approximately 52 ft.

3. You can determine the length of a side of any square by finding the square root of its area. Students may have used this strategy to find the side length of Square E in part (a) of Problem 1.

4. Possible answer: Form a right triangle whose hypotenuse is the line segment. The lengths of the legs are the positive difference in the $x$-coordinates of the endpoints and positive difference in the $y$-coordinates of the endpoints. Once you know the lengths of the legs, apply the Pythagorean Theorem to find the length of the hypotenuse, which is the line segment. If you forget the Pythagorean Theorem, you can build a square whose length is the given line segment. Find the area of the square and then take the square root of the area to find the length of the line segment.

5. a. The triangle is a right triangle. Therefore, the Pythagorean relationship applies: The sum of the area of the squares on the legs is equal to the area of the square on the hypotenuse.

b. Because the triangle in Figure 2 is not a right triangle, the Pythagorean Theorem does not apply.

6. a. The length of the diagonal of a square is the square root of the sum of the squares of two of the side lengths. If $d$ is the length of the diagonal and $s$ is the side length, then $d = \sqrt{s^2 + s^2} = \sqrt{2s^2} = s\sqrt{2}$.

b. The length of the diagonal of a rectangle is the square root of the sum of the squares of the length and width. If $d$ is the length of the diagonal and $s$ and $t$ are the width and length, then $d = \sqrt{s^2 + t^2}$.

c. The length of the hypotenuse of a right triangle is the square root of the sum of the squares of the lengths of the legs. If $c$ is the length of the hypotenuse and $s$ and $t$ are the lengths of the legs, then $c = \sqrt{s^2 + t^2}$.

d. The height of an equilateral triangle is the square root of the difference of the square of a side length and the square of half a side length. If $h$ is the height and $s$ is the side length, then $d = \sqrt{s^2 - \left(\frac{s}{2}\right)^2}$ (or $d = \frac{s}{2}\sqrt{3}$ based on 30-60-90 triangle properties).

e. The length of one side of a right triangle is the square root of the difference of the squares of the hypotenuse and the other side length. If $a$ is unknown leg length, $t$ is the known leg length, and $h$ is the length of the hypotenuse, then $a = \sqrt{h^2 - t^2}$.

For the Teacher In Problem 6, students may describe each process as three steps. For example in part (a), they may say:

- Take the square of two side lengths of the square.
- Add these two squares.
- Take the square root of the sum.