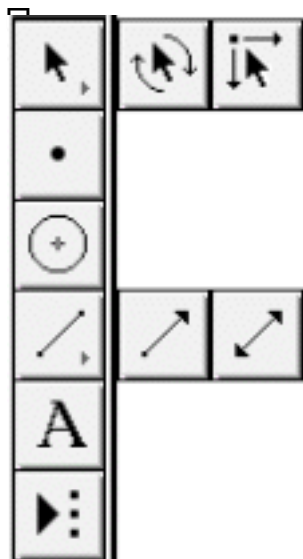


## GEOMETER'S SKETCHPAD 4.0

This program is the mathematician's toolbox.



**Selection Arrow tools:** used to select and drag objects in your sketch. Variations are drag-translate, drag-rotate, drag-dilate.

**Point tool:** used to construct points.

**Compass tool:** used to construct circles.

**Straightedge tool:** used to construct straight objects. Variations are segment, ray, and line.

**Text tool:** used to create and edit text and labels.

**Custom tools icon:** used to define, use, and manage custom tools.

**File Edit Display Construct Transform Measure Graph Help**

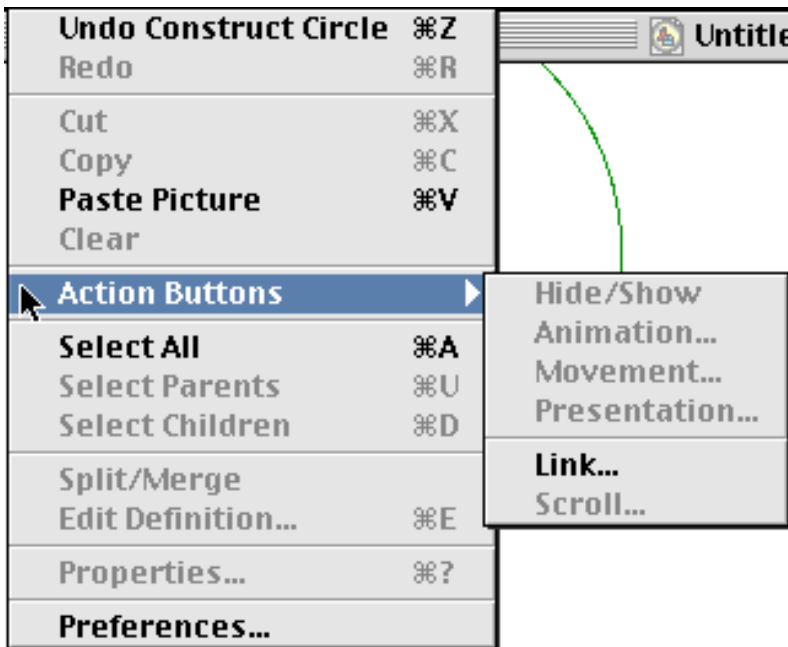
<b>New Sketch</b>	⌘N
<b>Open...</b>	⌘O
<b>Save</b>	⌘S
<b>Save As...</b>	
<b>Close</b>	⌘W
<b>Document Options...</b>	
<b>Page Setup...</b>	
<b>Print Preview...</b>	
<b>Print...</b>	
<b>Quit</b>	⌘Q

The menu bar contains the following choices:

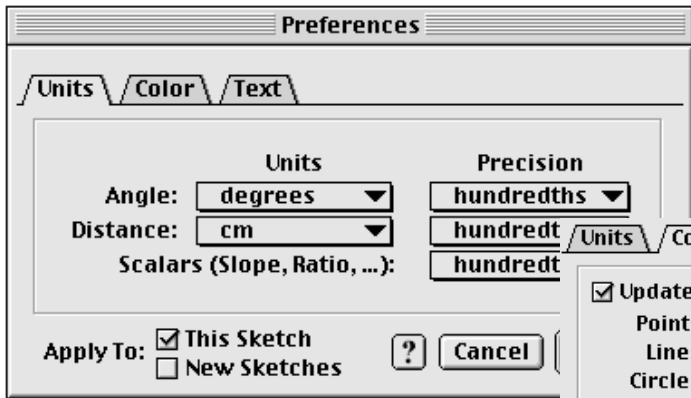
The **File menu** has the usual Macintosh or Window choices of Open, Save, Print, etc. Print Preview is very useful because it lets you scale your drawing to fit the printed page.

The **Edit menu** has the usual Cut, Copy, Paste and Clear options. The Undo allows you to undo continuously, back to your very first step — very useful. This menu also contains Action Buttons, which are used to create buttons for movement, animation, hide/show, presentation, link and scroll. This menu also contains items called Select, Select Parents and Select Children. (See below)

**Select:** If you select the Arrow tool and then the choice under the Edit menu is Select All. If the Point tool is selected, then the choice under the Edit menu is Select All Points. If the Compass tool is selected, then the choice under the Edit menu is Select All Circles. Likewise, if the Straightedge tool is selected, then the choice under the Edit menu is Select All Segments (or Rays or Lines).

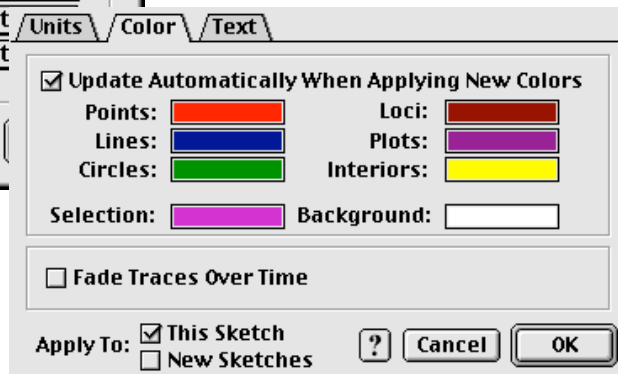


**Parents and Children:** When you select the necessary objects and then construct something, the construction is the child and the original objects are the parents. You can move a parent, or change its size, and thereby modify the child. You may not move a child in such a way that it changes the parent. (Moving a child will slide the entire figure on the page.) When you point at an object, the pointer turns sideways and the object is identified in the lower right corner of the screen.

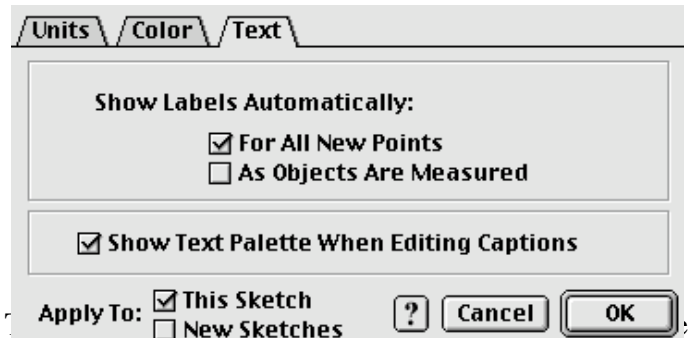


The **Preferences** item is very important. Here you can set the units and precision for measurements and computations.

You can choose colors.

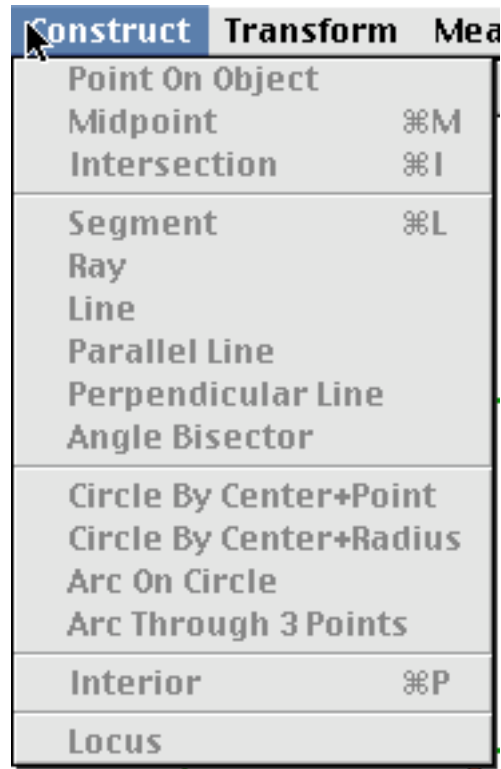


You can make choices about labeling.



top part contains choices to change line thickness, color, and text size and font. The next two sections deal with hiding and showing objects and labels. This allows you to hide constructions or parts of constructions. The next choice allows you to trace the path of an object (locus). The Animate choice lets you move an object. If a point is constructed on an object, then it will move it along this path (segment or circle). If the object is free, then it will move it randomly around the screen. This animation differs from the animation under the Action Buttons on the Edit menu, which creates a button you can turn off and on.

**Constructi  
on Tools:**  
Under the  
**Construct**



menu, you have the usual constructions from which to choose. Unlike the other geometry programs, you must first select the required points, lines, etc. before you choose the construction. For example, if you select two points and then go to the construction menu, you will see Segment, Ray, Line and Circle By Center+Point darkened. These are the constructions possible with two points. (Since the circle choice lists center+point, the first point will be the center and the second a point on the circle.)

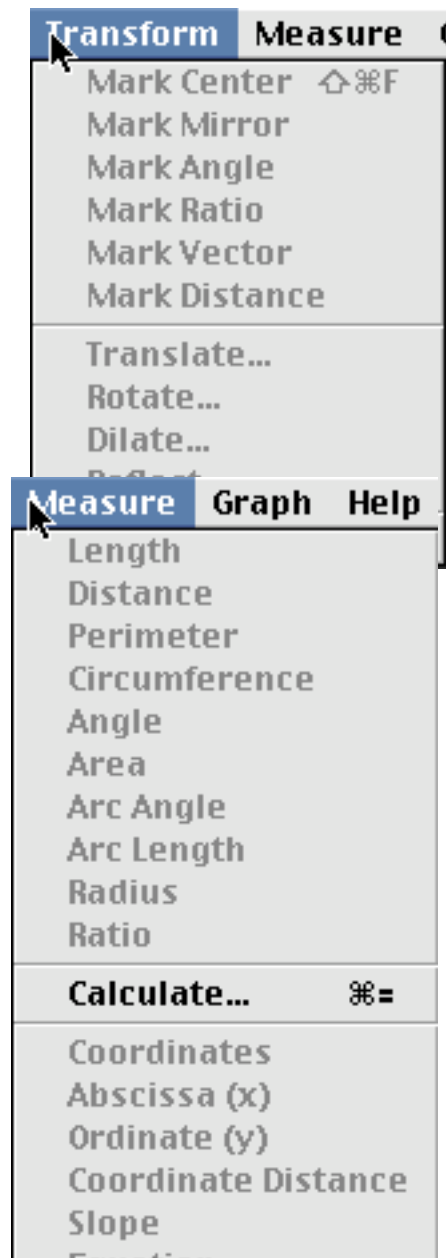
If you select an object, such as a circle or line, then you can choose Point On Object. This gives you a point that you can then drag to any location on the object by the point-click-and-drag method. This point

cannot be moved off the original object, unlike a point created by the Freehand tool. You can, however, split the point from the object using the choice under the Edit menu.

To construct an Arc On Circle, you first select the center, the first point on the arc, then the second point going counter-clockwise. This enables you to construct both major and minor arcs. You can also select 3 points and construct an arc through those points. Once an arc is selected, you can construct the sector or segment interior.

To construct a polygon, select the vertices in order, then choose segment under the Construct menu. To measure the area of a circle or polygon, you must first construct its interior. To construct the interior of a circle, simply select the circle. To construct the interior of a polygon, select the vertices in order.

If you select something you don't want, you can unselect it by clicking on it again. To unselect everything that is currently selected, simply point at a blank area of the screen and click or press **escape**. To select several objects, just click on them, but remember for certain constructions, order matters.



**Transformation Tools:** The **Transform** menu has choices to Translate, Rotate, Dilate, and Reflect. For the first three transformations, before you click the Transform button, you will see a sketch of what the result will look like.

You can translate by a vector in polar notation (distance & angle), rectangular notation ( $\Delta x$ ,  $\Delta y$ ), or a marked vector (from one point to another).

In order to rotate, you must first designate a center point, then you can rotate by a fixed angle (enter degrees) or marked angle (designated by 3 points).

To dilate, you must designate a center point, then enter a fixed ratio or a marked ratio (designate 2 segments).

To reflect, you must first select a straight object for the mirror.

**Measurement Tools:** You can select objects, then go to the **Measure** menu to get the measurement. You can select two points to get Distance or a segment to get Length. You need three points for the measure of an Angle. Angles are measured counter-clockwise. To measure the circumference of a circle, select the circle. To measure the Perimeter or Area of an object, you must first construct the interior (see Construction tools).

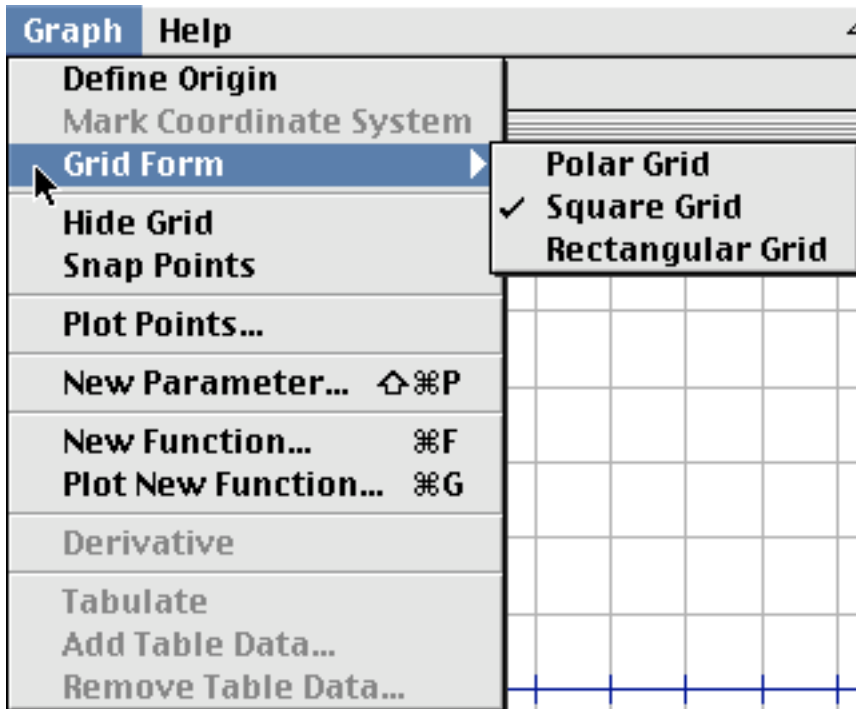
If you select the circle and 2 points on it, you can measure the minor arc with those end points, with either the arcangle or arclength. If you select the arc itself, you can get those measures for either the major or minor arc.

Unfortunately, when selecting a segment, circle, or arc, the program calls them by their names, such as j (for segment), c1 (for circle), or a2 (for arc). You can change these names by using the Text tool.

You can measure the Slope of a line, line segment, or ray. You can also obtain the Coordinates of a point or the Equation of a line or circle. These selections will automatically put coordinate axes on the screen using the unit set under preferences. Equation will not give the equation of a line segment, so a line must be constructed collinear with the segment.

If you change things that have been measured, the measurements will also change. This is the dynamic part of this program.

Notice that the measurements are right on the graphics screen. You can select them and move them by the point-click-and-drag maneuver. You can do calculations with several measurements by selecting them (multiple selections), then choose Calculate under the Measure menu. You will see a calculator with rectangle labeled Value. This rectangle is a pull-down menu containing the measurements you selected, plus  $\pi$  and  $e$ . If you have selected the coordinates of a point before going to Calculate, you can return just the x or y coordinate. You calculate by selecting values and operations. You can also enter numbers by clicking on them, just like buttons on a calculator. There is also a Functions menu for trig functions, logs, etc. Okay completes the calculation and returns to the graphics screen.



□

**Graph Tools:** The first choice is to define a coordinate system.

Select one point, then the choice is Define Origin and you get a square coordinate system centered at the selected point with the scale selected under Preferences.

Select a circle, then the choice is Define Unit Circle and you get a coordinate system with the center of the circle at the origin and 1 unit = radius.

Select a segment, then the choice is Define Unit Distance and you get a square coordinate system centered on the screen with 1 unit = length of segment.

Select a point and a segment, then you also get Define Unit Distance, but the coordinate system is centered at the selected point.

Select two segments, then the choice becomes Define Unit Distances and you get a rectangular coordinate system with the first segment the horizontal unit and the second segment the vertical unit.

Select two segments and a point, then the choice is the same as above and the rectangular coordinate system is centered at the point.

Select nothing, or anything else, and the choice is Define Coordinate System and you get a square coordinate system centered at the middle of the screen with the default unit.

You can Show Grid or hide the grid. Snap Points causes points to snap to the nearest grid point.



This can have weird consequences if dragging items, so it may be preferred to turn this off by clicking on this selection. You can plot points. You can change the grid form between square, rectangular, and polar.

You can select several measurements and put them in a table by choosing Tabulate under the Graph menu. The last entry in the table will dynamically change as you move objects.

To add additional lines, either double-click on the table or select the table and choose Add Table Data under the Graph menu. To delete the last entry in the table, select the table and press the  key or select the table and choose Remove Table Data from the Graph menu.

The **Help** menu in GSP 4.0 will open up in Internet Explorer. This is an applet, so it takes a little time to load the first time it is accessed in a session.

The menus and information are fairly comprehensive, but by no means as complete as you may want it to be.

This is a **NEW** Program, and I do not know all the answers. Please feel free to experiment. If you learn anything I don't know, please share your knowledge.

Before you start, go to the **Edit** menu and choose **Preferences**. Click on the **Text** tab. Make sure that there is a  $\surd$  in the box to **Show Labels Automatically For All New Points**. If there is not, click in this box. Click on the **Units** tab. Select centimeter and hundredths for measurements.

**INITIAL INVESTIGATIONS ON TRIANGLES**  
**(Beginning Sketchpad)**

**Problem #1 Attributes of triangles from three points.**

**Objective:** To construct a triangle from 3 points and explore angle measures.

- a. Select the **Point** tool. Make three points by moving to three locations on the screen and clicking the mouse button. Select the arrow tool to continue.
- b. Construct a triangle by selecting all three points. [Point at each one and click.] Now under the **Construct** menu choose **Segment**. The points will be connected in the order chosen.
- c. Measure the angles of the triangle. [Select three points, then under the **Measure** menu choose **Angle**. Now unselect the three points by clicking anywhere on the screen or pressing `esc`.]

Repeat this for the two remaining angles by choosing the three points in different orders.

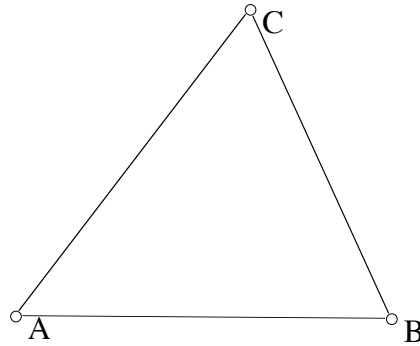
- d. Sum the angles of the triangle by choosing **Calculate** under the **Measure** menu. Choose the first measure, click on the +, choose the second measure, +, choose the third measure, then **Okay**.]

- e. Change the size and shape of the triangle. Point at one of the vertices (the arrow turns sideways). Hold down the mouse button and drag the point on the screen. Release the button.

Notice that the measurements for the angles have changed. What stayed the same?

Write two possible theorems you could discover from this construction.

$$\begin{aligned}m\angle ACB &= 61.98^\circ \\m\angle CBA &= 65.04^\circ \\m\angle BAC &= 52.98^\circ \\m\angle BAC + m\angle CBA + m\angle ACB &= 180.00^\circ\end{aligned}$$



**figure 1**



## Problem #2 A triangle using a segment and a point on a parallel line

**Objective:** To explore the attributes of skewed triangles.

a. Get a clean screen. Under the **File** menu choose **New Sketch**. This gives you a new screen to work with. Alternatively, you can go to the **Edit** menu and choose **Select All** then **Clear**.

b. Construct the base of the triangle using the **Segment** tool to draw a segment AB on the screen.

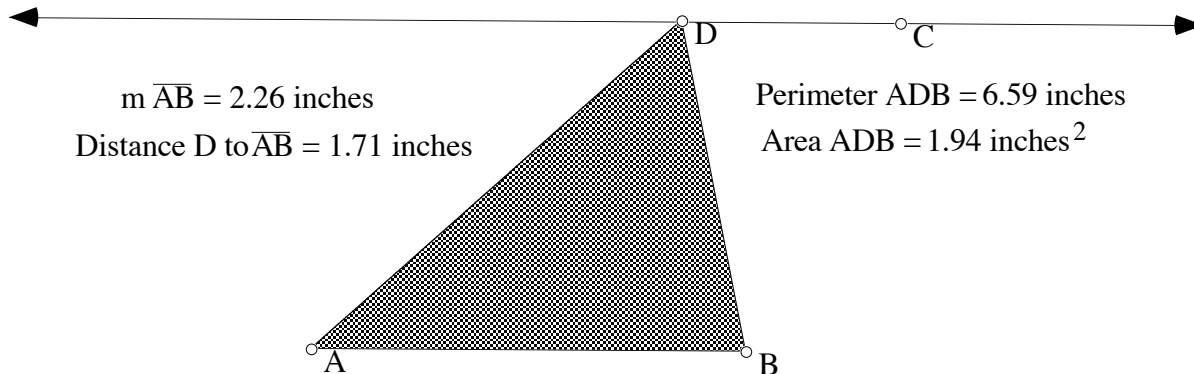
c. Construct the third vertex on a line parallel to the base as follows: use the **Point** tool to place a point C anywhere on the screen, not on the segment; choose the arrow tool; select the point and the line segment; then under the **Construct** menu choose **Parallel Line**.

The parallel line should now be selected (has stripes on it) because it was the last object created. Under the **Construct** menu choose **Point On Parallel Line**. You should now have a dark point, D, on the parallel line. This is a movable point — practice dragging it. I usually ask students to hide point C because dragging it causes the entire line to move.

d. Construct the rest of the triangle. [Select the movable point on the parallel line, D, then the two endpoints of the segment. Now under the **Construct** menu choose **Segments**.]

e. Measure the length of segment AB (base of the triangle). Measure the altitude of the triangle by selecting D (movable point) and segment AB, then choose **Distance** under the **Measure** menu.

Measure the area of the triangle as follows: first select points A, B, and D, then under the **Construct** menu, choose **Triangle Interior**. This interior will now be selected (colored), so under the **Measure** menu choose **Area**.



**figure 2**

Measure the perimeter of the triangle. Select the interior, then under the **Measure** menu choose **Perimeter**.

f. Move vertex D along the parallel line. Select point D, then hold down the mouse button and drag it. Since it is a child of the parallel line, it is restricted to moving along this line. This is called skewing the triangle.

What changes? What stays the same? Why?

Write a theorem.

Can you change the area without changing the perimeter? Describe how you did it.

### CONSTRUCTING A PARALLELOGRAM USING THE DEFINITION (Intermediate Sketchpad)

#### Problem #3 A quadrilateral with both pairs of opposite sides parallel

**Objective:** To find the parallelogram theorems through investigation.

- Construct the two sides of the parallelogram,  $\overline{AB}$  and  $\overline{BC}$ .
- Select C and segment AB, under the **Construct** menu choose **Parallel Line**.  
Select A and segment BC, then under **Construct** choose **Parallel Line**.
- Place a point at the intersection of the two constructed parallel lines. This will label the point of intersection, D.

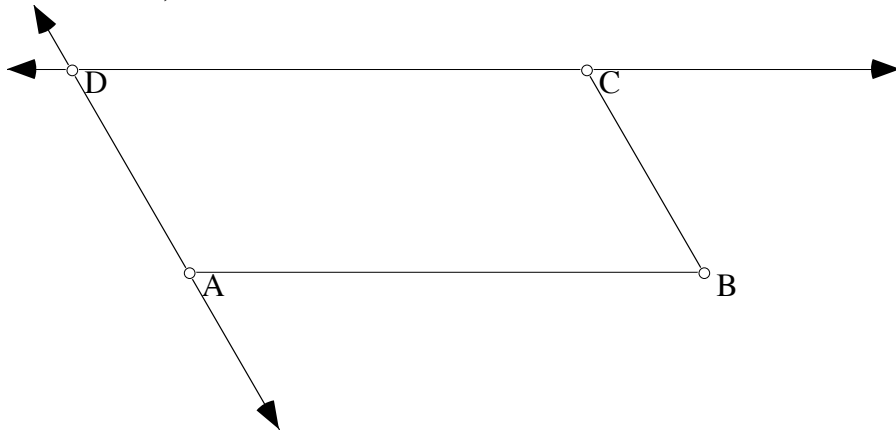
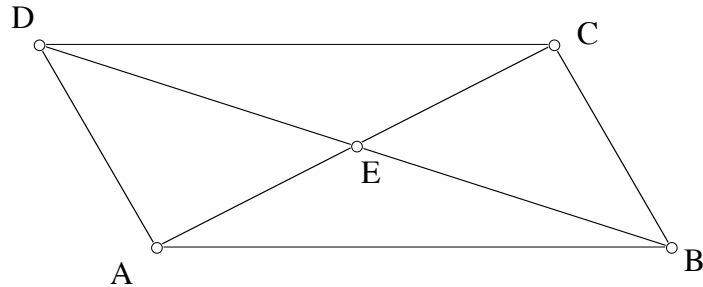


figure 3

- We want to define the parallelogram and get rid of the rest, so hide lines  $\overleftrightarrow{DC}$  and  $\overleftrightarrow{AD}$  (but not point D). Construct segments  $\overline{DC}$  and  $\overline{AD}$ .
- Draw diagonals  $\overline{AC}$  and  $\overline{BD}$ . Label the intersection.  
Measure segments and angles.



**figure 4**

Write a conjecture about segments  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CD}$ , and  $\overline{DA}$ .

Write a conjecture about angles  $\angle ABC$ ,  $\angle BCD$ ,  $\angle CDA$ , and  $\angle DAB$ .

Write a conjecture about segments  $\overline{AC}$ ,  $\overline{AE}$ ,  $\overline{EC}$ ,  $\overline{BD}$ ,  $\overline{BE}$ , and  $\overline{ED}$ .

Write a conjecture about angles  $\angle AEB$ ,  $\angle BEC$ ,  $\angle CED$ , and  $\angle DEA$ .

Write a conjecture about angles  $\angle EAB$ ,  $\angle EBA$ ,  $\angle EBC$ ,  $\angle ECB$ ,  $\angle ECD$ ,  $\angle EDC$ ,  $\angle EDA$  and  $\angle EAD$ .

- f. Move point B so that  $AB = BC$ . Now what is true? Name this shape.  
What happened to angles  $\angle AEB$  and  $\angle BEC$ ?
- g. Move point C so that  $m\angle ABC = 90^\circ$ . Now what is true? Name this shape.  
What happened to angles  $\angle ABC$ ,  $\angle BCD$ ,  $\angle CDA$ , and  $\angle DAB$ ?
- h. What happened to segments  $AC$ ,  $AE$ ,  $EC$ ,  $BD$ ,  $BE$ ,  $ED$ ?
- i. Write at least ten theorems you think might be true about parallelograms in general and special parallelograms, such as rectangles, squares, and rhombi.

#### Problem #4 Basic Transformations

**Objective:** To discover the attributes of the basic transformations of reflection, translation, and rotation.

Doing transformations and sequences of transformations are really easy with dynamic software. You can use the axes and grid to place points at specific locations. Once you have constructed the transformation, you can move the center of rotation, mirror line, original object, etc. to see the impact on the final image.

Construct a non-regular, preferably non-convex, polygon. The more unusual the better so you can tell if it has been reversed or changed. Place points on the screen using the Point tool. Select them in order, then under the Construct menu choose Segment to create the polygon. You may have a polygon similar to the one at the right.

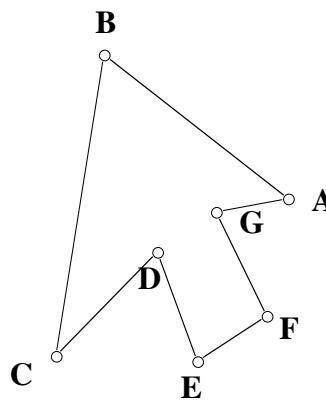


figure 5

1. **Reflection:**

A reflection requires an object and a line.

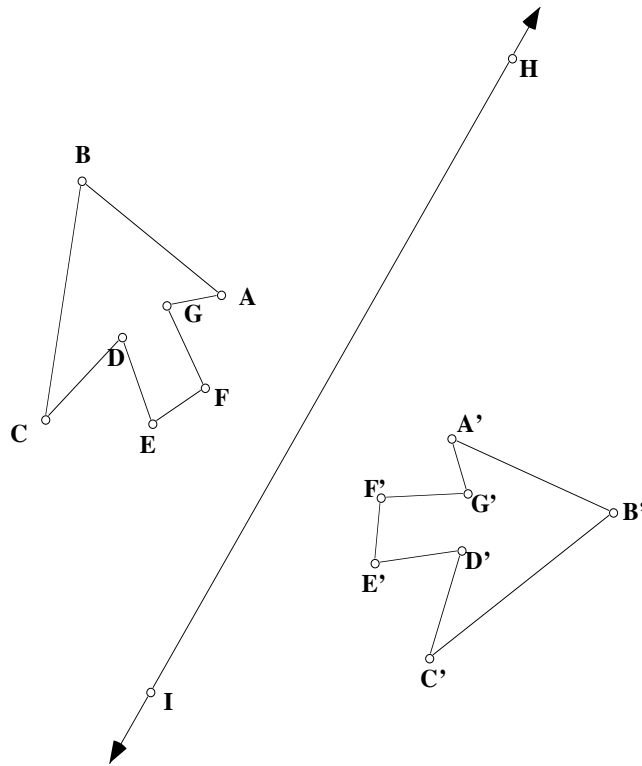
- a. Draw a line. Select the Segment tool, then drag to the last icon to select the line (two arrows). Now draw a line on the screen. While the line is selected, under the Transform menu choose Mark Mirror.
- b. Select the polygon. You can use a marquee to encompass the entire polygon, including points and segments. Using the **Arrow** tool, position at the upper left of the polygon, hold down the mouse button and drag to the bottom right of the polygon. A dotted rectangle (marquee) will be drawn. Everything inside the rectangle will be selected. If the line is also selected, click on it to deselect.
- c. Under the **Transform** menu choose **Reflect**. (Your result should look something like figure 6.)

You can measure lengths and angles of the original and the new polygon.

Does reflection change the length of the sides?

Does reflection change the size of the angles?

Is the new polygon named in clockwise or counter-clockwise order? (This is called orientation.) What about the original polygon?



**figure 6**

Check your observations by moving points on the original polygon and moving the line.

Under reflection, circle what remains unchanged: lengths    angle measure    orientation

**2. Translation:**

For a translation, you need an object and a vector or change in x and y. For this example, we will use a line segment where the order in which the endpoints are named indicates the direction and the length.

- a. Get rid of everything except the original polygon. Under the Edit menu choose Undo... repeatedly until you are back to your original polygon. (Or press  $\square$  - Z.)
- b. Change the Line tool back to a segment and draw a segment. Choose the endpoints of the segment. Under the Transform menu, choose Mark Vector.
- c. Select the polygon.
- d. Under the **Transform** menu, choose **Translate**. A dialog box will appear and the button next to **Marked** should be selected. Choose **Translate**.

Does translation change the length of the sides?

Does translation change the size of the angles?

Is the new polygon named in clockwise or counter-clockwise order? What about the original polygon?

Check your observations by moving points on the original polygon, changing the line segment.

Under translation, circle what remains unchanged: lengths    angle measure  
orientation

3. **Rotation:**

For a rotation you need an object, a point for the center of the rotation, and an angle of rotation.

- a. Undo back to the original polygon.
- b. Select point A on the polygon. Under Transform choose Mark Center.
- c. Choose the entire polygon. Under Transform choose Rotate. For the angle enter  $-127$ .

(This would be a clockwise rotation about  $\frac{1}{3}$  around the circle.) Your figure should look something like figure 7.)

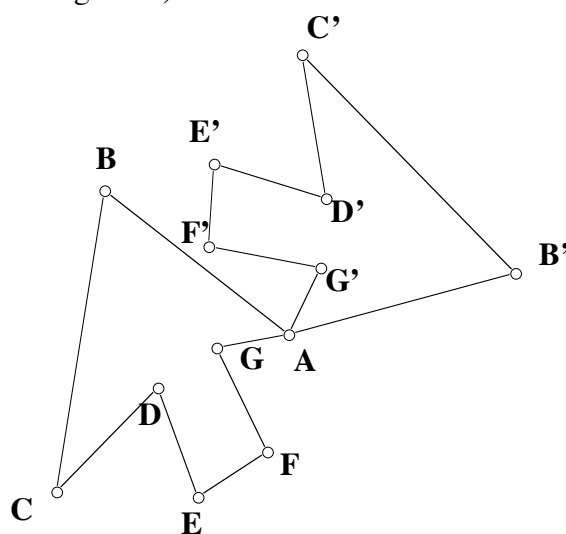


figure 7

- d. Undo the rotation.
- e. Select the angle ABC in that order. Under **Transform** choose **Mark Angle**.
- f. Select the entire polygon. Under **Transform** choose **Rotate**. This time the button next to **Marked Angle** should be selected. Choose **Rotate**.

Does rotation change the length of the sides?

Does rotation change the size of the angles?

Is the new polygon named in clockwise or counter-clockwise order? What about the original polygon?

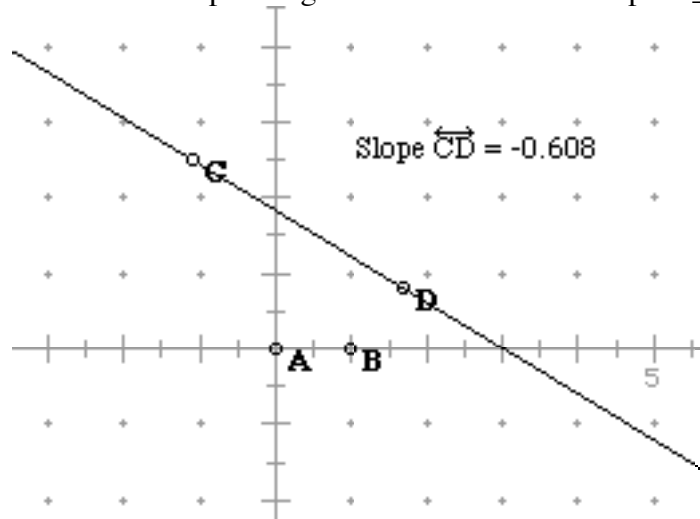
Check your observations by moving points on the original polygon.

Under rotation, circle what remains unchanged: lengths    angle measure    orientation

### Problem #5 Connecting Algebra & Geometry — Lines

**Objective:** To illustrate geometric representations of lines and the equations of lines.

1. In a new sketch, choose Define Coordinate System from the Graph menu.
2. Draw any line. Measure its slope using the Measure menu. Slope = \_\_\_\_\_.



**figure 8**

3. Select the line itself and drag it around. Observe the effect on its slope.
4. Select one of the points on the line and drag this point to rotate the line. How does this affect the slope?
  - a. Which lines have a positive slope and which have a negative slope?
  - b. What is the slope of a horizontal line?
  - c. How can you tell a steeper slope from a shallower slope?
  - d. What is the slope of a vertical line?
5. Select one of the points on the line, and the line, and construct a perpendicular line. Measure the slope of the perpendicular. How do the slopes compare?
6. Select the two slope measurements and, using Calculate under the Measure menu, multiply these two numbers. What is the product?

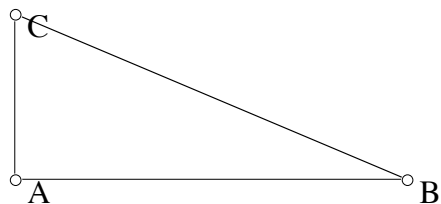
Drag around one of the points of the original line. What happens to the product?
7. Place a point on the screen that is not on any line. Select the point and your original line and construct a parallel line. Measure the slope of the parallel line. How does the slope compare to the original line?

8. In a new sketch, choose Define Coordinate System. Select the x-axis and construct a point on it. Select the y-axis and construct a point on it.
9. Select the two points and construct a line (not a segment) through these two points.
10. Select the point on the y-axis and get its coordinates (under the Measure menu). Select the line and measure its slope. Also measure its equation.
11. Drag your line and compare the three measurements. One type of linear equation is called the slope-intercept equation. A linear equation in this form looks like  $y = mx + b$ . Describe what  $m$  and  $b$  represent, using observations from your sketch.

### Problem #6 Creating a Tool for the Pythagorean Theorem

**Objective:** To investigate the Pythagorean Theorem as it applies to right and non-right triangles.

Tools are recordings of the steps you did to create a construction. The purpose is to allow you to repeat constructions on the same figure or new figures. One method of creating a tool is to do a construction, highlight everything, and then under the **Custom Tool icon** choose **Create New Tool**.



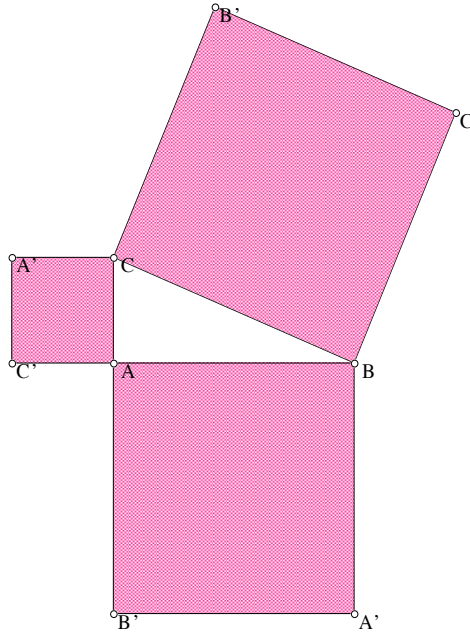
1. Open a new sketch. Construct a right triangle. (One method is to construct segment AB, construct a  $\perp$  at A, construct a point C on the  $\perp$ , hide the  $\perp$ , connect C to the endpoints of the segment.)

**figure 9**

2. Now construct a square on side AB in the following manner. Select A and mark as a center. Select B and rotate  $-90^\circ$  (clockwise).



3. Select B and mark as a center. Select A and rotate  $90^\circ$  (counter-clockwise). Select A, B, A', B' and construct segments to make a square. Click on A, B, A', B' to construct a quadrilateral interior.
4. Now to create the tool, select points A, B, A', B', segments BA', A'B', B'A, and the quadrilateral interior. Click on the Custom Tool icon and choose **Create New Tool**. You can name your tool. It will be added to the menu.
5. To construct the other two squares, we will just use your tool.



**figure 10**

Be careful though. If you select **Show Script View** in the Custom Tool menu, you can see the order in which the points are named in the **Given** section. Mine says "1. Point B, 2. Point A" which means that I have to name the points in clockwise order. If I reverse the order, the square will be constructed on the inside of the triangle.

Click on the Custom Tool icon and select your new tool. When you move the cursor to the screen, it looks like an arrow with a dot on the tip. Point to A and click, now when you move to C you can see the square forming. Repeat for side CB.

6. Now click on each of the interiors and measure the areas of the squares on the legs and the area of the square on the hypotenuse. Add the two areas of the squares on the legs. How does the sum compare to the area of the square on the hypotenuse?

Move A, B, and C to determine if this relationship will always hold.

7. Get a new sketch. Now construct any triangle ABC, then run the script to construct squares on the sides. You can show that  $AB^2 + AC^2 = BC^2$  if and only if  $m\angle CAB = 90^\circ$ . For a non-right triangle, you would have  $AB^2 + AC^2 > BC^2$  or  $AB^2 + AC^2 < BC^2$ .

If  $m\angle CAB > 90^\circ$  then  $BC^2 < = > AB^2 + AC^2?$  (circle one)

If  $m\angle CAB < 90^\circ$  then  $BC^2 < = > AB^2 + AC^2?$  (circle one)

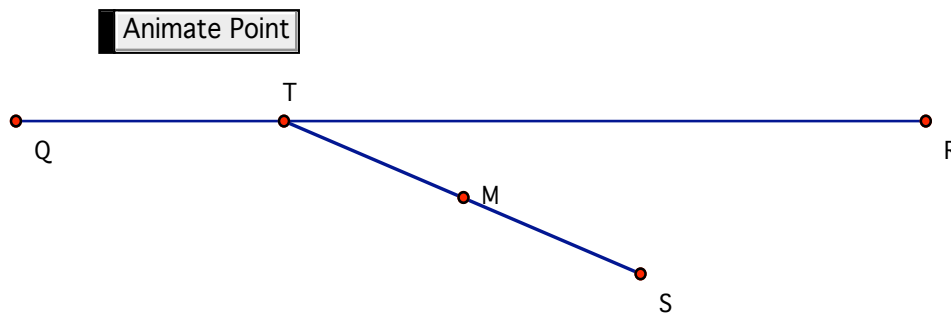
These inequalities will eventually lead to the Law of Cosines.

### **Problem #7 Locus Through Animation**

**Objective:** To find the locus of points when the construction is animated.

In order to do animation, you need to have an object move along a path, usually a point moving on a line segment or circle, or randomly around the screen. You then have the following choices: object moves forward, backward, bi-directional, or random and object moves at a slow, medium, or fast speed. You can trace the locus of some objects in order to see the pattern that occurs.

1. Construction: Draw segment  $\overline{QR}$ . Construct point  $T$  on  $\overline{QR}$ . Place a point  $S$  somewhere not on  $\overline{QR}$ . Construct segment  $\overline{TS}$ . Construct  $M$ , the midpoint of  $\overline{TS}$ . If point  $T$  is animated along segment  $\overline{QR}$ , what is the locus of point  $M$ ?

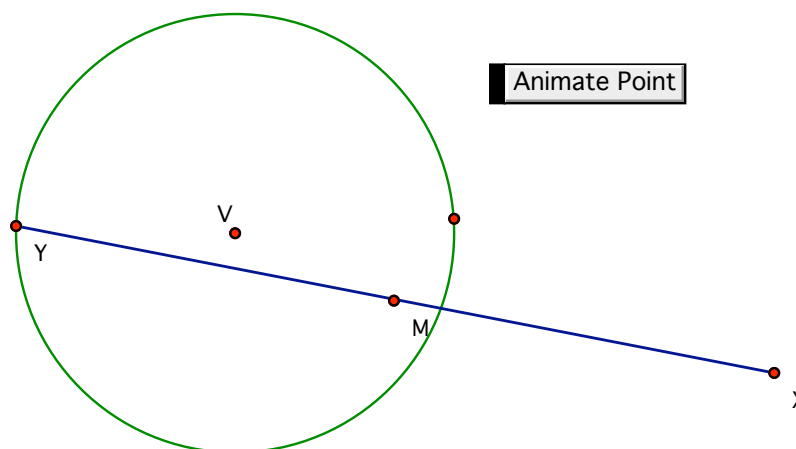


**figure 11**

First select point  $M$ , then under the Display menu choose the Trace Midpoint option. Now select point  $T$ . Under the Edit menu choose Action Buttons, then Animation. Now click on OK to accept the defaults.

Describe the tracing that results from this animation. What is the shape? How is it related to  $\overline{QR}$ ?

2. Construction: Draw a circle with center labeled  $V$ . Construct a point  $Y$  on the circle (do not use the point that defines the radius). Draw a point  $X$ , not on the circle, and construct segment  $\overline{XY}$ . Construct  $M$ , the midpoint of  $\overline{XY}$ .



**figure 12**

Select point M, then under the Display menu choose the Trace Midpoint option. Now select point Y and under the Edit menu choose Action Buttons, then Animation. Now click on OK.

As point Y is animated around the circle, what is the locus of point M? How is the locus related to the circle and to the distance XV?

## Extension Problems

### I. Conic Sections

#### A. Parabolas constructed by locus definition:

The definition of a parabola is the set of points that are equidistant from a line (directrix) and a point (focus). In figure 13, line AB is the directrix and D is the focus. If point F is on the parabola, then  $FD = FC$ .

To construct the locus of all points F, we need to observe that  $\triangle FDC$  would be isosceles and  $\overline{FC} \perp \overline{AB}$ . The following steps will construct the locus of point F.

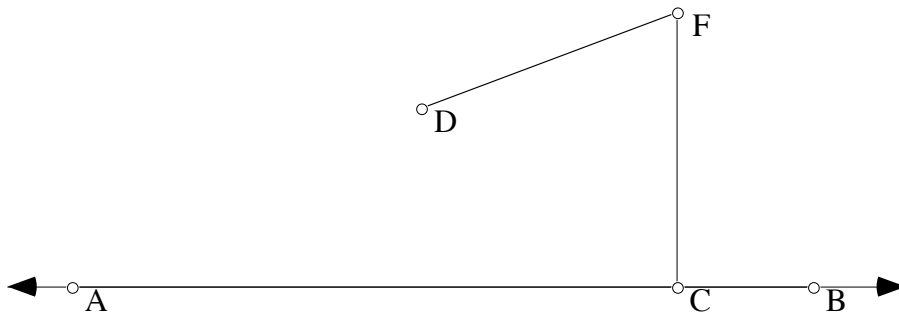


figure 13

1. Get a new sketch.

Draw a horizontal line, AB, at the bottom of the screen. [Hold down the shift key while using the line tool to make a horizontal line.] Select the line and construct a point on the line. (C)

2. Place a point about an inch above the line. (D)

Select the point C and the line AB and construct a Perpendicular line.

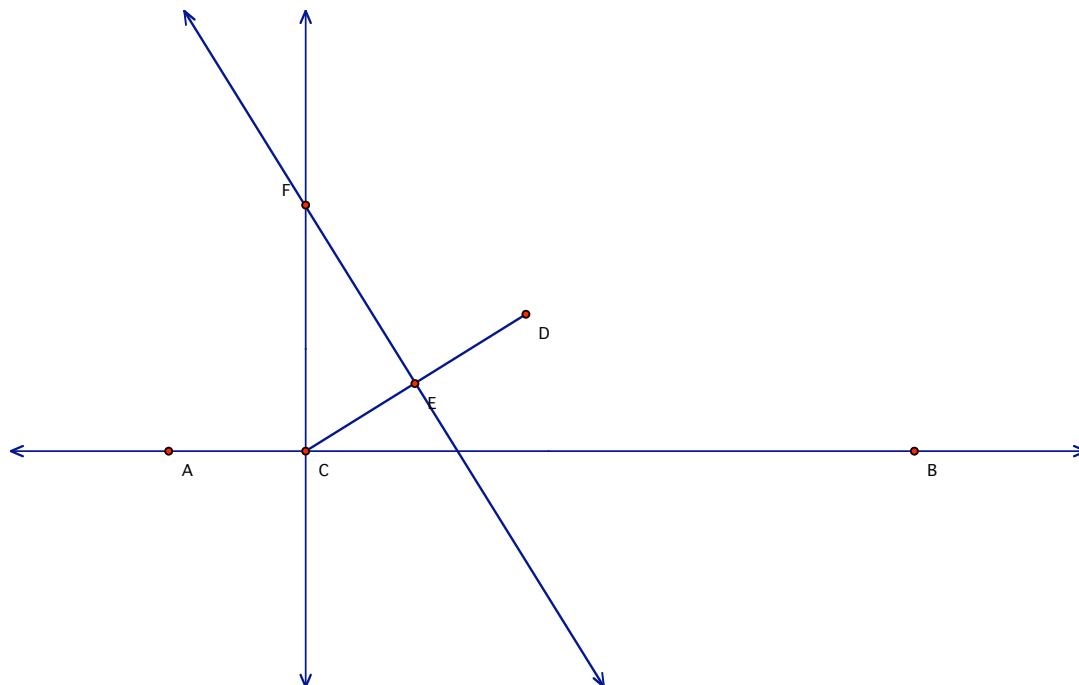
[The desired point F is located on that perpendicular. Since this point is also the vertex of an isosceles triangle, then it is located on the perpendicular bisector of the base,  $\overline{CD}$ .]

- Construct a segment joining the focus D and the point on the line C. ( $\overline{CD}$  in my sketch.) With the segment selected, construct a point at the midpoint. (E)
- Select the midpoint and the line segment CD, then construct a perpendicular. Construct the point F where the perpendiculars intersect. (See figure 14.)
- Select the perpendicular,  $\overleftrightarrow{EF}$ , then under Display choose Trace Perpendicular Line. If you drag C along  $\overleftrightarrow{AB}$  you will see shadows left of the locus of the perpendicular.
- We are now ready to animate this sketch.

Select the point on the horizontal segment (C) and under the Display menu choose Animate Point.

What does the locus of the perpendicular bisector outline? This is called an “envelope.” Click on the || button to stop. Under the Display menu, choose Erase Traces.

figure 14



- Select the two perpendiculars ( $\overleftrightarrow{CF}$  and  $\overleftrightarrow{EF}$ ),  $\overline{CD}$ , and point E, then under the Display menu choose Hide Objects. Choose point F, and then point C, and under Construct choose Locus.

Now you can see that as you move point C along line AB, the locus of the point will outline the parabola — the set of all points where  $FD = FC$ . Your screen should look like the figure 15.

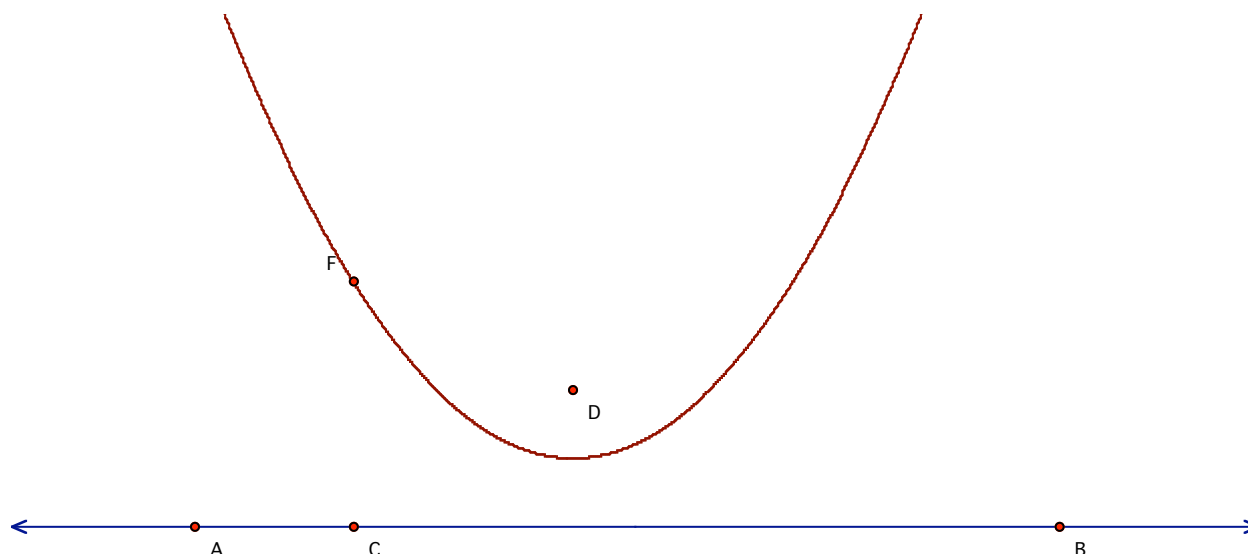


figure 15

8. You can change the location of the focus in relation to the directrix to determine the effect this distance has on the shape of the parabola. As you move the focus  $D$  away from the directrix  $\overleftrightarrow{AB}$ , how is the parabola changed?

As you move the focus  $D$  closer to the directrix  $\overleftrightarrow{AB}$ , how is the parabola changed?

### B. Graphing Parabolas using the function equation

A parabola is defined by the function  $f(x) = a(x - h)^2 + k$ , where the vertex is  $(h, k)$ . The value of  $a$  determines the shape and direction. (Note: the following directions can easily be modified to illustrate the equation  $f(x) = ax^2 + bx + c$ .)

1. Open a new sketch and choose Define Coordinate System from the Graph menu.
2. From the Graph menu choose New Parameter... Change the name to  $a$  and change the unit to cm.  
Repeat this step to create parameters  $h$  and  $k$ .
3. Select the three parameters, then under the Graph menu choose New Function... From the Values submenu and the keypad, build the equation  $f(x) = a(x - h)^2 + k$ .
4. Select the function and choose Plot Function. (See figure 16.)
5. Select parameter  $a$ . Click on the + key to increase the value of  $a$ . Click on the - key to decrease the value of  $a$ .

What happens to the parabola when parameter  $a$  is negative?

What happens to the parabola when parameter  $a$  is positive?

What happens to the parabola when parameter  $a$  is very large?

What happens to the parabola when parameter  $a$  is between 0 and 1?

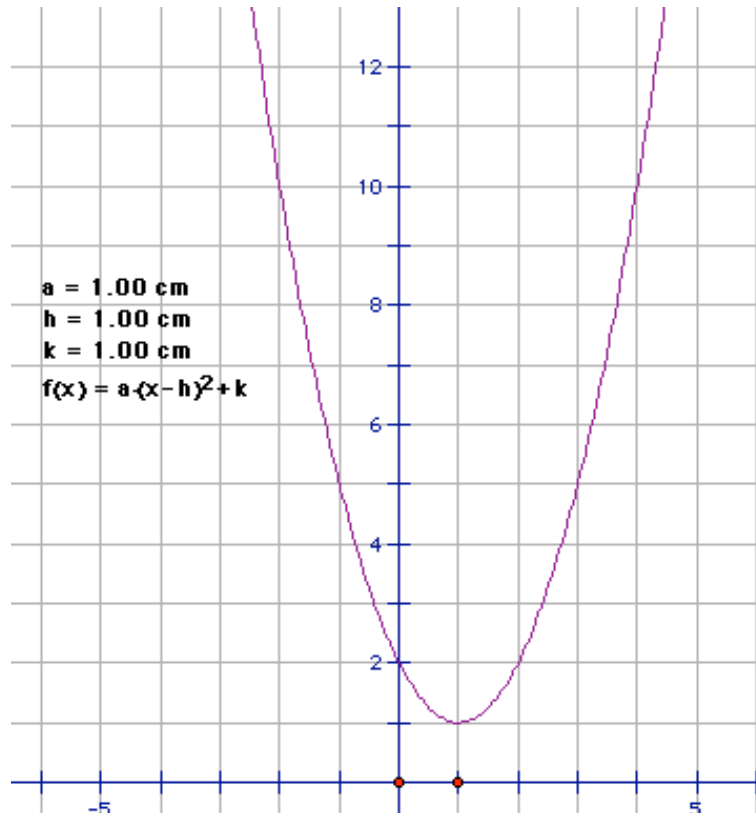
What happens to the parabola when parameter **a** is zero?

6. Select parameter **h**. Click on the + key to increase the value of **h**. Click on the – key to decrease the value of **h**.

What happens to the parabola when parameter **h** is negative?

What happens to the parabola when parameter **h** is positive?

What happens to the parabola when parameter **h** is zero?



**figure 16**

7. Select parameter **k**. Click on the + key to increase the value of **k**. Click on the – key to decrease the value of **k**.

What happens to the parabola when parameter **k** is negative?

What happens to the parabola when parameter **k** is positive?

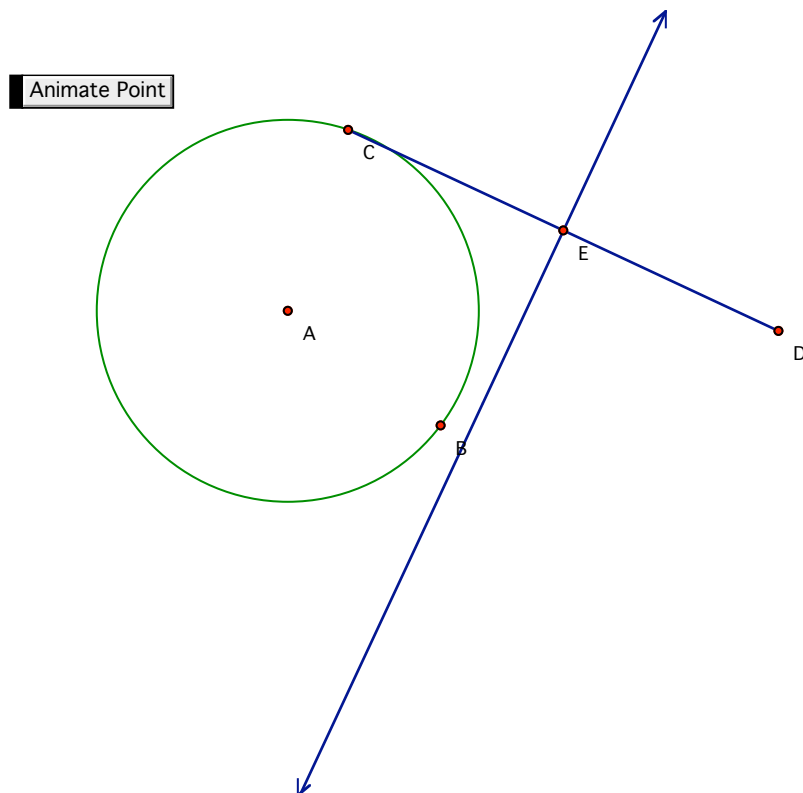
What happens to the parabola when parameter **k** is zero?

Repeat this construction for  $f(x) = ax^2 + bx + c$ . When parameter **a** is positive, as you change the value of parameter **b**, how does the vertex of the parabola change? When parameter **a** is negative, as you change the value of parameter **b**, how does the vertex of the parabola change?

**C. Ellipses and hyperbolas — emulating wax paper folding:**

This method was originally done with wax paper. Students drew a circle on the wax paper with the center indicated, then chose a point outside the circle. They then placed about 20 or more points on the circle. The point outside the circle was matched with a point on the circle and the wax paper was creased. By continuing in this fashion, the hyperbola envelope was formed. If the point were placed inside the circle and folding done as before, the ellipse envelope was formed.

1. Get a new sketch. Draw a circle in the middle of the screen. Select the circle and construct a point on the circle. (C)
2. Place a point outside the circle. (D)
3. Construct a segment joining the point on the circle and the point outside the circle. ( $\overline{CD}$  in my sketch.) With the segment selected, construct a point at the midpoint. (E)
4. Select the midpoint and the line segment CD, then construct a perpendicular.



**figure 17**

5. Select the perpendicular, then under Display choose Trace Perpendicular Line.
6. Since we want to reposition D and animate this several times, we will create an “Animation button”. Select the point C on the circle. Under Edit choose the Action Buttons submenu, then choose Animation. Click on Once Only and Fast, then OK. You will now have a button on the screen.

7. Click on the button to animate the sketch. What does the locus of the perpendicular bisector outline?
8. Move point D inside the circle. Click the animation button. Now what does the locus of the perpendicular bisector outline?
9. Move point D on top of the center of the circle. Animate again. Now what do you have?

## II. TRANSFORMATIONS

### 1. Glide Reflections — An iterated transformation

A glide reflection is a combination of a translation and a reflection in parallel lines. You can define your own compound transformations, which consist of a sequence of one or more basic transformations. Follow the steps outlined below to define a glide reflection.

1. In one corner, out of the way, draw a horizontal segment. (This is the vector, or delta-x and delta-y for our translation.) Click on the left endpoint of the segment, then the right endpoint. Under Transform choose Mark Vector.
2. Place a point on the left edge of the screen, halfway down. Select the point and the line segment and construct a parallel line. While the line is selected, under the Transform menu choose Mark Mirror.
3. Draw a polygon, similar to the one in Problem 4. Construct a polygon interior.

Now you have a figure, a line for reflection, and a translation defined by a vector. (You can also define a translation by a distance and an angle.) Remember for a glide reflection, the glide should be parallel to the line of reflection.

4. Perform the reflection followed by the translation (or vice versa).

#### Reflection:

- a. Select the polygon. If you have selected anything extra, click to deselect.
- b. Under Transform choose Reflect.

#### Translation

- c. The reflected polygon should now be selected.
  - d. Under Transform choose Translate.
  - e. You will get a dialog box. Choose Marked vector and click on Translate.
5. Select the middle polygon. (If the polygons overlap, drag the segment to make the vector longer.) Under the Display menu choose Hide Objects.
  6. Select the vertices of the initial image, and then from the Transform menu choose Iterate. From the dialog box match D to D'' by clicking on point D'', match E to E'' by clicking on the point E'', and so on for each vertex. Then click on Iterate.



You now have a sequence of shapes illustrating a glide reflection. (See figure 18.)

Under glide reflection, circle what remains unchanged: lengths    angle measure    orientation

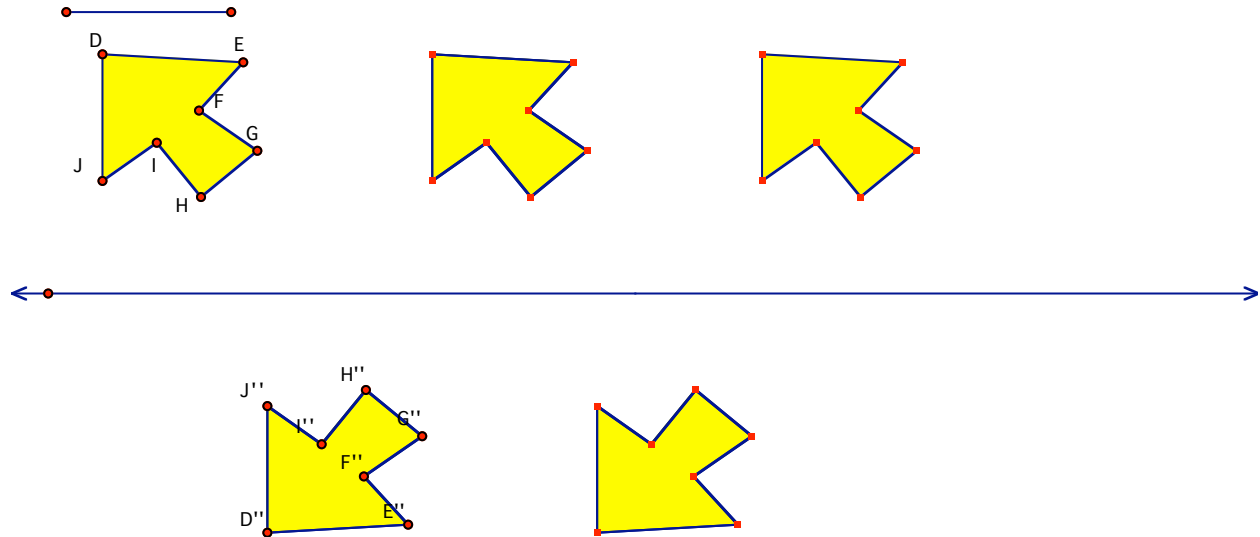


figure 18

### 2. Your own.

Create another two or three step transformation using some combination of reflections, translations, dilations, rotations, etc. Iterate and print out the result.

### III. Fractals — The Sierpinski Gasket

This is a fractal that is defined as follows: divide a triangle into fourths by connecting the midpoints of the three sides, then remove the middle triangle. This step is repeated on each of the three remaining triangles. Create a Sierpinski gasket with the following steps.

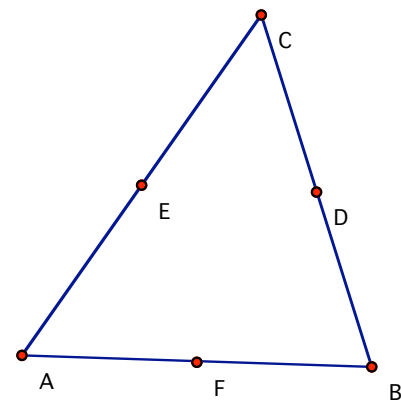
1. Get a new sketch.

Draw three points on the screen. Select all three points and construct a triangle. While the triangle sides are selected, from the Construct choose Point at Midpoint.

You should now have a triangle with three midpoints, D, E, F.

2. Select the three vertices, A, B, C.

From the Transform menu choose **Iterate**.



From the dialog box map  $A \rightarrow A$ ,  $B \rightarrow F$ ,  $C \rightarrow E$ .

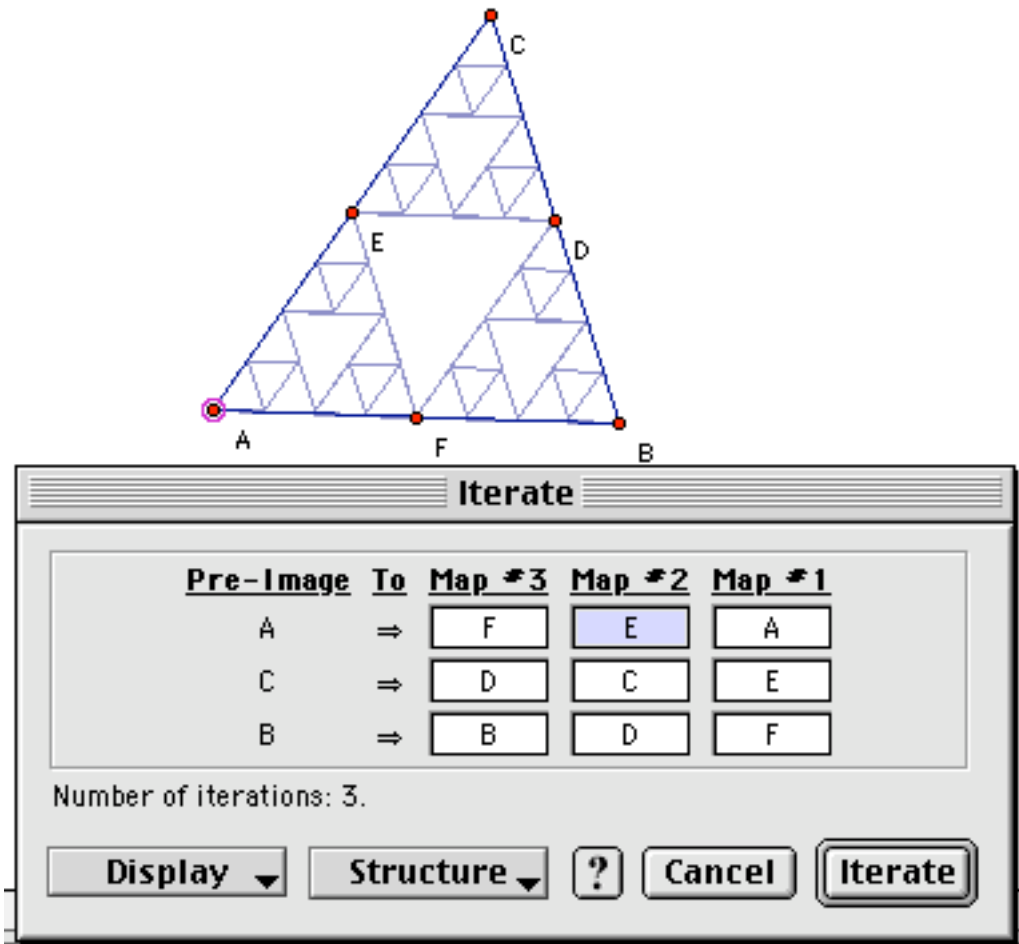
Then from the **Structure** submenu, choose **Add New Map**.

From the new set of mappings, map  $A \rightarrow E$ ,  $B \rightarrow D$ ,  $C \rightarrow C$ .

**figure 19**

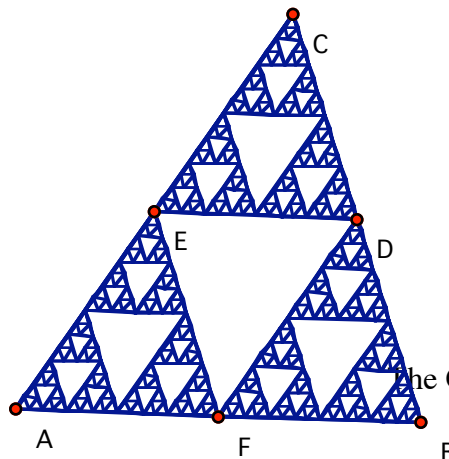
From the **Structure** submenu, choose **Add New Map**.

From the new set of mappings, map  $A \rightarrow F$ ,  $B \rightarrow B$ ,  $C \rightarrow D$ .



Then click on **Iterate**. (See figure 19.)

Select the fractal and press + to increase the number of iterations. Press - to reduce the



iterations. (See figure 20.)

## figure 20

### Koch Snowflake

1. Start with a new sketch. Construct a segment.  
Select point A and under the Transform menu, choose Mark Center.  
Choose point B and then under Transform choose Dilate. The ratio should be 1 to 3.  
Repeat this choosing a ratio 2 to 3.
2. Choose the  $1/3$  point and mark as a center. Rotate the  $2/3$  point by  $60^\circ$ .  
Hide segment AB. Connect the points as in figure 21a.

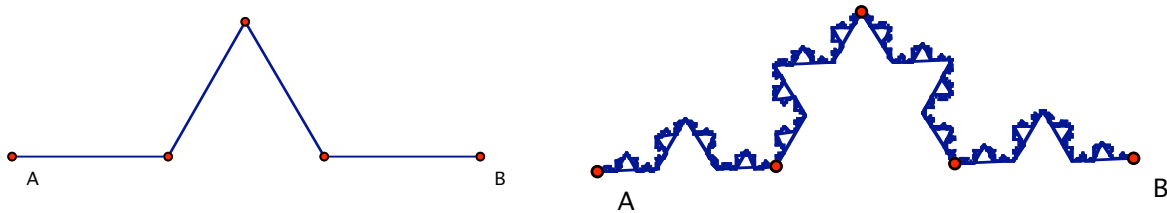


figure 21 a

b

3. Select points A and B. Under Transform, choose Iterate. Then select each of the four segments in turn.  
Increase and decrease the number of iterations using the + and - keys. (See figure 21b.)