Chapter 7
PROPERTIES OF MULTIPLICATION

7-1. Multiplication of Real Numbers

Now let us decide how we should multiply two real numbers to obtain another real number. All that we can say at present is that we know how to multiply two non-negative numbers.

Of primary importance here, as in the definition of addition, is that we maintain the "structure" of the number system. We know that if \( a, b, c \) are any numbers of arithmetic, then

\[
ab = ba,
\]

\[
(ab)c = a(bc),
\]

\[
a \cdot 1 = a,
\]

\[
a \cdot 0 = 0,
\]

\[
a(b + c) = ab + ac.
\]

(What names did we give to these properties of multiplication?) Whatever meaning we give to the product of two real numbers, we must be sure that it agrees with the products which we already have for non-negative real numbers and that the above properties of multiplication still hold for all real numbers.

Consider some possible products:

\[
(2)(3), (3)(0), (0)(0), (-3)(0), (3)(-2), (-2)(-3).
\]

(Do these include examples of every case of multiplication of positive and negative numbers and zero?) Notice that the first three products involve only non-negative numbers and are therefore already determined:

\[
(2)(3) = 6, \ (3)(0) = 0, \ (0)(0) = 0.
\]

Now let us try to see what the remaining three products will have to be in order to preserve the basic properties of multiplication listed above. In the first place, if we want the multiplication
property of 0 to hold for all real numbers, then we must have
(-3)(0) = 0. The other two products can be obtained as follows:

\[ 0 = (3)(0) \]
\[ 0 = (3)(2 + (-2)), \quad \text{by writing } 0 = 2 + (-2); \quad \text{(Notice how this introduces a negative number into the discussion.)} \]
\[ 0 = (3)(2) + (3)(-2), \quad \text{if the distributive property is to hold for real numbers;} \]
\[ 0 = 6 + (3)(-2), \quad \text{since } (3)(2) = 6. \]

We know from uniqueness of the additive inverse that the only real number which yields 0 when added to 6 is the number -6. Therefore, if the properties of numbers are expected to hold, the only possible value for (3)(-2) which we can accept is -6.

Next, we take a similar course to answer the second question.

\[ 0 = (-2)(0) \quad \text{if the multiplication property of } 0 \text{ is to hold for real numbers;} \]
\[ 0 = (-2)(3 + (-3)), \quad \text{by writing } 0 = 3 + (-3); \]
\[ 0 = (-2)(3) + (-2)(-3), \quad \text{if the distributive property is to hold for real numbers;} \]
\[ 0 = (3)(-2) + (-2)(-3), \quad \text{if the commutative property is to hold for real numbers;} \]
\[ 0 = (-6) + (-2)(-3), \quad \text{by the previous result, which was } (3)(-2) = -6. \]

Now we have to come to a point where (-2)(-3) must be the opposite of -6; hence, if we want the properties of multiplication to hold for real numbers, then (-2)(-3) must be 6.

Let us think of these examples now in terms of absolute value. Recall that the product of two positive numbers is a positive number. Then what are the values of |3||2| and |-2||-3|? How do these compare, respectively, with (3)(2) and (-2)(-3)? Compare (-3)(4) and -(|-3||4|); (-5)(-3) and |-5||-3|; (0)(-2) and |0||-2|.

This is the hint we needed. If we want the structure of the number system to be the same for real numbers as it was for the
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PART I

SCHOOL MATHEMATICS STUDY GROUP

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14. Last year a boy earned more than one hundred twenty dollars on his paper route. What were his average monthly earnings?

15. Mr. Smith is 4 times as old as his son. In 16 years he will be only twice as old. What are their ages now? (Hint: If the son is \( x \) years old now, how old will he be in another 16 years?)

16. In a particular triangle one angle is twice as large as another. The third angle is three times as large as the smaller of the two other angles. How many degrees are there in the measure of each angle? (Hint: How many degrees are there in the sum of the measures of the angles of a triangle?)

17. The sum of three consecutive whole numbers is 123. What are the three numbers?

18. Bob has $1.25 in nickels and dimes. He has three times as many nickels as dimes. Find how many of each he has.

---

2-3. Finding Solution Sets

We know that there are many different ways to express any number. For instance, \( 25 = \frac{50}{2} = \frac{75}{3} = 30 - 5 = 5^2 = (31 - 6) \). Of all these ways of expressing the number twenty-five, 25 is the simplest. Consider the following equations:

\[
\begin{align*}
  x + 3 &= 8, & x + 1 &= 6, & 2x &= 10, & x + (-2) &= 3, & 10 &= 2x, & 8 &= 3 + x.
\end{align*}
\]

Each of these equations has the solution \( x = 5 \); that is, if in each equation we replace \( x \) by 5 we have a true sentence and if we replace \( x \) by any number different from 5, the sentence is false. These equations are called \textit{equivalent} because all the solution sets are the same. In fact, the equation \( x = 5 \) could also be included in the list. Just as 25 is the simplest way to express the various numbers indicated at the beginning of this section, so \( x = 5 \) is the simplest equation equivalent to the list of equations given above.
Class Exercises 2-3a

1. Find six equations equivalent to the equation $x + 1 = 4$.
2. Find six equations equivalent to the equation $x = 3$.
3. Find six equations equivalent to the equation $2x = 12$.
4. What methods can you discover to get from a given equation one or more equations equivalent to it?
5. If one equation is equivalent to a second equation, is the second equivalent to the first? Why?
6. Are the following two equations equivalent: $x = x + 3$, $x = x - 1$?

Consider, for instance, the equation

$$x + 3 = 7.$$

If $x$ is replaced by a solution of this equation, $x + 3$ and 7 are two different names for the same number. Hence, for instance, $(x + 3) + 4$ and $7 + 4$ must be names for the same number, whenever $x$ is replaced by a solution of the given equation. Since it is probably easier to see this in terms of numbers first, consider this:

$$10 = (15 - 5).$$

Hence,

$$10 + 5 = (15 - 5) + 5.$$

We have added the same number to a given number, 10, expressed in two ways. These are examples of the

Addition Property of Equality. If two numbers, $a$ and $b$, are equal (that is, if $a$ and $b$ are two different names for the same number), then if you add the same number to each of them, the two sums will be equal.

That is,

if $a = b$, then $a + c = b + c$.

[sec. 2-3]
Graphically, we would have the following figures.

If $c > 0$

\[ \begin{array}{c}
0 \\
\downarrow \\
b \to b+c \\
\hline \hline \\
(a+c) \\
\downarrow \\
b+c \\
\end{array} \quad \begin{array}{c}
0 \\
\downarrow \\
a+c \to a+b \\
\hline \hline \\
b+c \\
\downarrow \\
c \to b \\
\end{array} \]

To see how this applies to the equation: $x + 3 = 7$, notice that if $x$ denotes a solution of this equation, $x + 3$ and 7 are two names for the same number. Hence in the additive property,

$x + 3$ plays the role of the letter $a$,

7 plays the role of the letter $b$,

if $x$ is a solution of the equation. Hence, if $x$ is a solution of

\[ x + 3 = 7 \]

it is also a solution of

\[ (x + 3) + c = 7 + c, \]

no matter what number $c$ is. A brief way of describing this is to say that we get the second equation from the first by "adding the same number, $c$, to both sides of the equation."

The most useful choice of $c$ in the above is $-3$, for

\[ (x + 3) + (-3) = 7 + (-3). \]

Then, by the associative property for addition, this is equivalent to

\[ x + (3 + (-3)) = 7 + (-3). \]

Hence

\[ x + 0 = 4, \]

\[ x = 4. \]

Thus, by the addition property of equality, if any number is a solution of $x + 3 = 7$, it is a solution of $x = 4$. Conversely, if any number is a solution of $x = 4$, we may add 3 to both sides of the equation to get

\[ x + 3 = 7. \]
and see that if any number is a solution of \( x = 4 \), it is also a solution of \( x + 3 = 7 \). Hence the equations:

\[
x = 4 \quad \text{and} \quad x + 3 = 7
\]

are equivalent equations; that is, their solution sets are the same.

By this means we can show that:

\[
a = b \quad \text{and} \quad a + c = b + c
\]

are equivalent equations; that is, if \( a = b \), then

\[
a + c = b + c,
\]

and if \( a + c = b + c \), then \( a = b \).

Using this result it also follows that \( x = 3 \) and \( x + 7 = 10 \) are equivalent equations since we got the second equation from the first by adding 7 to both sides of the equation \( x = 3 \).

**Class Exercises 2-3b**

1. Find the solution set of each of the following equations and check your solutions:

   (a) \( x + 4 = 10 \)  
   (b) \( x + 2 = 5 \)  
   (c) \( 3 = x + 4 \)  
   (d) \( -2 = x + 3 \)  

2. Use the distributive property to simplify each of the following:

   (a) \( 2x + 3x = ? \)  
   (b) \( x + 5x = ? \)  
   (c) \( -2x + 3x = ? \)  
   (d) \( x + 7x = ? \)  
   (e) \( -x + (-2)x = ? \)  
   (f) \( (-2)x + (-5)x = ? \)

Suppose instead of adding a known number to both sides of the equation \( x + 7 = 10 \) we add an unknown, \( 2x \). Then we would have

\[
2x + (x + 7) = 2x + 10.
\]

We can write the left side, using the associative property, as

\[
(2x + x) + 7.
\]

[sec. 2-3]
Now, \( x = 1 \), so \( x \) and hence \( 2x + x = 2x + 1 \cdot x = (2 + 1)x = 3x \), using the distributive property. Thus, if we add \( 2x \) to both sides of the equation \( x + 7 = 10 \), we have

\[
3x + 7 = 2x + 10.
\]

This is all very well, but suppose we were given the last equation. How could we get back to the first equation? Since we got the last equation by adding \( 2x \) to both sides of the first, we should be able to get the first equation by adding \(-2x\) to both sides of the last. Let us see if this is so. Then

\[
-2x + 3x + 7 = -2x + 2x + 10.
\]

Now \(-2x + 2x = 0\) since \(-2x\) is the additive inverse of \(2x\). But what is \(-2x + 3x\) equal to? We find it this way:

\[
-2x + 3x = -2x + 3x = -2 + 3x = 1 \cdot x = x,
\]

using the distributive property. Thus, by adding \(-2x\) to both sides of the equation \(3x + 7 = 2x + 10\) we get \(x + 7 = 10\) as an equivalent equation. Since the solution of this equation is \(3\), the solution of \(3x + 7 = 2x + 10\) is also \(3\). You should check this to show that we have made no mistake.

Now we can find the solution of an equation like

\[
4x + 5 = 3x + 2.
\]

We wish to find an equivalent equation in which \(x\) occurs on one side only. To do this we can add \(-3x\) to both sides to get

\[
-(3x) + 4x + 5 = -(3x) + 3x + 2.
\]

This results in the sentence

\[
x + 5 = 2.
\]

(You should fill in the steps needed to show this.) Then, if we add \(-5\) to both sides of this equation (since we want to have \(x\) by itself on one side) we have

\[
(x + 5) + -5 = -5 + 2
\]

\[
x + (5 + -5) = -5 + 2
\]

\[
x + 0 = -3
\]

\[
x = -3
\]

[sec. 2-3]
Thus \( x = -3 \) is equivalent to the equation \( 4x + 5 = 3x + 2 \), which shows that \( -3 \) is its solution.

The solution of the equation \( x = -3 \) is obvious and, since it is equivalent to the equation \( 4x + 5 = 3x + 2 \), this equation also has the solution \( -3 \). In fact, a method of solving an equation is to find an equivalent equation which has an obvious solution—that is, of the form \( x = \) some number. Let us go back over the process we used. We first added \( -(3x) \) to both sides of the equation in order to get an equivalent equation in which \( x \) occurred on only one side: \( x + 5 = 2 \). Then, since we wanted an equation of the form \( x = \) some number, we added \( -5 \) to both sides.

To make sure that we have made no mistake, let us check to see that \( -3 \) is really a solution of the equation:

\[
4x + 5 = 3x + 2.
\]

If \( x = -3 \), the left side of the equation becomes

\[
(\ -3\ ) \cdot 4 + 5 = 12 + 5 = -7.
\]

If \( x = -3 \), the right side of the equation becomes

\[
3(\ -3\ ) + 2 = -9 + 2 = -7.
\]

Hence for this value of \( x \), the number on the left side is equal to that on the right. This is our check.

Of course there are equations which have no solutions. One such equation is \( x + 3 = x \). This may be considered to be obvious since no number can be \( 3 \) greater than itself. But let us find what this equation is equivalent to. We may add \( -x \) to both sides and have

\[
-x + (x + 3) = -x + x
\]

or

\[
(-x + x) + 3 = -x + x
\]

\[
0 + 3 = 0
\]

\[
3 = 0.
\]

(Remember that just as \( -3 \) is the number with the property that \( -3 + 3 = 0 \), so \( -x \) is the number with the property that \( -x + x = 0 \).)

So the given equation is equivalent to \( 3 = 0 \). This has no
solution and hence the given equation has no solution.

Other equations have many solutions. Consider \(2x = x + x\).
This is true for all values of \(x\). You might like to show that
this is equivalent to \(0 = 0\).

Exercises 2-3a

1. Using the methods of the previous section, find four equations
equivalent to each of the following equations:

(a) \(x + 7 = 13\)
(b) \(17 = x + -3\)
(c) \(x = 7\)

2. Use the addition property to solve the following equations.
Check your results.

Example: \(x + (-3) = 11\). First use the addition property and
add 3 to both sides of the equation. This gives

\[(x + -3) + 3 = 11 + 3.\]

By the associative property of addition this is equivalent to

\[x + (-3 + 3) = 14,\]
\[x + 0 = 14\]
\[x = 14.\]

To check this see that if \(x = 14\), \(x + -3\) is \(14 + -3\) which
is equal to 11.

(a) \(x + 5 = 6\) \hspace{1cm} (g) \(-2 = -4 + x\)
(b) \(x + 6 = 5\) \hspace{1cm} (h) \(x + \frac{3}{2} = 10\)
(c) \(x + -7 = 7\) \hspace{1cm} (i) \(y + -\left(\frac{3}{2}\right) = \frac{5}{2}\)
(d) \(x + -7 = -7\) \hspace{1cm} (j) \(u + 14 = \frac{9}{2}\)
(e) \(t + 6 = -13\) \hspace{1cm} (k) \(\frac{13}{7} = 1 + x\)
(f) \(4 = x + 3\) \hspace{1cm} (l) \(x + -\left(\frac{4}{9}\right) = -\left(\frac{7}{13}\right)\)

[sec. 2-3]
3. Apply the addition property to these equations, adding the indicated number, and write the resulting equation.

Example: \(3x + 4 = 5\) (add \(-4\))

\[(3x + 4) + (4 - 4) = 5 + (4 - 4)\]

by the addition property.

\[3x + (4 + (4 - 4)) = 1\]

by the associative property.

The resulting equation is: \(3x = 1\).

(a) \(2x + 5 = 10\) (add \(-5\))
(b) \(3x + 10 = 5\) (add \(-10\))
(c) \(5x + 2 = -2\) (add \(-2\))
(d) \(10x + -1 = 9\) (add 1)
(e) \(2u + 1 = 11\) (add \(-1\))
(f) \(2x + -3 = 9\) (add 3)
(g) \(-4y + -3 = \frac{9}{2}\) (add 3)

4. (a) What number do you add (using the addition property) to solve \(x + 3 = 2\)?
(b) What number do you add (using the addition property) to solve \(x + (\text{7}) = 4\)?
(c) What is the relation between 3 and \(-3\) relative to addition?
(d) What is the relation between 7 and \(-7\) relative to addition?

5. (a) If \(x = 3\), what is \(-x\)?
(b) If \(x = \frac{1}{2}\), what is \(-x\)?
(c) If \(x = -3\), what is \(-x\)? Answer: By \(-x\), we mean that number with the property that \(x + (-x) = 0\). So for \(x = -3\), we have

\[-3 + (-x) = 0.\]

But we know that \(-3 + 3 = 0\). Hence \(-x\) must be equal to 3. In other words, \((-(-3)) = 3.\n
[sec. 2-3]
(d) If \( x = -\left(\frac{1}{2}\right) \), what is \( x \)?
(e) If \( x = -7 \), what is \( x \)?

6. Earlier in this section we solved the equation

\[ 4x + 5 = 3x + 2 \]

by adding \(-(3x)\) to both sides of the equation. Suppose we had begun by adding \(-(4x)\) to both sides. Could the solution be found this way? Do you think this is a simpler method of solution?

7. Simplify each of the following:
(a) \( \bar{x} + 3x = ? \)
(b) \( 3x + \bar{x} = ? \)
(c) \( -(2x) + 3x = ? \)

8. Solve the equation: \( 2x + 7 = x \).
9. Solve the equation: \( 2x + 3 = x + 2 \).
10. Solve the equation: \( x = 2x + 6 \).
11. Solve the equation: \( 3x + 5 = 2x \).

Adding the same number to both sides of an equation is not the only way to get an equivalent one. We may also multiply both sides by the same number. For example:

\[ 5 + 2 = 7, \]

is a true sentence. If we multiply both sides by 3, we get

\[ 3(5 + 2) = 3 \cdot 7. \]

This is also true. This is an example of the multiplication property of equality.

The Multiplication Property of Equality: If \( a \) and \( b \) are two equal numbers, then \( ca = cb \).

Is it true that \( ca = cb \) implies that \( a = b \)? Before
answering this question, let us consider the solution of an equation using the multiplication property. Since our method of solution is very much like that for an equation involving addition, in the left-hand column below we shall solve an equation using the addition property and in the right-hand column a similar equation using the multiplication property.

<table>
<thead>
<tr>
<th>Problem: Solve (3 + x = 6)</th>
<th>Problem: Solve (3x = 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First add the additive inverse of 3, to both sides of the equation to get</td>
<td>First multiply both sides of the equation by (\frac{1}{3}), the multiplicative inverse of 3, to get</td>
</tr>
<tr>
<td>(-3 + (3 + x) = -3 + 6).</td>
<td>(\frac{1}{3}(3x) = \frac{1}{3}(6)).</td>
</tr>
<tr>
<td>Use the associative property of addition:</td>
<td>Use the associative property of multiplication:</td>
</tr>
<tr>
<td>((-3 + 3) + x = -3 + 6)</td>
<td>((\frac{1}{3} \cdot 3)x = \frac{1}{3} \cdot 6).</td>
</tr>
<tr>
<td>(0 + x = 3)</td>
<td>(1 \cdot x = 2)</td>
</tr>
<tr>
<td>(x = 3)</td>
<td>(x = 2)</td>
</tr>
</tbody>
</table>

We can use this same parallel treatment to show that

if \(ca = cb\), and \(c \neq 0\), then \(a = b\).

Problem: Prove that, if \(c + a = c + b\), then \(a = b\).

First add \(-c\), the additive inverse of \(c\), to both sides of the equality to get

\[\begin{align*}
-c + (c + a) &= -c + (c + b) \\
0 + a &= 0 + b \\
a &= b
\end{align*}\]

Using the associative property of addition, we have

\(\frac{1}{c} \cdot (ca) = \frac{1}{c} \cdot (cb)\)

Using the associative property of multiplication, we have

\(\frac{1}{c} \cdot c) = \frac{1}{c} \cdot c\)

\(1 \cdot a = 1 \cdot b\)

\(a = b\)

[sec. 2-3]
Thus we have shown that if \( c \neq 0 \), \( ca = cb \) and \( a = b \) are equivalent equations. This means that if \( c \neq 0 \) and \( ca = cb \), then \( a = b \); also if \( c \neq 0 \) and \( a = b \), then \( ca = cb \).

Now let us apply this result to the solution of the equation,

\[ 3x + 1 = 13. \]

We wish to have \( 3x \) by itself on one side of the equation and, as in the first part of this section, we can accomplish this by adding \(-1\) to both sides. This gives us

\[ 3x + 1 + (-1) = 13 + (-1), \]
\[ 3x + 0 = 12, \]
\[ 3x = 12. \]

Since we wish to have an equation of the form \( x = \) some number we can accomplish this by multiplying both sides by \( \frac{1}{3} \), the reciprocal (that is, the multiplicative inverse) of 3. Thus the equation

\[ 3x = 12 \]

is equivalent to

\[ \frac{1}{3} \cdot (3x) = \frac{1}{3} \cdot (12), \]

or

\[ (\frac{1}{3} \cdot 3) \cdot x = 4, \]

or

\[ x = 4. \]

We have shown that the equation \( 3x + 1 = 13 \) is equivalent to \( x = 4 \). Since \( x = 4 \) has the obvious solution \( 4 \), so is \( 4 \) the solution of \( 3x + 1 = 13 \). From what we have done, we can be sure that if we did not make a mistake \( x = 4 \) is the solution of the equation \( 3x + 1 = 13 \). But it is reassuring and is also good policy for us to check this answer to see if it is indeed a solution. If we replace \( x \) by \( 4 \) in \( 3x + 1 \) we get \( 3 \cdot 4 + 1 \) which is equal to 13. This is the check we wanted.

Class Exercises 2-3c

1. Indicate which property, the addition or the multiplication property of equality, and which number is to be added or used as a multiplier in solving the following equations.
(a) \[ x + 10 = 22 \]  
(b) \[ 6.2 + x = 1.12 \]  
(c) \[ -2 + x = -10 \]  
(d) \[ 5x = 15 \]  
(e) \[ 6 = \frac{x}{15} \]  
(f) \[ 14 - x = 0 \]  
(g) \[ \frac{1}{2}x = 17 \]  
(h) \[ 18 + y = 8.6 \]  
(i) \[ y + 6 = 5 + 3 \]  
(j) \[ 0.08d = 73 \]  
(k) \[ 19 = 6 - y \]  
(l) \[ \frac{2}{3}n = 15 + 0.4 \]  
(m) \[ 45 - b = 1 \]  
(n) \[ \frac{7}{c} = 1, c \neq 0 \]

2. Find the reciprocals of each of the following numbers:
(a) \( 7 \). Answer: \( \frac{1}{7} \) is the reciprocal since \( 7 \left( \frac{1}{7} \right) = 1 \).
(b) \( 5 \)
(c) \( -3 \)
(d) \( \frac{1}{2} \). Answer: the reciprocal of a number \( b \) is defined to be that number \( \frac{1}{b} \) such that \( b \left( \frac{1}{b} \right) = 1 \). Since \( \frac{1}{2} \left( \frac{2}{1} \right) = 1 \), \( 2 \) is the reciprocal of \( \frac{1}{2} \).
(e) \( \frac{1}{4} \)
(f) \( \frac{2}{3} \)
(g) \( -\left( \frac{1}{2} \right) \)

3. Find the additive inverse and the multiplicative inverse of each of the following, being careful to state which is which:
(a) \( 3 \)  
(b) \( \frac{1}{2} \)  
(c) \( \frac{5}{7} \)

Exercises 2-3b

1. Find the additive inverse and the multiplicative inverse of each of the following, being careful to state which is which:
(a) \( 7 \)  
(b) \( \frac{3}{4} \)  
(c) \( -2 \)  
(d) \( \frac{1}{2} \)

[sec. 2-3]
2. Solve each of the following equations and state where you use the addition property and where the multiplication property of equality.

(a) $3x + 2 = 14$
(b) $7x = 2$
(c) $-3x + 7 = 22$
(d) $\frac{1}{2}x = 7$
(e) $-(\frac{1}{2})x = 14$.

Let us try our methods on a more complicated equation:

$$-(\frac{1}{2})x + 2 = 2x + \frac{1}{2}.$$  

We wish first to find an equivalent equation in which only one side has a term in $x$. Here we use the addition property and add $\frac{1}{2}x$ to both sides:

$$\frac{1}{2}x + \left(-\left(\frac{1}{2}\right)x + 2\right) = \frac{1}{2}x + (2x + \frac{1}{2}).$$

Using the associative property for addition we have:

(A) $$\left(\frac{1}{2}x + -\left(\frac{1}{2}\right)x\right) + 2 = \left(\frac{1}{2}x + 2x\right) + \frac{1}{2}.$$  

Now $\frac{1}{2}x + -\left(\frac{1}{2}\right)x = 0$ since $-\left(\frac{1}{2}\right)x$ is the additive inverse of $\frac{1}{2}x$. Also by the distributive property,

$$\frac{1}{2}x + 2x = \left(\frac{1}{2} + 2\right)x.$$  

Hence the equation (A) above is equivalent to

$$0 + 2 = \left(\frac{1}{2} + 2\right)x + \frac{1}{2};$$

that is,

$$2 = \frac{5}{2}x + \frac{1}{2}.$$  

Since we wish the term in $x$ to be by itself on one side of the equation, we can again use the addition property and add $-\left(\frac{1}{2}\right)$
to both sides of the equation to get

$$\frac{3}{2} = \frac{5}{2}x.$$

Since we want an equation of the form, a number = x, we can use the multiplication property and multiply both sides by \(\frac{\frac{2}{5}}{\frac{5}{2}}\), the reciprocal of \(\frac{5}{2}\), and get

$$\frac{2}{5} \cdot \frac{3}{2} = \frac{2}{5} \cdot \frac{5}{2}x.$$

We have, finally (you will need to write in some steps),

$$\frac{3}{5} = x.$$

Thus \(\frac{3}{5}\) is the solution of the equation \(-\left(\frac{1}{2}\right)x + 2 = 2x + \frac{1}{2}\).

Now getting this result was rather long and there were many opportunities for mistakes. We should check our result.

If \(x = \frac{3}{5}\), the left side of the given equation, \(-\left(\frac{1}{2}\right)x + 2\), becomes \(-\left(\frac{1}{2}\right) \cdot \frac{3}{5} + 2 = -\frac{3}{10} + 2 = -\frac{3}{10} + 2 = \frac{17}{10}\).

If \(x = \frac{3}{5}\), the right side of the given equation, \(2x + \frac{1}{2}\), becomes \(2 \cdot \frac{3}{5} + \frac{1}{2} = \frac{6}{5} + \frac{1}{2} = \frac{12}{10} + \frac{5}{10} = \frac{17}{10}\).

This shows that for \(x = \frac{3}{5}\), \(-\left(\frac{1}{2}\right)x + 2\) is equal to \(2x + \frac{1}{2}\).

There are other methods of solving this equation. One would be to begin by multiplying both sides of the equation by 2, to get rid of the fractions. You are asked to try this out in an exercise below.

**Class Exercises 2-3d**

1. What property is used, and how is it used, to get the second equation from the first?

Example: (1) \(2x + 4 = 7\)

(2) \(2x = 3\) addition property, adding \(-4\).
2. Solve and check each of the following equations:

(a) \( \frac{1}{2}x + 1 = 1 \)  
(b) \( 1.6 = 4y \)  
(c) \( \frac{2(m + 5)}{3} = 6 \)  
(d) \( -x = 5 \)  
(e) \( 2(y + 4) = 8 \)  
(f) \( \frac{2}{5}x = 10 \)  
(g) \( 1 \)  
(h) \( 0.14 + x = 5.28 \)  

2. Solve and check each of the following equations:

(a) \( 3y + 2 = 7 \)  
(b) \( 7 = 3x + 1 \)  
(c) \( 6 = 3w \)  
(d) \( \frac{1}{2}t - 1.7 = -1.3 \)  
(e) \( 2 = \frac{x}{18} \)  
(f) \( 0.14 + x = 5.28 \)  
(g) \( 5x - 7 = 2x \)  
(h) \( x = 7 - 2x \)  

Exercises 2-3c

1. Solve the following equations by using the properties of "equals." Give your reason for each step.

(a) \( 2x + 1 = 7 \)  
(b) \( y - 2 = 6 \)  
(c) \( \frac{t}{2} - 3 = -4 \)  
(d) \( 3x - 5 = -4 \)  

2. Solve the following equations.

(a) \( x + 3 = 5 \)  
(b) \( 3 + y = -5 \)  
(c) \( 2v + 3 = 5 \)  
(d) \( 3 + 2m = -5 \)  
(e) \( y - 3 = 5 \)  
(f) \( 3 - u = -5 \)  
(g) \( 2w - 3 = 5 \)  
(h) \( 3 - 2s = -5 \)  

[sec. 2-3]
3. (a) In solving the equation $9x = 27$ what number would you use as a multiplier?

(b) In solving the equation $\frac{1}{3}x = 4$ what number would you use as a multiplier?

(c) In solving the equation $\frac{4}{5}x = \frac{1}{2}$ what number would you use as a multiplier?

(d) What is the relation between 9 and $\frac{1}{9}$, relative to multiplication?

(e) What is the relation between 3 and $\frac{1}{3}$, relative to multiplication?

(f) What is the relation between $\frac{4}{5}$ and $\frac{5}{4}$, relative to multiplication?

4. In solving an equation such as $3x + 1 = 9$, you have learned to use the addition property first (to find $3x$) and the multiplication property second (to find $x$). Sometimes you will find it best to reverse the order in which you use these properties. Solve the following equations by using the multiplication property first.

(a) $4(x + 1) = 12$

(b) $7(x - 2) = 13$

(c) $\frac{x + 0}{3} = 5$

(d) $0.6(x - 0.3) = 0.2$

(e) $\frac{3x + \frac{4}{2}}{2} = 7$

(f) $\frac{4x + 1}{0.12} = 3$

5. To solve the equation $\frac{1}{2}(-x) + 2 = \frac{1}{2} + 2x$, begin by multiplying both sides by 2 and get the equivalent equation

$-x + 4 = 1 + 4x$.

Then find the solution of this equation. Does it agree with that in the previous discussion? Do you think this method is easier than the one used?

[sec. 2-3]